Mathematics and Statistics Undergraduate Handbook

Supplement to the Handbook

Honour School of Mathematics and Statistics
Syllabus and Synopses for Part A 2014–2015
for examination in 2015

Contents

Notice of misprints or errors of any kind, and suggestions for improvements in this booklet should be addressed to the Academic Administrator in the Department of Statistics.

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1. Honour School of Mathematics and Statistics

Please see the current edition of the Examination Regulations for the full regulations governing these examinations.

In Part A each candidate shall be required to offer 8 written papers. These papers **must** include:

- A1 Algebra 1 and Differential Equations 1 (3 hours)
- A2 Metric Spaces and Complex Analysis (3 hours)
- A8 Probability (1.5 hours)
- A9 Statistics (1.5 hours)
- ASO Short Options (1.5 hours)

and **three** papers from the Long Options (each 1.5 hours long)

- A3 Algebra 2: Rings and Modules
- A4 Integration
- A5 Topology
- A6 Differential Equations 2
- A7 Numerical Analysis
- A10 Waves and Fluids
- A11 Quantum Theory
- A12 Simulation and Statistical Programming

Paper ASO will examine the seven Short Options (Number Theory, Group Theory, Projective Geometry, Multivariable Differentiation, Calculus of Variations, Graph Theory, Special Relativity). Students are recommended to take three of these Short Options.

Part A shall be taken on one occasion only (there will be no resits). At the end of the Part A examination, a candidate will be awarded eight 'University Standardised Marks' (USMs) for their performance in Part A – one USM for each paper taken in Part A. The USMs from papers A1 and A2 will have twice the weight of the USMs awarded for papers A8, A9, ASO and the Long Options. A weighted average of the USMs will be carried forward for the classification awarded at the end of the third year, with this average from the second year papers counting for 40%.

All students who complete the first three years of the course will be classified, and those wishing to graduate at this point may supplicate for a BA.

Students wishing to take the four-year course should register to do so at the beginning of their third year and will be permitted to do so on the basis of an upper second class performance, or better, in the third year classification. They will take Part C in their fourth year, be awarded a separate classification and, if successful, be allowed to supplicate for an MMath.

Examination conventions can be found at:
http://www.stats.ox.ac.uk/current_students/bammath/examinations

Syllabus and Synopses

The syllabus details in this booklet are those referred to in the Examination Regulations and have been approved by the Statistics Academic Committee for examination in Trinity Term 2015. The synopses in this booklet give some additional detail, and show how the material is split between the different lecture courses. They also include details of recommended reading.

Actuarial exemption

A student who obtains an average USM of at least 60 in the Part A papers A8 Probability and A9 Statistics will be eligible to apply for an exemption from the Institute of Actuaries' paper CT3 Probability and Mathematical Statistics, which is a compulsory paper in their cycle of professional actuarial examinations. (This applies both to Mathematics and Statistics students, and to Mathematics students.) Students wishing to be considered for an exemption should complete an exemption request form in the year in which they take Part B.

2. CORE MATERIAL

2.1 Syllabi

The examination syllabi of the core papers A1 and A2 shall be the mathematical content of the synopses for the courses

Linear Algebra
Differential Equations
Metric Spaces and Complex Analysis

For the synopses for these courses, see those for Part A of the Honour School of Mathematics, which are available on the web at
https://www1.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/handbooks-synopses

The examination syllabi of the core papers A8 and A9 shall be the mathematical content of the synopses for the courses

Probability
Statistics

2.2. Synopses of lectures

This section contains the lecture synopses associated with the two core papers A8 and A9.

2.2.1 A8 Probability – 16 lectures MT

Aims and objectives
The first half of the course takes further the probability theory that was developed in the first year. The aim is to build up a range of techniques that will be useful in dealing with mathematical models involving uncertainty. The second half of the course is concerned with Markov chains in discrete time and Poisson processes in one dimension, both with developing the relevant theory and giving examples of applications.

Synopsis
Continuous random variables. Jointly continuous random variables, independence, conditioning, functions of one or more random variables, change of variables. Examples including some with later applications in statistics. Moment generating functions and applications. Statements of the continuity and uniqueness theorems for moment generating functions. Characteristic functions (definition only).

Convergence in distribution and convergence in probability. Markov and Chebyshev inequalities. Weak law of large numbers and central limit theorem for independent identically distributed random variables. Statement of the strong law of large numbers.

Discrete-time Markov chains: definition, transition matrix, n-step transition probabilities, communicating classes, absorption, irreducibility, periodicity, calculation of hitting probabilities and mean hitting times. Recurrence and transience. Invariant distributions, mean return time, positive recurrence, convergence to equilibrium (proof not examinable), ergodic theorem (proof not examinable). Random walks (including symmetric and asymmetric random walks on Z, and symmetric random walks on Z^d).

Poisson processes in one dimension: exponential spacings, Poisson counts, thinning and superposition.

Reading

G. R. Grimmett and D. R. Stirzaker, *Probability and Random Processes* (3rd edition, OUP, (2001). Chapters 4, 6.1-6.5, 6.8.

G.R. Grimmett and D. R. Stirzaker, *One Thousand Exercises in Probability* (OUP, 2001).

