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Non-stationary transition matrices: An overlooked issue in intra-distribution dynamics☆

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ABSTRACT

Previous papers dealing with the dynamics of income distributions have generally assumed stationary transition probabilities, effectively ruling out the possibility that intra-distribution *dynamics* change over time. This paper shows that the Chapman–Kolmogorov equation can be used to capture this added complexity. The paper also emphasizes that a test for stationarity should be carried out routinely because the stationarity assumption could lead to misleading conclusions.

1. Introduction

Since the widespread application of sigma and beta convergence in the early 1990s, the main advance in the economics of income convergence has been intra-distribution dynamics (e.g. López-Bazo et al., 1999; Fingleton, 2003; Le Gallo, 2004; Maza and Villaverde, 2004; Tortosa-Ausina et al., 2005; Ezcurra et al., 2005; Ertur et al., 2006; Ezcurra et al., 2006a,b; Fotopoulos, 2006). This approach generally involves estimating a *transition matrix*, whose elements represent the probability that an economy will move from one state to any other in a given time step.[1] As López-Bazo (2003) puts it, the matrix reveals "how economies transit from any point in the distribution to any other".

Nevertheless, to the best of our knowledge, the vast majority of studies have either ignored or deliberately ruled out changes in intra-distribution dynamics when it comes to estimating transition matrices. This means assuming that an economy's likelihood of transiting from a state to any other is the same today as it will be tomorrow, what unduly constrains the form of any long-period transition matrix. In academic jargon, these papers assume that the transition probabilities are stationary.

With this in mind, the main contribution of this paper is twofold. Firstly, it shows that the Chapman–Kolmogorov equation provides a straightforward way of introducing non-stationarity in the transition matrix estimation procedure. Secondly, through an illustrative example, the paper demonstrates that the stationarity assumption can lead to highly misleading estimates; for this reason, it highlights that a test for stationarity should be carried out routinely if the researcher wants to make this assumption.

2. Non-stationary transition matrices: the Chapman–Kolmogorov equation

The traditional way to estimate a transition matrix in intra-distribution dynamics analysis is by comparing the distribution at the initial and final years of the period considered; that is, one assumes that the probabilities are stationary (examples of this approach can be found in papers by Quah, 1993, 1996 and 1997; Magrini, 1999; and Le Gallo, 2004, among other works). Bearing this in mind, let suppose that the distribution under analysis in divided into a exhaustive set of m mutually exclusive states (usually intervals of income). If the distribution is known at two non-consecutive times t_0 and t_n, then it is possible to obtain the corresponding probability distributions denoted F^{t_0} and F^{t_n}, which give the proportion of economies at each state at t_0 and t_n, respectively. The transformation of the distribution between t_0 and t_n ($F^{t_0} \to F^{t_n}$)

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[1] Note that from a continuous perspective, intra-distribution dynamics can be analysed through the estimation of a stochastic kernel (Stokey and Lucas, 1989). A stochastic kernel is just the continuous counterpart of a transition probability matrix with an infinite number of rows and columns.

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is quantified by transition probabilities p_{ij}, that is, the probability that an economy is member of state j at time t_n, conditional on membership in state i at time t_0 ($i, j = 1,\ldots, m$). The set of transition probabilities is organized in a square stochastic matrix $P \in \mathfrak{R}_+^{m \times m}$. When P is estimated using only F^{t_0} and F^{t_n}, the matrix must be stationary (not time-dependent).[2]

This estimation method, although very popular, has a serious problem: it ignores the possibility of shifts in the transition probabilities between t_0 and t_n. In order to solve this drawback, the Chapman–Kolmogorov equation can be used. This equation incorporates information from all years in the estimation period.

We thus consider a sequence of transition matrices $P(t_0, t_1)$, $P(t_1, t_2)$,…,$P(t_{n-1},t_n)$. According to the Chapman–Kolmogorov equation, a pair of non-stationary transition matrices between t_0 and t_n must satisfy the following equation:

$$P(t_0, t_n) = P(t_0, t_{n-s}) \cdot P(t_{n-s}, t_n), \tag{1}$$

for all $s = 1, 2,\ldots, n$-1 (for more details, see Parzen, 1962). It is easy to prove by iteration that for an arbitrary number of transitions,

$$P(t_0, t_n) = P(t_0, t_1) \cdot P(t_1, t_2) \cdot \cdots \cdot P(t_{n-1}, t_n). \tag{2}$$

According to expression (2), the non-stationary transition matrix between t_0 and t_n is equivalent to the product of all yearly transition matrices.

