Title: **Random forests on weighted vertices**
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Brief Description: Consider the forests F on the vertex set $V = \{1, .., n\}$. Given positive integer weights $w(v)$ on the vertices v in V, let $\mu(F)$ be the product over the edges uv of F of $w(u)w(v)$. This can also be written as the product over the vertices v in V of $w(v)^{deg(v)}$, where $deg(v)$ denotes the degree of v in F. Thus

$$\mu(F) = \prod_{\text{edges uv}} w(u)w(v) = \prod_{v \in V} w(v)^{deg(v)}.$$

The aim is to investigate the properties of the random forest R, where the probability $P(R = F)$ is proportional to $\mu(F)$, in particular in relation to the uniform case when each weight $w(v) = 1$. A natural focus is on the probability that R is connected, and on the open question:

Conjecture $P(R \text{ is connected}) \geq P(F \text{ is connected})$, where F is a uniformly random forest on $\{1, .., n\}$.

If the conjecture is false then it might be possible to disprove it by considering modest values of n: such values of n should be checked.

Prerequisite courses: none, though graph theory and combinatorial optimisation would help.
theoretical project/simulation/computational project

References:

Louigi Addario-Berry, Colin McDiarmid and Bruce Reed, Connectivity of Addable Monotone Graph Classes, manuscript 2007.

Colin McDiarmid, Angelika Steger and Dominic Welsh, Random planar graphs, J. Combinatorial Theory B 93(2005) 187 – 206.

Colin McDiarmid, Angelika Steger and Dominic Welsh, Random graphs from planar and other addable classes. In Topics in Discrete Mathematics (M. Klazar, J. Kratochvil, M. Loebl, J. Matousek, R. Thomas, P. Valtr eds), Algorithms and Combinatorics 26, Springer, 2006, 231 – 246.