

Network Evolution

Networks in Cellular Biology

- A.** Metabolic Pathways
- B.** Regulatory Networks
- C.** Signaling Pathways
- D.** Protein Interaction Networks - PIN

E. Other Networks

The Internet

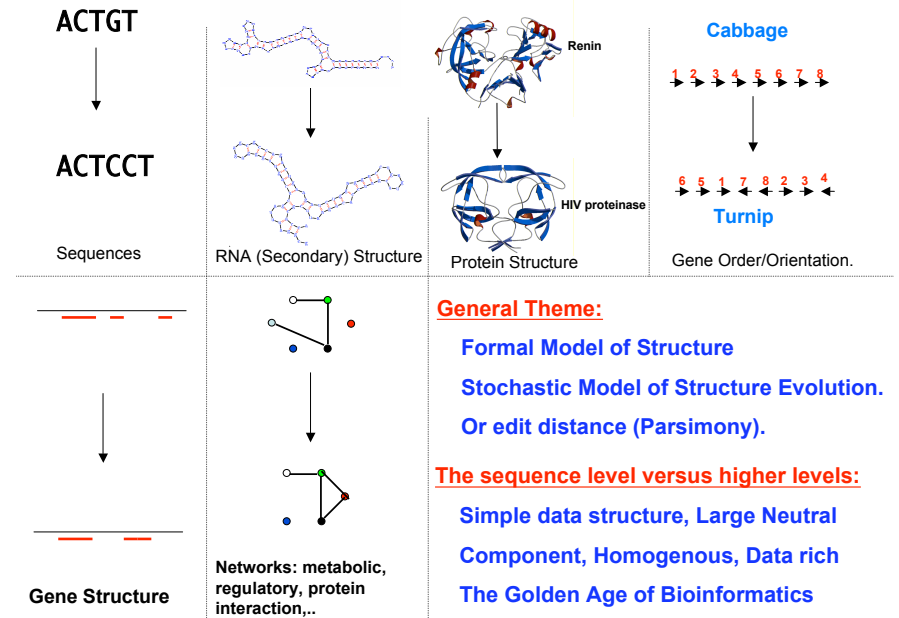
Statistics of Networks

Comparing Networks

- Network Matching
- Stochastic Models of Network

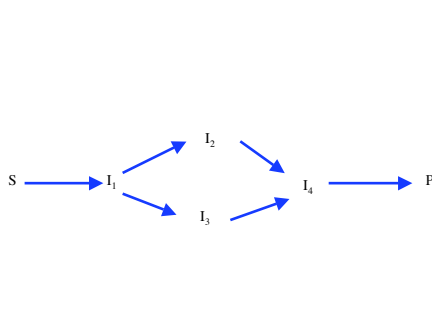
Examples of Comparison and Evolution

Comparative Biology

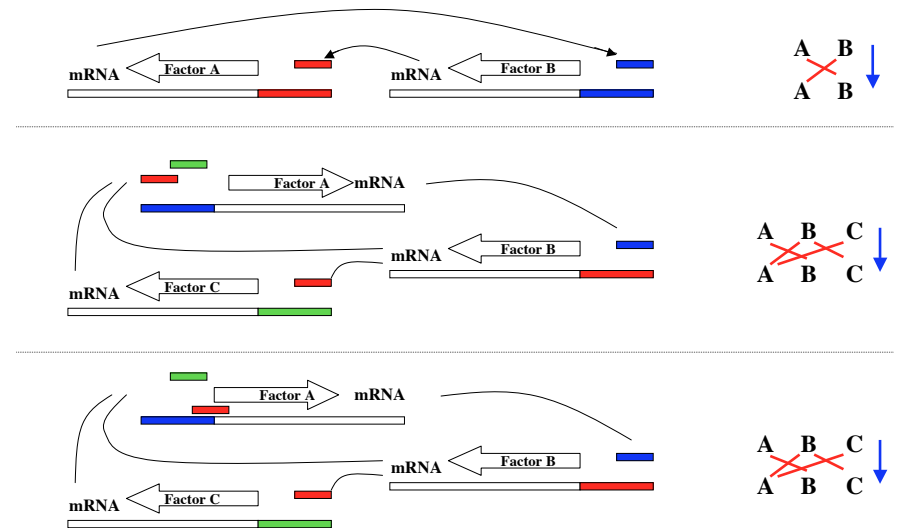


A. Metabolic Pathways

- Flux Analysis
- Metabolic Control Theory
- Biochemical Systems Theory
- Kinetic Modeling



B. Regulatory Networks



Boolean functions, Wiring Diagrams and Trajectories

	A	B	C
	A	B	C
Inputs	2	1	1
Rule	4	2	2

A activates B
 B activates C
 A is activated by B, inhibited by (B>C)

Point Attractor

A	B	C
1	1	0
1	1	1
0	1	1
0	0	1
0	0	0
0	0	0



