3 Changes in risk and insurance demand

1. Consider two lotteries L_a and L_b. The outcomes of lottery L_a are uniformly distributed on the unit interval. The probability density function of L_b is given by $g(x) = c\left(x - \frac{1}{2}\right)^2$ for $x \in [0,1]$, and $g(x) = 0$ otherwise.

   (a) Show that lottery L_b is a mean-preserving spread of lottery L_a.

   (b) Does one of the lotteries L_a and L_b first-order stochastically dominate the other?

2. An investor has wealth to invest in a set of n independent and identically distributed lotteries, $\tilde{x}_1, \ldots, \tilde{x}_n$. Let $\alpha_i \in [0,1]$ denote the share of wealth invested in lottery i, where $\sum_{i=1}^n \alpha_i = 1$. Show that the distribution of final wealth generated by the perfect diversification strategy $\alpha = (\frac{1}{n}, \ldots, \frac{1}{n})$ second order stochastically dominates the distribution of final wealth generated by any feasible strategy.
   [Hint: Consider adding $\sum_{i=1}^n (\alpha_i - \frac{1}{n})\tilde{x}_i$ to each potential outcome of the perfect diversification strategy.]

3. Consider an economy where each agent faces a statistically independent risk of losing 100 with probability p. A *pool* of N agents decide to create a mutual agreement where the aggregate loss in the pool will be equally split among its members.

   (a) Describe the possible losses and associated probabilities for each member of the pool when $N = 2$ and $N = 3$.

   (b) Show that an increase in the pool size from 2 to 3 would be preferred by all members as long as they are risk averse
   (Hint: Denoting $\tilde{x}$ as the loss borne by a member of a pool with $N = 2$ and $\tilde{y}$ as the loss borne by a member of a pool with $N = 3$, show that $\tilde{x} \sim \tilde{y} + \tilde{\varepsilon}$ for some white noise $\tilde{\varepsilon}$ where $\mathbb{E}[\tilde{\varepsilon}|\tilde{y}] = 0$.)

4. An agent with current wealth X has the opportunity to bet any amount on the occurrence of an event that she believes will occur with probability $p \in (0,1)$. If she wagers w, she will receive (the gross amount) $2w$ if the event occurs and 0 if it does not. Her utility function is given by $u(x) = -e^{-rx}$ with $r > 0$. How much should she wager?

5. Jude has a logarithmic utility function. She owns an asset with value £12 million, but which is subject to a potential loss of £8 million with probability $\frac{1}{4}$. Suppose that Jude can purchase coinsurance with level $\beta \in [0,1]$ and that insurance is priced with a loading of 0.2.

   (a) What is the insurance premium if Jude chooses $\beta = 1$?

   (b) Show that expected utility is concave in β and calculated the optimal value of β, denoted β^*.

   (c) Compute Jude's expected utility for $\beta = 0$, $\beta = 1$ and $\beta = \beta^*$.

   (d) What would happen to β^* if the loading fell to zero?

6. An individual owns assets of value $W_0 = 10$, which may suffer a random loss $\tilde{x}$ described by $\left(0, \frac{7}{10}; 4, \frac{1}{10}; 8, \frac{1}{10}; 10, \frac{1}{10}\right)$.

   (a) Compute the actuarially fair premium for full insurance.

   (b) What is the actuarially fair premium when a deductible $D = 3$ is selected? What happens to the premium when $D = 6$? Why does the premium not fall by 50%?

   (c) For each deductible compute the coinsurance rate β that yields the same actuarially fair premium.

   (d) Draw the cumulative distribution of final wealth first if $D = 6$ and then if the policy is characterized by the coinsurance rate β that yields the same premium as $D = 6$. By reference to the Rothschild-Stiglitz integral condition, show that the policy with a coinsurance rate induces a riskier distribution of final wealth.

7. A farmer must choose whether to purchase crop insurance or not. The farmer will make a decision that maximises her expected utility of wealth at harvest time, where utility of wealth w at harvest time is given by:

$$u(w) = \begin{cases} \frac{w^{1-\rho}-1}{1-\rho} \text{ for } \rho \neq 1 \\ \ln(w) \text{ for } \rho = 1 \end{cases}$$

   Without insurance there is a 50% chance that her wealth is 65 and a 50% chance that her wealth is 15. Insurance pays only in the bad state, and the farmer may purchase zero, partial or full insurance. With partial insurance there is a 50% chance that her wealth is 45 and a 50% chance that her wealth is 20. With full insurance she is guaranteed wealth of 25.

   (a) Calculate the premium for full insurance and the loading.

   (b) For $\rho = 2$ calculate the farmer's expected utility in the case of zero, partial and full insurance. Which of the three options offers the highest expected utility?

   (c) There exist constants α, β such that for $\rho \leq \alpha$ it is optimal to purchase zero insurance, for $\alpha \leq \rho \leq \beta$ it is optimal to purchase partial insurance and for $\rho \geq \beta$ it is optimal to purchase full insurance. Find α and β.