Expectation Particle Belief Propagation
Thibaut Lienart, Yee Whye Teh and Arnaud Doucet

NIPS'2015
Montreal

Context
Inference on undirected graphical models (MRF) with continuous random variables
pairwise interactions

Objective: computing (approximation of) the marginals

Loopy Belief Propagation is a well known message-propagation algorithm to obtain good approximations to the marginals
Applications in tracking, sensor networks, computer vision, distributed inference, ...

Adaptive Proposals
Build approximation of messages and node potentials in exponential family

\[ \eta_u(x_u) \approx \tilde{\eta}_u(x_u) \]

Build proposal copying structure of the beliefs

\[ q_u(x_u) \approx \eta_u(x_u) \prod_{w \in \text{neigh}(u)} \tilde{\eta}_w(x_w) \]

Expectation propagation [4,7] framework can be used to find approximations and update the proposals

for \( t = 1, \ldots \),

for each \( u \in V \)

sample from current proposal \( x_u^t \sim q_u(\cdot) \); \( i = 1, \ldots, N \)

for each \( v \in \Gamma_u \)

message approximations \( \tilde{m}_{uv}^t(x_v) = \sum_{x_u} \tilde{\eta}_u(x_u) \psi_{uv}(x_u, x_v) \frac{\tilde{h}_u^{-1}(x_u)}{\tilde{h}_u^{-1}(x_u) \prod_{w \in \text{neigh}(u)} \tilde{\eta}_w(x_w)} \)

exponential family approximation \( \eta_{uv} \) s.t. \( \frac{\eta_{uv}}{\tilde{\eta}_v} \approx \frac{\psi_{uv}(x_u, x_v)}{\tilde{m}_{uv}^t(x_v)} \frac{\tilde{h}_u^{-1}(x_u)}{\prod_{w \in \text{neigh}(u)} \tilde{\eta}_w(x_w)} \)

(same step for \( \tilde{\eta}_u \))

update proposal and approximations

\[ q_{u,v} \leftarrow \tilde{q}_{u,v} \frac{\tilde{\eta}_{uv}}{\tilde{\eta}_v}, \quad \tilde{\eta}_u \leftarrow \tilde{\eta}_u \]

Discussion

Bottleneck: computation of the importance weights

pointwise evaluation of \( \tilde{h}_u^{-1} \) is \( \mathcal{O}(|\Gamma_u|N) \)
can be accelerated by computing cheaper unbiased estimator [6], this will work well if the incoming messages at \( u \) have similar effective support
cost then \( \mathcal{O}(|\Gamma_u|N) \) where \( N \) can be chosen (e.g., \( \log(N) \))

Theoretical guarantees: consistency of message estimators
Empirically performs well on mildly multimodal targets and outperforms vanilla PBP
Applied the accelerated version successfully on larger graphical models where PBP would have been prohibitive
Applied successfully on models with non-integrable potentials

Future work
Convergence analysis (in line with [3])
Exploit links with the Generalized two filter smoother [6]
Analysis in the case of Max Product BP (as in e.g., [8])

References

[8] F. Besse et al., PatchMatch Belief Propagation for correspondence field estimation, 2014