G. R. Grimmett and D J A Welsh, *Probability: An Introduction* (OUP, 1986). Chapters 6, 7.4, 8, 11.1-11.3.

J. R. Norris, *Markov Chains* (CUP, 1997). Chapter 1.

D. R. Stirzaker, *Elementary Probability* (Second edition, CUP, 2003). Chapters 7-9 excluding 9.9.

2.2.2 **A9 Statistics – 16 lectures HT**

Overview

Building on the first year course, this course develops statistics for mathematicians, emphasising both its underlying mathematical structure and its application to the logical interpretation of scientific data. Advances in theoretical statistics are generally driven by the need to analyse new and interesting data which come from all walks of life.

Learning Outcomes

At the end of the course students should have an understanding of: the use of probability plots to investigate plausible probability models for a set of data; maximum likelihood estimation and large sample properties of maximum likelihood estimators; hypothesis tests and confidence intervals (and the relationship between them). They should have a corresponding understanding of similar concepts in Bayesian inference.

Synopsis

Order statistics, probability plots.

Estimation: observed and expected information, statement of large sample properties of maximum likelihood estimators in the regular case, methods for calculating maximum likelihood estimates, large sample distribution of sample estimators using the delta method.

Hypothesis testing: simple and composite hypotheses, size, power and p-values, Neyman-Pearson lemma, distribution theory for testing means and variances in the normal model, generalized likelihood ratio, statement of its large sample distribution under the null hypothesis, analysis of count data.

Confidence intervals: exact intervals, approximate intervals using large sample theory, relationship to hypothesis testing.

Probability and Bayesian Inference. Posterior and prior probability densities. Constructing priors including conjugate priors, subjective priors, Jeffreys priors. Bayes estimators and credible intervals. Statement of asymptotic normality of the posterior. Model choice via posterior probabilities and Bayes factors.

Examples: statistical techniques will be illustrated with relevant datasets in the lectures.

Reading

F Daly, D J Hand, M C Jones, A D Lunn and K J McConway, *Elements of Statistics*, Addison Wesley (1995) Chapters 7-10 (and Chapters 1-6 for background)
J A Rice*, Mathematical Statistics and Data Analysis*, 2nd edition, Wadsworth (1995)
Sections 8.5, 8.6, 9.1-9.7, 9.9, 10.3-10.6, 11.2, 11.3, 12.2.1, 13.3, 13.4.
T Leonard and J S J Hsu *Bayesian Methods*, Cambridge, Chapters 2 and 3.

Further Reading

G Casella and R L Berger, *Statistical Inference,* 2nd edition, Wadsworth (2001)
A C Davison, *Statistical Models*, Chapter 11.

3 OPTIONS

3.1 Syllabi

The examination syllabi of the options papers A3-A7 and A10-A12 shall be the mathematical content of the synopses for the courses

- A3 Algebra 2: Rings and Modules
- A4 Integration
- A5 Topology
- A6 Differential Equations 2
- A7 Numerical Analysis
- A10 Waves and Fluids
- A11 Quantum Theory
- A12 Simulation and Statistical Programming

For the synopses of A3-A7 and A10-A11, see those for Part A of the Honour School of Mathematics, which are available on the web at https://www1.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/handbooks-synopses

3.2 Synopsis of Lectures

3.2.1 A12 Simulation and Statistical Programming – 14 lectures and 6 practicals HT

The workload of this course is equivalent to an 16-lecture course.

Aims and Objectives
Building on Part A probability and first year statistics, this course introduces Monte Carlo methods, collectively one of the most important toolkits for modern statistical inference. In parallel, students are taught programming in R, a programming language widely used in statistics. Lectures alternate between Monte Carlo methods and Statistical Programming so that students learn to programme by writing simulation algorithms.

Synopsis
Simulation: Transformation methods. Rejection sampling including proof for a scalar random variable, Importance Sampling. Unbiased and consistent IS estimators. MCMC including the Metropolis-Hastings algorithm.

Statistical Programming: Numbers, strings, vectors, matrices, data frames and lists, and Boolean variables in R. Calling functions. Input and Output. Writing functions and flow control. Scope. Recursion. Runtime as a function of input size. Solving systems of linear equations. Numerical stability. Regression. Monte Carlo and optimisation examples for elementary Bayesian inference.

Course Structure
The course will consist of fourteen lectures. Six of these will be held in a computer laboratory and are followed by an associated practical session. There will be four classes on problem sheets.

Reading
W J Braun and D J Murdoch, *A First Course in Statistical Programming with R*, CUP 2007
S M Ross, *Simulation,* Elsevier, 4th edition, 2006
J R Norris, *Markov Chains*, CUP, 1997

Reference
C P Robert and G Casella, *Monte Carlo Statistical Methods*, Springer, 2004
B D Ripley, *Stochastic Simulation*, Wiley, 1987

4. SHORT OPTIONS

4.1 Syllabi and Synopses

The examination syllabi of the short options paper ASO shall be the mathematical content of the synopses for the courses

Number Theory
Group Theory
Projective Geometry
Multivariate Differentiation
Calculus of Variations
Graph Theory
Special Relativity

For the synopses for these courses, see those for Part A of the Honour School of Mathematics, which are available on the web at
https://www1.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/handbooks-synopses