2 State Attractor

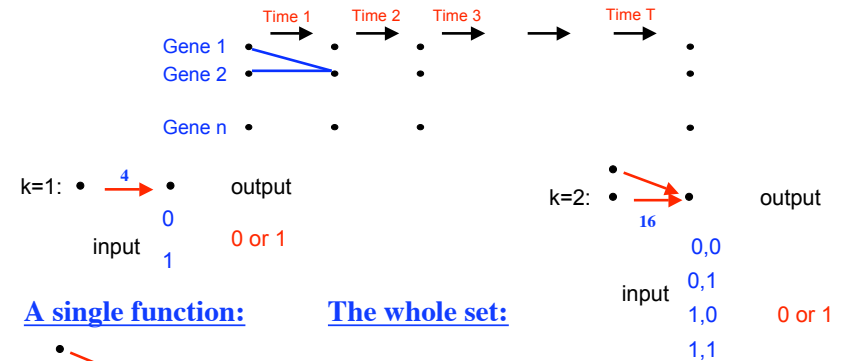
A	B	C
1	0	0
0	1	0
1	0	1
0	1	0



Remade from Somogyi & Sniegoski, 96, F4

Boolean Networks

R. Somogyi & CA Sniegoski (1996) Modelling the Complexity of Genetic Networks Complexity 1.6.45-64.



A single function:

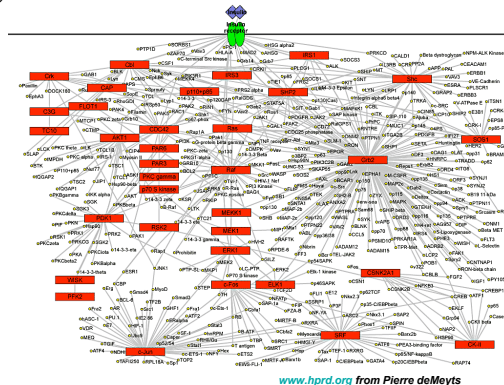
The whole set:

For each gene dependent on i genes: $\binom{k}{i}$ choices of dependent genes. Number of Boolean Rules $\left(\binom{k}{i} 2^i\right)^k$

Contradiction: Always turned off (biological meaningless) **Tautology:** Always turned on (household genes)

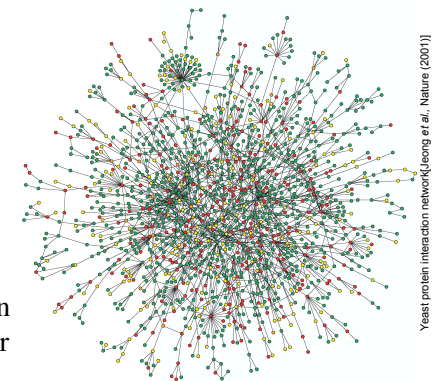
C. Signaling Pathways

- Transmits signals from membrane to gene regulation.
- Its function is enigmatic as some of the molecules involved are common to different functions and how cross-interaction is avoided is unknown.



