MATH 191, Winter 2015

Title: Graphs and Networks

Instructor: Mihai Cucuringu

Course Description: This is a project-based course about graphs and networks, that combines both mathematical and empirical approaches. We will consider a variety of graphs that have been studied extensively in the sciences and engineering literatures. We will study social networks, the web and internet graphs, protein interaction networks, geometric and random graphs, preferential attachment and small-world models, but also topics such as random walks on graphs, spectral graph theory, isoperimetric ratio and connection with the spectrum of the graph Laplacian, measures of centrality in networks including the PageRank algorithm, diffusion, gossip and rumor spreading, the evolution of graphs, electrical flows and their algorithmic implications. Finally, we will consider various notions of similarity and distance between the nodes of a graph, clustering and community detection, and graph drawing.

Prerequisites: discrete mathematics (at the level of MATH 61), linear algebra (at the level of MATH 33A) and elementary probability. Some familiarity with a programming language like MATLAB (or other related graph analysis software packages) would be useful, but this is also something you can pick up along the way. Please contact the instructor if unsure about the prerequisites.

Textbook:
Networks: An Introduction
by Mark Newman (Author)
Hardcover: 720 pages
Publisher: Oxford University Press; 1 edition (May 20, 2010)
Language: English
ISBN-10: 0199206651
ISBN-13: 978-0199206650

Topics covered

- Overview of various types of real-world networks
- Basic definitions from graph theory
- Weighted graphs; Cocitation and Bibliographic Coupling; Acyclic graphs and their properties
- Bipartite graphs and one-mode projections; Hypergraphs
- Connectivity; Trees and Forests
- Planar graphs; Chromatic number; Graph minors
- Counting walks on graphs
- Geodesic paths; Diameter; Girth
- Eulerian paths and cycles; Hamiltonian paths and cycles
- Laplacian Eigenmaps and their application to network analysis
- Spectral graph theory; Eigenvalues of the adjacency matrix; Bounds on the average and max degree
- Spectral properties of the Random-Walk Laplacian and the Combinatorial Laplacian; d-regular graphs, complete graphs K_n, bipartite graphs, bipartite complete graphs $K_{m,n}$
- Isoperimetric ratio, conductance and the spectral gap
- Network centrality measures: degree, closeness centrality, betweenness centrality, eigenvector centrality, Katz centrality, subgraph centrality
- Graph similarity, Louvain method by Blondel et al.
- Authority Hub Score (Kleinberg, 1999)
- Structural equivalence vs. regular equivalence; cosine similarity, Pearson coefficients
- Complexity of computing eigenvectors
- Graph partitioning and community detection, the Kernighan-Lin algorithm, spectral partitioning, the Network Community Profile Plot (NCP)
- Erdős-Rényi random graphs: mean degree, degree distribution, clustering coefficient, giant component, connectivity
- Power-law graphs
- Small World models, clustering coefficient, navigation in a small world (Kleinberg's paper)
- Preferential Attachment Models; Price Model, Barabási Albert Model

- Graphs in Algorithmic Game Theory
- Additional topics covered by the final research projects:
  - spectral-based vertex similarity and random-walk betweenness centrality
  - a local-to-global approach for vertex and graph similarity via the generalized condition number
  - a gravity model for spatial networks, and re-drawing the map of the US via constrained clustering
  - a centrality measure via the effective resistance embedding
  - global alignment of networks
  - co-clustering and bipartite clustering
  - constrained clustering and forecasting locations in Twitter data
  - vertex similarity via angular synchronization
  - correlation clustering