1. Histograms of the log-likelihood error

We examine for the Gaussian autoregressive model and for the Huang-Tauchen stochastic volatility model histograms of the log-likelihood error. Details of the autoregressive model can be found in Section B.5 of the paper "Efficient Implementation of MCMC When Using An Unbiased Likelihood Estimator" whereas details for the stochastic volatility model are given in Section 4 of the same paper.

We run the Particle marginal Metropolis Hastings for different values of T ($T = 40, 300$ and 2700); see the main paper for details of the proposal. For both models, we adjusted N so that the variance of the log-likelihood estimator, evaluated at the posterior mean $\overline{\theta}_T$, is approximately one. We present histograms of the log-likelihood errors evaluated at $\overline{\theta}_T$ and for the log-likelihood errors evaluated at 100 thinly spaced samples from the posterior $\pi(\theta)$. For the Gaussian autogressive model, this log-likelihood error can be computed straightforwardly as the exact log-likelihood is given by the Kalman filter. For the stochastic volatility model, the exact log-likelihood is evaluated by running the particle filter $S = 500$ times at each parameter value θ_j and by computing the log of the average of the likelihood estimates over these run; i.e.

$$\log\left(S^{-1}\textstyle\sum_{k=1}^{S}\widehat{p}_N(y|\theta_j, u^{kj})\right).$$

The results are displayed in Figure 1 and Figure 2.

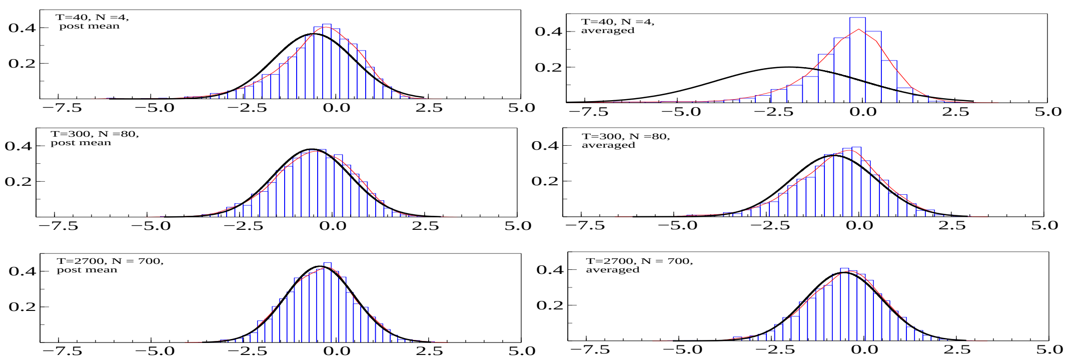

Fig. 1: *Huang and Tauchen model for varying T. Top to bottom: T=40, 300 and 2700 and N = 4, 80 and 700 respectively. Left: The histograms of Z (see text) calculated at the posterior mean. Right: The histograms of Z averaged over the 100 posterior draws of θ. Overlaid (in black) is the theoretical distribution $N(-\sigma^2/2; \sigma^2)$ where σ^2 is calculated as the variance of Z.*

The results indicate that as T increases to moderate or large values the marginal distribution of the log-likelihood error is very close to the distribution of the log-likelihood error at $\overline{\theta}_T$ and both distributions are very well approximated by our assumed Gaussian distribution.

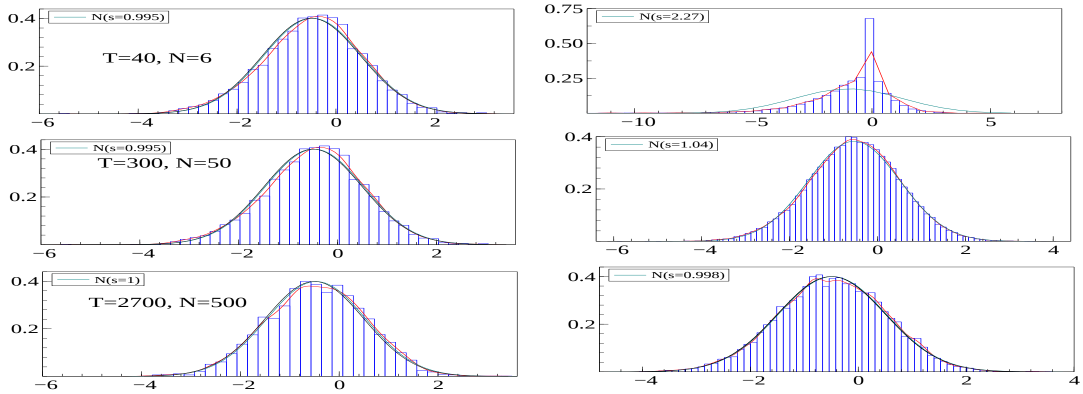

Fig. 2: *AR1 plus noise model for varying T. Top to bottom: T=40, 300 and 2700 and $N = 6$, 50 and 500 respectively. Left: The histograms of Z (see text) calculated at the posterior mean. Right: The histograms of Z averaged over the 100 posterior draws of θ. Overlaid (in black) is the theoretical distribution $N(-\sigma^2/2; \sigma^2)$ where σ^2 is calculated as the variance of Z.*