Honour School of Mathematics and Statistics

Supplement to the Undergraduate Handbook Syllabus and Synopses for Part B 2007–2008 for examination in 2008

Contents

Revised 23/11/07

1. Honour School of Mathematics and Statistics

See the current edition of the Examination Regulations for the full regulations governing these examinations.

In Part B each candidate shall offer a total of four units from the schedule of units and half-units.

(a) Each candidate shall offer the unit BS1 (Applied Statistics).
(b) Each candidate shall offer a total of at least one unit from the units and half-units available under BS2 (Statistical Inference) and BS3 (Stochastic Modelling).
(c) Each candidate may offer a total of at most one unit from BS4 (Actuarial Science) and the schedule of 'Other units and half units' (Ambassador's Scheme, half-unit) .
(d) Units from the schedule of 'Mathematics Department units and half-units' for Part B of the Honour School of Mathematics are also available.

Students staying on to take the four-year course will take two units from Part C in their fourth year, and will also offer a dissertation on a statistics project. Of the two units from Part C, at least half a unit will be from the schedule of 'Statistics' units for Part C.

In the classification awarded at the end of the third year, unit paper marks in Part A will be given a 'weighting' of 2, and unit paper marks in Part B will be given a 'weighting' of 3. For those students staying on to do the fourth year, a separate class will be awarded on the basis of the Part C marks.

We ask that you register by the end of week 9 Trinity Term 2007 for classes for the Mathematics/ Statistics courses that you wish to take. A registration form is attached to these synopses. Some combinations of subjects are not advised and lectures in these subjects may clash. However, when timetabling lectures we will aim to keep clashes to a minimum.

1.1 Units and half-units and methods of examination

Most subjects offered have a 'weight' of one unit, and will be examined in a 3-hour examination paper. In some of these subjects it will also be possible to take the first half, or either half, of the subject as a 'half-unit'. Where this is the case, a half-unit will usually be examined in an examination paper of 1 ¾ hours.

Rubrics on 3-hour examination papers
The rubric on 3-hour examination papers will usually be: "Submit answers to a maximum of five questions. Submit at least one answer in each section (half-unit) of the paper. The best answer in each section will count, along with the two best of the remaining answers".

Rubrics on 1 ¾-hour examination papers
The rubric on 1 ¾-hour examination papers will usually be: "Submit answers to a maximum of three questions. The best two answers will be counted".

BS1 Applied Statistics
This will be examined via a 2-hour examination paper plus assessed practical assignments.

2 Statistics units and half units

2.1 BS1 Applied Statistics

Level: H-level
Method of Assessment: 2-hour examination plus assessed practical assignments. The practical assignments contribute 1/3 of the marks for BS1. As the practicals are a formal part of the examination for BS1, it is compulsory to submit all practicals.

Prerequisites: Part A Probability and Statistics.
Weight: One unit.

Aims
The course aims to develop the theory of statistical methods, and also to introduce students to the analysis of data using a statistical package. The main topics are: Simulation, Practical aspects of linear models, Logistic regression and generalized linear models, and Robust and computer-intensive methods.

2.1.1 Applied Statistics I – 16 MT

Synopsis
Simulation: pseudo-random numbers, inversion, rejection, composition, ratio-of-uniforms method, computational efficiency. Inference using simulation methods.
Practical aspects of linear models and analysis of variance: review of multiple regression, model selection, fit criteria, use of residuals, outliers, leverage, Box-Cox transformation, added-variable plots, model interpretation.
Logistic regression. Linear exponential families and generalized linear models, scale parameter, link functions, canonical link. Maximum likelihood fitting and iterated weighted least squares. Asymptotic theory: statement and applications to inference, analysis of deviance, model checking, residuals. Examples: binomial, Poisson and gamma models.

2.1.2 *Applied Statistics II* – 10 HT

Synopsis
Nonparametric inference. Permutation tests. Rank statistics. L- M- and R-estimation. Influence curve. Breakdown point. Robust and resistant regression. Smoothing methods (kernels, splines, local polynomials). Bootstrapping. Monte Carlo tests.

Reading (Michaelmas Term)

S. M. Ross, *Simulation*, 2nd edition, Academic Press (1996)
A. J. Dobson, *An Introduction to Generalized Linear Models*, Chapman and Hall (1990)
D. Lunn, *Notes* (2003)

Reading (Hilary Term)

Chapters III.2, III.5 and III.9 of
http://www.quantlet.com/mdstat/scripts/csa/html/

P. J. Rousseeuw and A. M. Leroy, *Robust Regression and Outlier Detection*, Wiley (1987), pp 1-194.
J. D. Gibbons, *Nonparametric Statistical Inference*, Marcel Dekker (1985), pp 1-193, 273- 290.
R. H. Randles and D. A. Wolfe, *Introduction to the Theory of Nonparametric Statistics*, Wiley (1979), pp 1-322.

