Problem Sheet 5 - Part B Actuarial Science I - Oxford MT 2007

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1. An investor purchases a bond 3 months after issue. The bond will be redeemed at par ten years after issue and pays coupons of 6% per annum annually in arrears. The investor pays tax of 25% on both income and capital gains (with no relief for indexation). Calculate the purchase price of the bond per £100 nominal to provide the investor with a rate of return (after tax) of 8% per annum effective.

2. A zero-coupon bond was purchased m years ago by an investor who is liable to capital gains tax at rate t. At the time of purchase the outstanding term of the bond was n years ($n > m$). The price paid by the investor will provide him with an annual yield of $i > 0$ (after tax) if he holds the bond until it is redeemed.

   The investor now wishes to sell the bond. He will be liable to capital gains tax on the excess of his selling price over his purchase price.

   (a) Derive an expression in terms of t, n, and i for the purchase price (per unit redemption money).

   (b) Derive also an expression in terms of t, n, m, and i for the price (per unit redemption money) at which he should now sell the bond in order to obtain a net annual yield of i on the completed transaction.

   (c) Assume that in fact the bond is sold to a second investor, who is also liable to capital gains tax at rate t, at a price which will provide the *new* purchaser with a net annual yield of i, if he holds the bond until it is redeemed.

   Derive an equation from which can be found the value of j, the net annual yield obtained by the first investor on the completed transaction.

   Find the value of j when $n = 10$, $m = 5$, $t = 40\%$, and $i = 10\%$.

3. A bond pays coupons twice yearly in arrears at nominal annual rate $j^{(2)} = 5\%$, and will be redeemed at par after 5 years.

   An investor will be liable to capital gains tax at 40% on the difference between the redemption price and the purchase price, adjusted for inflation over the 5-year period.

   Under the assumption of a constant inflation rate of 2% p.a., find the purchase price which provides the investor with a yield (after capital gains tax) of (i) 6% (ii) 8%. What is the corresponding *real* yield in each case?

4. Mrs Chang wants to borrow £4000. She will repay the loan with one single payment at the end of one year's time. The lending agency estimates that there is a 5% chance of default on this loan, in which case they receive nothing at all. How much is the "fair amount" for Mrs Chang to repay if the "risk-free" rate of interest is $i = 10\%$?

5. Mr Strauss dies and leaves an estate valued at £50,000 in a bank account earning interest at rate $i^{(12)} = 9\%$. He has three children: Jim, aged 7, Fred, aged 5, and Sandra, just turned 4. The estate will be divided among the surviving children 14 years from now, on Sandra's 18th birthday. Find the expected value of the inheritance for each child given the following, and assuming independence.

Age today	Probability of Survival for 14 years
7	0.95
5	0.97
4	0.98

6. The *expected yield* of a random cash-flow is used to mean the rate i such that

$$\mathbb{E}NPV(i) = 0;$$

that is, the yield of the mean cash-flow is zero (or, put otherwise, the rate under which the expected value of the cash-flow is zero — this is not in general the same as the expectation of the yield of the cash-flow).

A £1000 20-year bond has coupons at $j^{(2)} = 12\%$ redeemable at par. Find the purchase price which provides an expected annual yield of $i^{(2)} = 14\%$, under the assumption of a semi-annual default probability of 2%. (You may find it convenient to work with 6 months as the basic time-unit).

7. An annuity is payable continuously at a rate of $\rho(t)$ per annum at time t provided the holder, who is aged x at time 0, is still alive. T_x is a random variable which models the remaining lifetime in years of a person aged x.

   (a) Write down an expression, in terms of T_x, for the (random) present value at time 0 of this cash-flow, at a constant force of interest δ p.a., and show that the expected present value at time 0 of the cash flow is equal to

$$\int_0^\infty e^{-\delta s}\rho(s)P(T_x > s)ds.$$

   (b) An annuity is payable continuously during the lifetime of a person now aged 30, but for at most 10 years. The rate of payment at all times t during the first 5 years is £5,000 p.a., and thereafter £10,000 p.a. The force of mortality to which this person is subject is assumed to be 0.01 p.a. at all ages between 30 and 35, and 0.02 p.a. between 35 and 40. Find the expected present value of this annuity at a force of interest of 0.05 p.a.

Course webpage: `http://www.stats.ox.ac.uk/~martin/BS4a.html`