Problem Sheet 4 - Part B Actuarial Science I - Oxford MT 2007

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1. *Hardy's Formula.* Consider a fund F, and incomplete information as follows: the amounts of the fund $F_0 \in \mathbb{R}_+$ just before time 0 and $F_1 \in \mathbb{R}_+$ just after time 1 and the (net) total new money $N \in \mathbb{R}$ invested during $[0,1]$. Denote $I = F_1 - F_0 - N$, the total interest and capital gain earned by the fund during $[0,1]$.

   (a) Assume that N is received in two equal instalments at the beginning and end of the year. Show that the money-weighted rate of return is given by the formula

   $$i = \frac{2I}{F_1 + F_0 - I}.$$

   What is the time-weighted rate of return?

   (b) Assume that N is received in one instalment at $t = 1/2$. Find an expression for the MWRR. What about the TWRR?

   (c) Assume that N is received continuously at constant rate in $[0,1]$. Give the yield equation for the money-weighted rate of return i. Show that for small i, the formula in part (a) is still a good approximation for the MWRR.

2. (i) A fund had the following revenue account for 2004:

	£million	
Value of fund at 1 January 2004		30
Add: new investments received during the year	18	
Deduct: withdrawals and other payments	30	
Value of fund at 31 December 2004		21

   Assuming that the new investments received and the withdrawals and other payments are spread evenly over the year, use Hardy's Formula from the previous question to caluclate an approximate effective annual rate of interest for 2004.

   (ii) A second set of accounts was constructed to show the dates at which the new investments were received, the dates at which the withdrawals and other payments actually occurred, and the value of the fund at various times during the year.

	£million	
Value of fund at 1 January 2004		30
Value of fund at 31 March		36
Value of fund at 15 May		35
New investments at 16 May	18	
Value of fund at 30 June		51
Value of fund at 30 September		45
Withdrawals on 1 October	30	
Value of fund at 31 December 2004		21

   (a) Show that the annual yield (i.e. the annual money weighted rate of return) on the fund for the year ending 31 December 2004 is approximately 8.87%.

(b) Calculate the annual TWRR on the fund for the year ending 31 Dec 2004.

(iii) Explain the differences between your answers for (i), (ii)(a) and (ii)(b).

3. Calculate the time-dependent force of inflation between two successive index values if intermediate values are computed by (a) linear (b) exponential interpolation.

4. A loan of £25,000 was issued at a certain time T and was repaid at par after three years. Interest was paid on the loan at the rate of 8% per annum, payable annually in arrears. The value of the Retail Price Index at various times (measured in years) is given by $R(T) = 205.0$; $R(T+1) = 215.6$; $R(T+2) = 223.5$; $R(T+3) = 231.5$. Estimate the real rate of return of the loan.

5. In a particular accumulation fund income is retained and used to increase the value of the fund unit. The 'middle price' of the unit on 1 April in each of the years 1999 to 2005 is given in the following table:

Year 1 April	1999	2000	2001	2002	2003	2004	2005
Middle price of unit in £	1.86	2.11	2.55	2.49	2.88	3.18	3.52

(a) On the basis of the above prices and ignoring taxation and expenses:

(i) Find the time-weighted rate of return over the period 1 April 1999 to 1 April 2005,

(ii) Show that the yield obtained by an investor who purchased 200 units on 1 April in each year from 1999 to 2004 inclusive, and who sold his holding on 1 April 2005, is approximately 10.60%.

(iii) Show that the yield obtained by a person who invested £500 in the fund on 1 April each year from 1999 to 2004 inclusive, and who sold back his holding to the fund managers on 1 April 2005, is approximately 10.67% (You should assume that investors may purchase fractional parts of units.)

(b) Suppose that, in order to allow for expenses, the fund's managers sell units 2% above the published middle price and buy back units 2% below the middle price. On this basis find revised answers to (ii) and (iii) of (a).

6. On 15 May 1997 a government issued an index-linked bond of term 15 years. Coupons are payable half-yearly in arrears, and the annual nominal coupon rate is $D = 4\%$.

Interest and capital repayments are indexed by reference to the value of an inflation index with a time lag of 8 months. The index value in September 1996 was $Q(-8/12) = 200$ and in March 1997 was $Q(-2/12) = 206$.

The issue price of the bond was such that, if the index Q were to increase at a constant rate of 7% p.a. from March 1997, a tax-exempt purchaser of the bond at the issue date would obtain a real yield of $y = 3\%$ p.a. convertible half-yearly.

(a) Determine $Q(t)$ for all $t \geq -2/12$ and hence the coupon and redemption payments of the bond per 100 nominal, assuming inflation at constant rate 7%.

(b) Show that the issue price of the bond is £111.53 per £100 nominal.