Thus, there are two approaches to compute transition matrices, each one providing different benefits. Whereas the stationarity approach enables us to estimate the ergodic distribution as a faithful picture of the long-run equilibrium distribution, the non-stationary approach adds information about changes occurred at intermediate times. With this in mind, the use of an appropriate stationarity test, such as that suggested by Anderson and Goodman (1957), would be a good practice to discriminate between these two approaches.

3. An application to the European per capita income distribution

This section illustrates the differences between stationary transition matrices and non-stationary transition matrices, and the consequences of this choice in computational analysis. To this end, we present a case study: the dynamics of the European income distribution over the period 1980–2005. Specifically, we draw on the Cambridge Econometrics regional database for measures of the annual relative per capita income (per capita GAV in PPS) in 196 regions (NUTS2) of the fifteen-member European Union (EU-15).[3]

To compute the transition matrices, we divide relative per capita income into five mutually exclusive states. Taking 100 as the European average, the states are [0, 75), [75, 90), [90, 110), [110, 125), and [125, + ∞).[4] Table 1 reports transition matrices estimated for the full 1980–2005 period, using both the traditional stationary method and the Chapman–Kolmogorov equation.

As is evident from the matrices, the two procedures give different results.[5] Indeed, the root-mean-square difference between the matrices amounts to 0.12. To confirm the existence of non-stationary transition probabilities, we applied the aforementioned test of stationarity (Anderson and Goodman, 1957). This test results in a chi-square value of 12,448.47 with degrees of freedom 480 (p-value = 0.000), rejecting the null hypothesis of stationarity at standard confidence levels. As a feature of note, the greatest difference appears in the transition from the [90, 110) state to the [75, 90) state: the non-stationary method gives a probability of 0.20, while the stationary method provides a probability of 0.57. The difference must lie in shifts occurring in intermediate years, which are completely ignored in the stationary method.

Therefore, the main lesson to be learnt from this paper is that the stationarity assumption usually made in intra-distribution dynamics analysis is an aspect that should be properly tackled. The researcher must take into account that in case of important shifts at between times this assumption could cause a significant loss in estimation accuracy. However, in case of shifts being soft, the researcher should assess whether the advantages of assuming stationarity (as some long-run outcomes fall out from this assumption) are enough to offset the disadvantages associated with a wrong assumption.

Table 1
Alternative estimates of the Transition Matrix for the period 1980–2005.

a) Stationary transition matrix					
States	[0, 75)	[75, 90)	[90, 110)	[110, 125)	[125, + ∞)
[0, 75)	0.41	0.21	0.29	0.07	0.01
[75, 90)	0.01	0.20	0.36	0.25	0.18
[90, 110)	0.05	0.57	0.29	0.05	0.05
[110, 125)	0.09	0.01	0.54	0.09	0.27
[125, + ∞)	0.02	0.07	0.57	0.07	0.26
b) Non-stationary transition matrix					
States	[0, 75)	[75, 90)	[90, 110)	[110, 125)	[125, + ∞)
[0, 75)	0.34	0.24	0.34	0.05	0.03
[75, 90)	0.19	0.23	0.43	0.08	0.06
[90, 110)	0.08	0.20	0.49	0.11	0.12
[110, 125)	0.05	0.16	0.46	0.13	0.20

4. Conclusions

Stationarity is a recurrent hypothesis in empirical research on intra-distribution dynamics, particularly when estimating transition matrices. This paper emphasizes that a test for stationarity should be carried out routinely if this hypothesis is made. The reason is that any stationary estimation runs the risk of not accounting for changes in intra-distribution dynamics at intermediate times and, consequently, the results may be spurious. Using the case of European relative incomes as a test bed, this paper demonstrates that transition matrices can be estimated much more accurately if the Chapman–Kolmogorov is applied to include information on intermediate times.

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[2] For estimation purposes, the conditioned relative frequencies are maximum likelihood estimates of the corresponding transition probabilities (for more details, see Anderson and Goodman, 1957).

[3] Lacking complete data, we exclude from our analysis the French Overseas Departments and the *Länders* of former East Germany.

[4] The results obtained by this approach depend critically on the number and length of the intervals considered. Here, we adopt the usual classification employed in European analyses on regional topics (see Cuadrado-Roura et al., 2002; Ezcurra et al., 2006a).

[5] To check the robustness of our result, we repeated the analysis using other state definitions. In each case significant differences were found between the transition matrices.

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