Cohabitation dissolution model

Kiernan (2001)[1] provides data on rates of conversion of cohabitations into marriage or separation, measured by years since the birth of the first child. The following data relate to couples in the UK:

(a): % of cohabiting couples who marry within the stated time:

n	1 year	3 years	5 years
150	18	30	39

(b): from among those who do not marry within 5 years, % of cohabiting couples remaining together at the stated time:

n	after 3 years	after 5 years
106	61	48

We will try to estimate rates of marriage and separation based on this data. There are various peculiarities of the data which we need to be aware of:

(1) Note that the two tables are based on slightly different samples. The total number included in table (b), 106, does not correspond to the total number in table (a) who do not marry within 5 years (which would be $(1 - 0.39) \times 150$).

(2) There is no figure in table (b) corresponding to the 1-year point.

(3) The data in table (b) are relative frequencies (conditional on not marrying before year 5).

(4) Note that table (a) shows "decrements" while table (b) shows "survivals".

We'll start with a simpler analysis, where we ignore the 1-year figure in table (a). This corresponds to a multiple decrement model with two time-periods, $[0, 3]$ and $[3, 5]$. We will assume constant rates for marriage and for separation on each of these intervals, denoted by $\mu^M_{1.5}$, $\mu^S_{1.5}$, μ^M_4, μ^S_4.

First it will be helpful to rewrite table (b) in terms of absolute decrements rather than relative survivals:

[1] Kathleen Kiernan, *The rise of cohabitation and childbearing outside marriage in Western Europe* (International Journal of Law, Policy and the Family vol. 15, pp. 1-21, 2001)

(b'): % of cohabiting couples separating within stated time:

3 years	5 years
(1-0.39)×(1-0.61) =23.8%	(1-0.39)×(1-0.48) = 31.7%

Writing d_x^M and d_x^S for the numbers marrying and separating in a given period, we can now create a life-table as follows (based on a notional radix of 1000 couples at time 0):

x	ℓ_x	d_x^M	d_x^S	q_x^M	q_x^S
0-3	1000	300	238	0.3	0.238
3-5	462	90	79	0.195	0.171

Here q_x^M and q_x^S are calculated by d_x^M/ℓ_x and d_x^S/ℓ_x.

To calculate corresponding decrement rates μ_x^M and μ_x^S for the first period, we can proceed as follows. The first equation relates the total proportion of decrements to the sum of the decrement rates, while the second follows from the fact that the ratio of decrements of each type should be the same as the ratio of the decrement rates:

$$1 - e^{-3\left(\mu_{1.5}^S+\mu_{1.5}^M\right)} = {}_3q_0^M + {}_3q_0^S = 0.3 + 0.238$$
$$\frac{\mu_{1.5}^S}{\mu_{1.5}^S + \mu_{1.5}^M} = \frac{{}_3q_0^S}{{}_3q_0^M + {}_3q_0^S} = \frac{0.238}{0.3 + 0.238}.$$

From these two equations together we obtain $\mu_{1.5}^M = 0.144$, $\mu_{1.5}^S = 0.114$.

Using a similar calculation for the period $[3,5]$, we can extend the above table as follows:

x	ℓ_x	d_x^M	d_x^S	q_x^M	q_x^S	μ_x^M	μ_x^S
0-3	1000	300	238	0.3	0.238	0.144	0.114
3-5	462	90	79	0.195	0.171	0.121	0.106

Once we incorporate the year-1 marriage figure from table (a), we might decide to model with different marriage rates on $[0,1]$ and $[1,3]$, denoted by $\mu_{0.5}^M$ and μ_2^M, say, while keeping a single rate of separation $\mu_{1.5}^S$ on $[0,3]$.

Then we have the following relations:

$$\begin{aligned}
18\% &= P(\text{marry in } [0,1]) \\
&= \left[1 - \exp\left(-\left(\mu_{0.5}^M + \mu_{1.5}^S\right)\right)\right] \frac{\mu_{0.5}^M}{\mu_{0.5}^M + \mu_{1.5}^S}; \\
12\% &= P(\text{marry in } [1,3]) \\
&= \exp\left(-\left(\mu_{0.5}^M + \mu_{1.5}^S\right)\right) \left[1 - \exp\left(-2\left(\mu_{0.5}^M + \mu_{1.5}^S\right)\right)\right] \frac{\mu_2^M}{\mu_2^M + \mu_{1.5}^S}; \\
46.2\% &= P(\text{neither marry nor separate in } [0,3]) \\
&= \exp\left(-\left(3\mu_{1.5}^S + \mu_{0.5}^M + 2\mu_2^M\right)\right).
\end{aligned}$$

We have three equations in three unknowns, and we can solve them numerically to give $\mu^M_{0.5} = 0.211$, $\mu^M_2 = 0.103$, $\mu^S_{1.5} = 0.118$.

Note that the separation rate $\mu^S_{1.5}$ has increased compared to the one we computed when we had a single rate for marriage on the whole interval $[0, 3]$. The model with different marriage rates corresponds sees a bias towards marriages occurring earlier. As a result, the total "exposure to risk of separation" is lower, so the same number of separations corresponds to a higher implied rate.

If we had wanted to do the calculation above by hand, we could have used an iterative procedure. For example:

(1) calculate $\mu^M_{1.5}$, $\mu^S_{1.5}$ as above;

(2) keeping $\mu^S_{1.5}$ fixed, generate $\mu^M_{0.5}$ and μ^M_2 to give the right distribution of marriages at times 1 and 3;

(3) readjust $\mu^S_{1.5}$ to give the right proportion of separated couples at time 3;

(4) readjust $\mu^M_{0.5}$, μ^M_2 again using the distribution of marriages at times 1 and 3; and so on.

We would expect this procedure to converge very fast.

We can calculate the implied proportions of separations at time 1, and summarise the rates obtained in a fuller table:

x	ℓ_x	d^M_x	d^S_x	q^M_x	q^S_x	μ^M_x	μ^S_x
0-1	1000	180	101	0.18	0.101	0.211	0.118
1-3	719	120	137	0.167	0.191	0.103	0.118
3-5	462	90	79	0.195	0.171	0.121	0.106

We could extrapolate to produce an estimate of the probability that a cohabitation with children will end in separation. We need to decide what to do with the lack of observations after 5 years. Most simply, we could assume that the rates from then on remain constant at the same values observed in the final interval $[3, 5]$.

Under those rates, the proportion separating is $\mu^S_x/(\mu^M_x + \mu^S_x) = 0.467$, so the number of separations seen after time 5 would be $\ell_5 \times 0.467 = 293 \times 0.467 = 137$. The overall number of separations is then 101+137+79+137=454, corresponding to a proportion of 0.454 of cohabitations eventually ending in separation.

This extension to the model is far from realistic, however. In particular, we have ignored the substantial number of cohabitions which are maintained in the long term without either marriage or separation.