Actuarial Science Hilary Term Lecture 3

Asset/Liability Matching

Essence of an insurance company is that it is investing money (assets) to provide income to cover expected outgo (liabilities). Safest position is where the amount and timing of each item of income exactly matches each item of outgo. For example an annuity certain for 10 years in arrears could be exactly matched by 10 zero coupon bonds of terms 1,2.... 10. Annuity portfolios try to match as closely as possible but there are uncertainties about mortality and also the latest expected date of payment may well be after the longest dated loan stock. Currently the longest dated UK Government stock is Treasury 4.25% with a redemption date of 2055.

There are several undated stocks such as War Loan 3.5% but these could be repaid at any date and so are not suitable for exact matching.

First two weeks we considered other types of investment – equities, property, collective investment schemes and derivatives – all with some uncertain qualities. For this week we will return to fixed interest stock – and also assume that the rate of interest is constant across all terms. That is not normally the case as we shall see next week but it is useful for this introductory look at matching.

Effective Duration

Consider a series of cash flows $\{C_{t_k}\}$ for k = 1 to n.

Let A(i) be the present value of the series at rate i.

$$A(i) = \sum_{k=1}^{n} v^{t_k} C_{t_k}$$

Effective duration (or volatility) is defined as

$$\nu = -\frac{1}{A}\frac{dA}{di} = -\frac{A'}{A}$$

$$= \frac{1}{\sum_{1}^{n} v^{t_k} C_{t_k}} \left\{ \sum_{1}^{n} C_{t_k} \times t_k \times v^{t_k+1} \right\}$$

It is a measure of the rate of change in value for small changes in interest rate.

Macauley Duration

This is the mean term of the cash flows C, weighted by their present value.

$$\bar{t} = \frac{\sum_{k=1}^{n} t_k C_{t_k} v^{t_k}}{\sum_{1}^{n} C_{t_k} v^{t_k}}$$