D. Protein Interaction Network

- The sticking together of different protein is measured by mass spectroscopy.
- The nodes will be all known proteins.
- Two nodes are connected if they stick together. This can be indicator of being part of a functional protein complex, but can also occur for other reasons.



Yeast protein interaction network (Leong et al., Nature (2001))

E. Other Networks

More Sub-Cellular

- Alternative Splicing Graph



Cellular

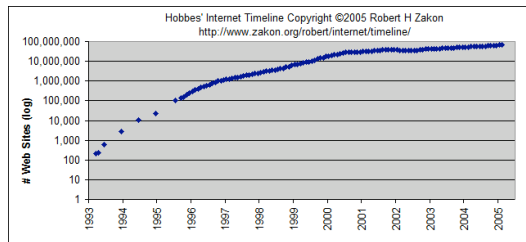
- Neural Networks
- Immunological Networks

Above the Cell

- Disease Networks
- Genealogical Networks

Non-biological Networks

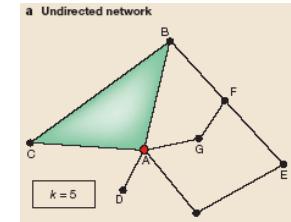
- Social Networks
- The Internet
- Collaboration Networks
- Semantic Networks
- Publications and references



Network Description and Statistics I

Barabasi & Oltvai, 2004

- Degree
- Shortest Path $Dist(i, j)$
- Mean Path Length
- Diameter: $Max_{i,j}\{Dist(i, j)\}$
- Clustering Coefficient - $C_1 = 2T_1 / n_1(n_1 - 1)$



$$C_A = 2/20$$

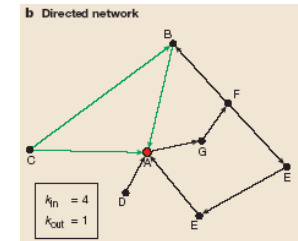
- Degree Distribution - $P(k)$
- Scale Free Networks $P(k) \sim k^{-\gamma}$ $\gamma > 2$

- Hubs: multiply connected nodes

The lower γ , the more hubs.

Small World Property:

Graph connected and path lengths small

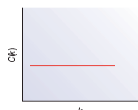
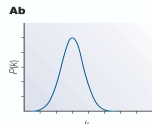
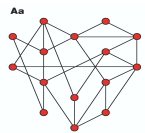


Remade from Barabasi, 2004

Network Description and Statistics II

Barabasi & Oltvai, 2004

A. Random Networks [Erdos and Rényi (1959, 1960)]

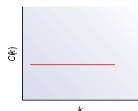
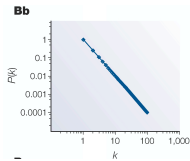
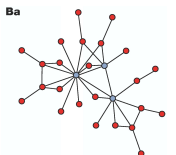


$$P(k) = \frac{e^{-\bar{k}} \bar{k}^k}{k!}$$

Mean path length $\sim \ln(k)$

Phase transition:
Connected if: $p \geq \ln(k)/k$

B. Scale Free [Price, 1965 & Barabasi, 1999]

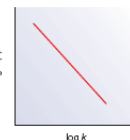
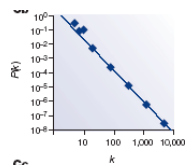
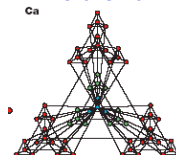


$$P(k) \sim k^{-\gamma}, k \gg 1, 2 < \gamma$$

Mean path length $\sim \ln \ln(k)$

Preferential attachment. Add proportionally to connectedness

C. Hierarchical

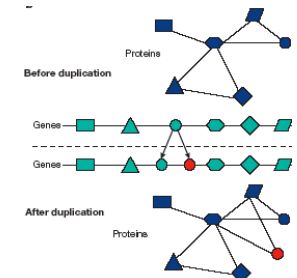


Copy smaller graphs and let them keep their connections.

