

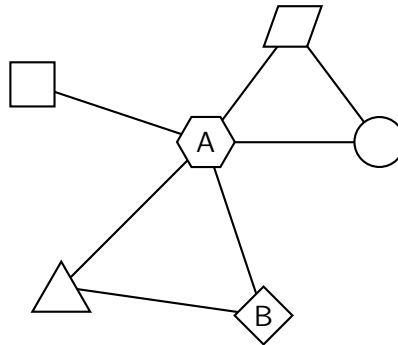
MS2a, Exercises Week 5

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1 Network Growth

Consider the network from [1, Fig. 3b]:



- Assume the network grows as described in [1, Fig. 3b], by duplication of a gene (node) chosen uniformly at random. For the nodes marked A and B, respectively, compute the expected number of connections a random descendant has after a single duplication. How does the expected fraction of all network nodes A and B are connected to change with the update?
- For node u let $P_u(i | n)$ denote the probability that a node u descendant is connected to exactly i other nodes when there are n nodes in the network. Write a recursion for $P_u(i | n)$, including boundary conditions.
- Plot or tabulate $P_u(\cdot | n)$ for $n = 9$ for the node marked A in the above figure.
- Let $c_{u,n} = \sum_{i=0}^{\infty} iP_u(i | n)/n$ denote the expected fraction of all nodes descendants of node u are connected to when there are n nodes in the network. Write an expression for $c_{u,n}$ in terms of $c_{u,6}$, i.e. the fraction of nodes u is connected to in the original network.

- e. Let c_n denote the expected average fraction of nodes a node is connected to once the network has grown to n nodes, e.g. $c_6 = 7/18$. Write a recursion for c_n (boundary condition was just given). Are your results concordant with your results for $c_{u,n}$?
- f. Connections between genes are not guaranteed eternal survival. Assume we extend the model such that in each round, after duplication of a gene, we choose a random edge uniformly at random and delete it. How will this affect the rich-get-richer nature of the model?
- g. Loss of connections will usually result from evolutionary drift. If we assume that connection loss probability is positively correlated with evolutionary distance, is the connection loss model proposed above realistic?

References

- [1] A.-L. Barabási and Z. N. Oltvai. Network biology: Understanding the cell's functional organisation. *Nature Reviews Genetics*, 5:101–113, 2004.