

SIMULATION, KRIGING, AND VISUALIZATION
OF CIRCULAR-SPATIAL DATA

by

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ABSTRACT

Visualization, Kriging, and Simulation
of Circular-Spatial Data

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The circular dataimage is defined by displaying direction as the color at the same direction in a color wheel composed of a sequence of two-color gradients with color continuity between gradients. The resulting image of circular-spatial data is continuous with high resolution. Examples include ocean wind direction, Earth's main magnetic field, and rocket nozzle internal combustion flow. The cosineogram is defined as the mean cosine of the angle between random components of direction as a function of distance between observation locations. It expresses the spatial correlation of circular-spatial data. A circular kriging solution is developed based on a model fitted to the cosineogram. A method for simulating circular random fields is given based on a transformation of a Gaussian random field. It is adaptable to any continuous probability distribution. Circular random fields were implemented for selected circular probability distributions. An R software package was created with functions and documentation.

(391 pages)

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Most of the figures were generated in R, versions 2.8.0 (R Development Core Team 2008). R was originally created by Ihaka and Gentleman (1996) and is now a collaborative worldwide effort. The binary distributions of R and R contributor packages are freely downloadable from <http://www.r-project.org/>, and are supported on Windows (NT, 95 and later) and in some versions for other operating systems. Figure 1-4 was produced using functions of R package Fields (Furrer, Nychka, and Sain 2009), software for simulation of random fields. Figure 1-5, Appendix Figure N-1 (b), and Figure N-2 (a) were produced using functions of R package CircStats (Lund and Agostinelli 2007), software for circular statistics, and manually enhanced. The functions of CircStats were used extensively in the codes written for this dissertation. Figures N-2 (b) and (c) were produced using a demo version of Oriana 2, software for the analysis and display of circular data (Kovach Computing 2004), and manually enhanced. Oriana is available at <http://www.kovcomp.co.uk/oriana/oribroc.html>. Oriana is supported on Microsoft

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Figures 2-15, 2-16, and N-4 were constructed using functions of R package RGL (Adler 2009), software for 3-D real time visualization. Other R packages used extensively include geoR (Ribeiro and Diggle 2001), and RandomFields (Schlather 2001).

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LIST OF SYMBOLS, TERMINOLOGY, ACRONYMS

Symbols

σ_{CK}^2	: Circular kriging variance
Θ	: Circular random variable (CRV)
θ	: Observation (realization) or simulation of a CRV
\mathbf{C}	: Matrix of cosines of angles between observations of direction
\mathbf{c}	: Vector of cosines of angles between observations and unobserved direction to be estimated
$\varsigma(d)$: Model of the mean cosine of the angle between random components of direction as a function of distance between observation locations
$\hat{\varsigma}(d)$: Cosineogram estimate of $\varsigma(d)$
κ	: Concentration parameter of the von Mises distribution
n_g	: Nugget
κ	: Population concentration about the mean direction for von Mises CRV
μ	: Population mean resultant vector direction
ρ	: Population resultant vector mean length and concentration about the mean direction
\overline{R}_n	: Sample resultant mean vector length
\mathbf{x}	: Vector of spatial coordinates

New Terminology

Circular Dataimage
 Cosineocloud
 Cosineogram
 Cosine Model
 Circular Random Field

Terminology from Linear Kriging

Covariogram
 Covariance Model
 Nugget
 Range
 Sill
 Variogram

LIST OF SYMBOLS, TERMINOLOGY, ACRONYMS

Acronyms

CDF : Cumulative Distribution Function
CRF : Circular Random Field
CRV : Circular Random Variable
CCW: Counterclockwise
GRV : Gaussian Random Variable
GRF : Gaussian Random Field
GUI : Graphical User Interface
GYRB: Green Yellow Red Blue
HSV : Hue Saturation Value
KBWR: Black Blue White Red
MAD : Mean Absolute difference
PDF : Probability Density Function
RF : Random Field
RGB : Red Green Blue
RV : Random Variable
YRGB: Yellow Red Green Blue