

Let's return to the fundamental question of matching assets and liabilities. Consider a fund with an asset cash flow  $\{A_t\}$  and an equivalent liability cash flow  $\{L_t\}$ .

Let the present values of the 2 flows be  $V_A(i)$  and  $V_L(i)$  respectively and similarly the volatilities  $v_A(i)$  and  $v_L(i)$ ; and the convexities  $C_A(i)$  and  $C_L(i)$

We assume that at interest rate  $i$  the fund is exactly balanced so that  $V_A(i) = V_L(i)$ .

If the interest rate moves to  $i + \epsilon$ , consider the surplus  $S = V_A - V_L$

$$S(i + \epsilon) = S(i) + \epsilon S'(i) + \frac{\epsilon^2}{2} S''(i) + \dots$$

For the fund to be "immunised" we require that  $S(i + \epsilon)$  is always positive whether  $\epsilon$  is positive or negative.

Therefore, to achieve this we require that  $S'(i) = 0$  i.e.  $v_A(i) = v_L(i)$

Effective duration is the same.

and also that  $S''(i) > 0$  i.e. convexity of assets is greater than convexity of liabilities.

#### *Practical Problems*

Requires constant rebalancing (not to mention dealing expenses)

There may well be options and other uncertainties (e.g. mortality) in both the assets and the liabilities.

Relevant assets may not exist (e.g. not long enough terms)

*Only works for small interest rate changes.*