Monte Carlo Objectives

- Treat log of unbiased estimator of \( p(x) \) as an objective
  \[
  q_k(z_{1:k}, z_{1:k-1}) \}
  \]
  \( \text{that can be optimized with stochastic gradients.} \)

Defining FIVO

- Given model \( p \) over a sequence of \( n \) observations \( x \) with \( n \) latents \( z \) that factorizes in tractable conditionals,
  \[
  \text{and a variational posterior,}
  \]
  \( q_k(z_{1:k}, z_{1:k-1}) \}
  \]
  \( \text{define the incremental importance weights}
  \]
  \( \alpha_k(z_{1:k}) = \frac{p_k(z_{1:k}, z_{1:k-1})}{q_k(z_{1:k}, z_{1:k-1})} \}
  \]
  \( \text{Simulate the particle filter, see figures to the right.}
  \]
  \( \text{Define FIVO objective as expected log-likelihood estimator}
  \]
  \( L_{N}^{FIVO}(x_{1:n}, p, q) = \mathbb{E} \left[ \sum_{k=1}^{n} \log \hat{p}_k \right] \leq \log p(x_{1:n}) \}
  \]
  \( \text{Tightness}
  \]
  \( \text{In many sequential settings FIVO is a tighter bound than}
  \]
  \( \text{IWAE, since relative variance of particle filter scales better}
  \]
  \( \text{than importance sampling:}
  \]
  \( \text{Proposition. Let } \hat{p}_N(x) \text{ be an unbiased positive estimator of } p(x). \text{ Let } g(N) = \mathbb{E}[\hat{p}_N(x) - p(x)^N] \text{ be the 6th central moment. If the 1st inverse moment } \limsup N g(N) < \infty \text{ is bounded, then}
  \]
  \( \log p(x) - \mathbb{E} \log \hat{p}_N(x) = \frac{1}{2} \text{var} \left( \hat{p}_N(x) \right) + O(\sqrt{g(N)}) \}
  \]

- VRNN [3] trained with ELBO, IWAE, FIVO, and evaluate log-likelihood (relative to ELBO for TIMIT) on heldout data:

Optimizing FIVO

- This is an unbiased gradient for reparameterized latents
  \[
  \sum_{k=1}^{N} \nabla \log \hat{p}_k + \sum_{k'=k+1}^{N} \log \hat{p}_{k'} \nabla \log w_{k'} \}
  \]
  \( \text{in practice, we found it better to drop resampling terms.} \)

ARXIV Link and Citations

https://arxiv.org/abs/1705.09279