Problem Sheet 2 - Part B Actuarial Science I - Oxford MT 2006

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1. For $m \in \mathbb{N}$, the prefix ${}_{m|}$ before an annuity symbol indicates that the sequence of payments concerned is *deferred* by an amount of time m. For example, the discounted present value (in the constant interest-rate model) of a *deferred annuity*, with unit payments per unit time payable from $m+1$ to $m+n$, is denoted by ${}_{m|}a_{\overline{n}|}$.

   (a) Express ${}_{m|}a_{\overline{n}|}$ in terms of the ordinary annuity symbols introduced in the lectures.

   (b) The case $m = -1$ corresponds to an annuity-due and is denoted by $\ddot{a}_{\overline{n}|}$. Write $\ddot{a}_{\overline{n}|}$ in terms of $a_{\overline{n}|}$.

   (c) The discounted present value of an *increasing annuity* with payments j at time $j = 1, \ldots, n$ is denoted by $(Ia)_{\overline{n}|}$. Express $(Ia)_{\overline{n}|}$ in terms of ordinary annuity symbols.

   (d) Consider a security redeemable at par, with term n and $p$thly coupon payments at nominal rate j. Express the accumulated (time n) and discounted (time 0) values in the constant i model in terms of annuity symbols.

2. Find, on the basis of an effective interest rate of 4% per unit time, the values of

$$a^{(4)}_{\overline{67}|}, \quad s^{(12)}_{\overline{18}|}, \quad \ddot{a}^{(4)}_{\overline{16.5}|}, \quad \ddot{s}^{(12)}_{\overline{15.25}|}, \quad {}_{4.25|}a^{(4)}_{\overline{3.75}|}, \quad (Is)_{\overline{4}|}.$$

   Describe the meaning of each of the symbols.

3. Show that in any constant i interest rate model, the cash flows $c_p = (k/p, i^{(p)}/p)_{k=1,\ldots,p}$ are equivalent for all $p \in \mathbb{N}$.

4. A loan of £30,000 is to be repaid by a level annuity payable monthly in arrears for 25 years, and calculated on the basis of an (effective) interest rate of 12% pa. Calculate the intitial monthly repayments.

   After ten years of repayments the borrower asks to:

   (a) pay off the loan which is outstanding. Calculate the lump sum which would be required to pay off the outstanding loan.

   (b) extend the loan by a further five years (i.e. to 30 years in total), and with repayments changed from monthly to quarterly in arrears. Calculate the revised level of quarterly repayments.

   (c) reduce the loan period by five years (i.e. to 20 years in total) and for repayments to be biannually (i.e. once every two years) in arrears. Calculate the revised level biannual repayments.

   [*Hint: calculate annual repayment levels first, then combine each two consecutive payments into an equivalent single payment.*]

5. Calculate the accumulated value on 1 March 1999 of level payments of £100 made on 1 March 1996, 1 March 1997 and 1 March 1998. The effective interest rate was initially 5.3% but changed on 3 February 1997 to 4.7%, on 5 March 1998 to 4.4% and on 31 January 1999 to 4.3%. Give an equivalent model for this cash flow that has constant interest rates between the payments.

6. (Corrected 25/10/06). Under the terms of a savings scheme an investor who makes an initial investment of £4,000 may receive either

   - £2,000 after 2 years and a further £3,000 after 7 years; or
   - £4,400 at the end of 4 years.

   Which of these options corresponds to a higher rate of interest on the investor's money?

Optional questions for further practice:

7. An insurance company issues an annuity of £10,000 p.a. payable monthly in arrears for 25 years. The cost of the annuity is calculated using an effective rate of 10% p.a.

   (a) Calculate the interest component of the first instalment of the sixth year.

   (b) Calculate the total interest paid in the first 5 years.

8. (a) An annuity-certain is payable annually in advance for n years. The first payment of the annuity is 1. Thereafter the amount of each payment is $(1 + r)$ times that of the preceding payment.

   Show that, on the basis of an interest rate of i per annum, the present value of the annuity is $\ddot{a}_{\overline{n}|j}$ where $j = (i - r)/(1 + r)$.

   (b) Suppose instead that the annuity is payable annually in arrear. Is its present value (at rate i) now equal to $a_{\overline{n}|j}$?

   (c) In return for a single premium of £10,000 an investor will receive an annuity payable annually in arrear for 20 years. The annuity payments increase from year to year at the (compound) rate of 5% per annum.

   Given that the initial amount of the annuity is determined on the basis of an interest rate of 9% per annum, find the amount of the first payment.

Course webpage: `http://www.stats.ox.ac.uk/~martin/BS4a.html`