Problem Sheet 6 - Part B Actuarial Science I - Oxford MT 2007

For some of the exercises on this sheet, you will need values of the standard normal distribution function. If $Z \sim N(0,1)$, then:

z	-3	-2.3263	-2.4778	-1.8349	-0.039	0.233
$P(Z \leq z)$	0.00135	0.01000	0.00661	0.03326	0.48445	0.59211

1. An insurer issues n identical policies. Let Y_j be the claim amount from the jth policy, and suppose that the random variables Y_j, $j = 1, \ldots, n$ are i.i.d. with mean $\mu > 0$ and variance σ^2. The insurer charges a premium of A for each policy.

   (a) Show that if $A = \mu + 10\sigma n^{-1/2}$, then the probability that total claims exceed total premiums is no more than 1%, for any value of n.

   (b) Use the Central Limit Theorem to show that if instead $A = \mu + 3\sigma n^{-1/2}$, then this probability is still less than 1%, provided n is large enough.

2. Suppose that the interest rate for the next six months is known to be 5.5% (effective rate per annum), while the rate for the six months after that is unknown and assumed to be uniformly distributed on the interval $(4\%, 6\%)$. Under this assumption, find the expectations of:

   (a) the accumulated value after one year of £100 invested now;

   (b) the discounted present value of a payment of £100 in a year's time.

3. Let I_j denote the effective rate of interest in the year $j-1$ to j. Suppose that, for $j \geq 1$

$$I_{j+1} = \begin{cases} I_j + 0.02 & & 0.25 \\ I_j & \text{with probability} & 0.5 \\ I_j - 0.02 & & 0.25 \end{cases}$$

   Given that $I_1 \equiv 0.06$, calculate the probability that an investment of 1 at time 0 accumulates to more than 1.2 at time 3.

4. Let $1 + I$ be a lognormal random variable with parameters μ and σ^2, mean $1 + j$ and variance s^2. Show that

$$\sigma^2 = \log\left(1 + \left(\frac{s}{1+j}\right)^2\right) \qquad \text{and} \qquad \mu = \log\left(\frac{1+j}{\sqrt{1 + \left(\frac{s}{1+j}\right)^2}}\right)$$

5. The rate of return on an investment in a given year is denoted by Y. Suppose $1+Y$ is lognormally distributed. The expected value of the rate of return is 5% and its standard deviation is 11%.

   (a) Calculate the parameters of the lognormal distribution of $1 + Y$.

(b) Calculate the probability that the rate of return for the year lies between 4% and 7%.

6. A company is adopting a particular investment strategy such that the expected annual effective rate of return from investments is 7% and the standard deviation of annual returns is 9%. Annual returns are independent and $(1+I_j)$ is lognormally distributed, where I_j is the return in the jth year.

   (a) Calculate the expected value and standard deviation of an investment of 1 unit over 10 years, deriving all formulae that you use.

   (b) Calculate the probability that the accumulation of such an investment will be less than 50% of its expected value in ten years' time.

   (c) The company has an outstanding debt and must make a payment of £140,000 in 10 years time. Calculate the probability that an investment of £120,000 now will provide sufficient funds to meet this liability.

7. Suppose the force of interest Δ_j during the year from $j-1$ to j is given by

$$\Delta_j = \mu + \frac{1}{\sqrt{2}}\,\epsilon_{j-1} + \frac{1}{\sqrt{2}}\,\epsilon_j,$$

   where ϵ_0, ϵ_1, $\epsilon_2, \ldots$ are i.i.d. random variables with distribution $N(0, \sigma^2)$.

   (a) Show that $\Delta_j \sim N(\mu, \sigma^2)$ for all j.

   (b) Write an expression for $\Delta_1 + \Delta_2 + \cdots + \Delta_n$ in terms of the random variables ϵ_j. Hence show that the accumulated value at time n of 1 unit invested at time 0 has a lognormal distribution, and find its parameters.

Course webpage: `http://www.stats.ox.ac.uk/~martin/BS4a.html`