Network Evolution

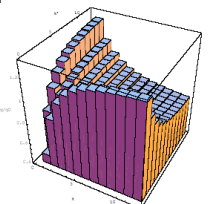
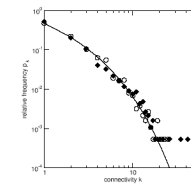
Barabasi & Oltvai, 2004 & Berg et al., 2004

- A gene duplicates
- Inherits its connections
- The connections can change



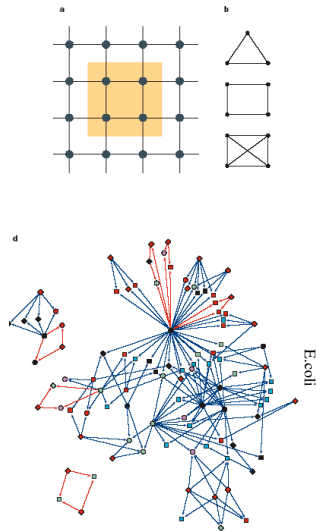
Berg et al., 2004

- Gene duplication slow $\sim 10^{-9}$ /year
- Connection evolution fast $\sim 10^{-6}$ /year
- Observed networks can be modeled as if node number was fixed.



Network Alignment & Motifs

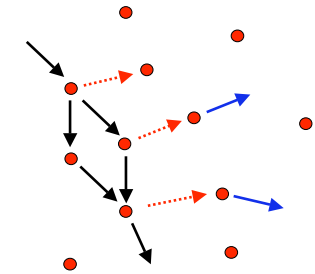
Barabasi & Oltvai, 2004



- Global Network Matching
- Network integration
- Network Search
- Motifs

A Model for Network Inference I

- A given set of metabolites: ●
- A given set of possible reactions - arrows not shown.
- A core metabolism: →
- A set of present reactions - **M**
black and red arrows



Restriction R:

A metabolism must define a connected graph
M + R defines

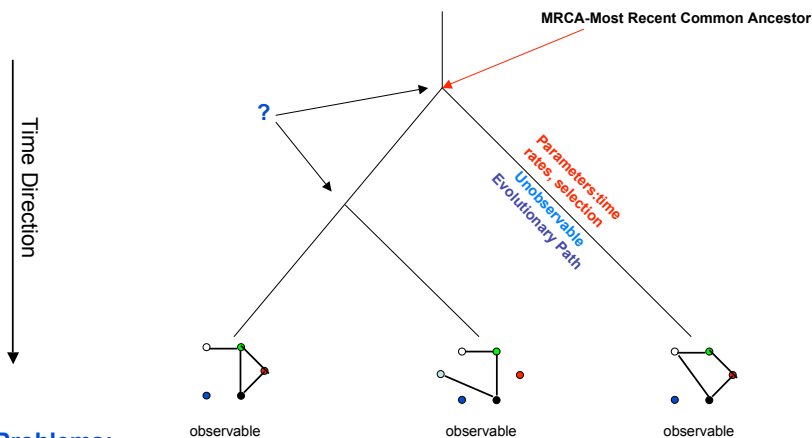
1. a set of deletable (dashed) edges **D(M)**: - - - - -
2. and a set of addable edges **A(M)**: →

Let μ be the rate of deletion
 λ the rate of insertion

Then

$$\frac{dP(M)}{dt} = \lambda \sum_{M' \in D(M)} P(M') + \mu \sum_{M'' \in A(M)} P(M'') - P(M)[\lambda|D(M)| + \mu|A(M)|]$$

A Model for Network Inference II



3 Problems:

- Test all possible relationships.
- Examine unknown internal states.
- Explore unknown paths between states at nodes.

Recommended Literature

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