Further Reading
F. L. Ramsey and D. W. Schafer, *The Statistical Sleuth: A Course in Methods of Data Analysis*, 2nd edition, Duxbury (2002)
W. N. Venables and B. D. Ripley, *Modern Applied Statistics with S*, Springer (2002)

Practicals
In addition to the lectures there will be four supervised practicals, each containing one or more problems whose written solutions will be assessed as part of the unit examination. Similar practical applications will be used as illustrations in lectures.

2.2 **BS2 Statistical Inference**

Level: H-level
Method of Assessment: 3-hour or 1¾-hour examination
Weight: One unit, or the first 16 lectures can be taken as a half-unit BS2a.
Prerequisites: The Part A Statistics course. BS2b cannot be taken alone.
BS2a is a prerequisite for BS2b.

2.2.1 BS2a *Foundations of Statistical Inference* – 16 MT

[Option BS2a if taken as a half-unit.]

Level: H-level
Prerequisite: The Part A Statistics course.

Learning outcomes
Understanding how data can be interpreted in the context of a statistical model. Working knowledge and understanding of key-elements of model-based statistical inference, including awareness of similarities, relationships and differences between Bayesian and frequentist approaches.

Synopsis
Exponential families: Curved and linear exponential families; canonical parametrization; likelihood equations. Sufficiency: Factorization theorem; sufficiency in exponential families.

Frequentist estimation: unbiasedness; method of moments; the Cram´er-Rao information inequality; statement of the large sample distribution of the MLE; proof for curved exponential families assuming consistency.

The Bayesian paradigm: subjective probability; prior to posterior analysis; conjugacy; examples from exponential families. Choice of prior distribution: proper and improper priors; Jeffreys' and maximum entropy priors. Hierarchical Bayes models, graphical representation.

Computational techniques: Markov chain Monte Carlo methods; sampling importance resampling; data examples.

Decision theory: risk function; randomized decision rules; admissibility. Rao-Blackwell theorem: Rao-Blackwellization; illustration with squared error loss. Minimax rules, Bayes rules and admissibility. Hypothesis testing as decision problem.

Empirical Bayes methods. James Stein estimator. Shrinkage.

Reading
G.A.Young and R.L. Smith, *Essentials of Statistical Inference*, Cambridge University Press, 2005.
T. Leonard and J.S.J. Hsu, *Bayesian Methods*, Cambridge University Press, 2005.
Further reading
D. R. Cox, *Principles of Statistical Inference*, Cambridge University Press, 2006.
D. Sorensen and D. Gianola, *Likelihood, Bayesian and MCMC Methods in Quantitative Genetics*, Springer, NY, 2002.
Y. Pawitan, I*n All Likelihood: Statistical Modelling and Inference using Likelihood*, OUP, 2001.

2.2.2 BS2b: *Further Statistical Inference* – 16 HT

Level: H-level – Prerequisite: BS2a

Learning outcomes
Awareness of the increasing complexity of data sets in the modern world
and the need to develop appropriate statistical methodology. Understanding of fundamental methods associated with statistical inference for multi-parameter models and high-dimensional data. Understanding of Bayesian and frequentist methods for the statistical analysis of data that arrive sequentially.

Synopsis
Ancillarity; conditional inference; dealing with nuisance parameters. Comparison of Bayesian and frequentist approaches.

Generalized linear models and exponential families: Newton-Raphson iteration and the method of scoring for multi-parameter problems.

Approximating integrals that arise in statistical applications: saddle-point expansions; Laplace's approximation.

Model comparison and model selection: Bayes factors; asymptotic approximation of Bayes factors; AIC vs. BIC.

The multivariate normal distribution; Wishart and inverse Wishart distributions. Frequentist inference for multivariate normal models: Wilks' test and Hotelling's T2. Bayesian inference for multivariate normal models.

Sequential frequentist methods: sequential probability ratio tests. Sequential Bayesian methods: Kalman filter.

Reading
G.A.Young and R.L. Smith, *Essentials of Statistical Inference*, Cambridge University Press, 2005.
T. Leonard and J.S.J. Hsu, *Bayesian Methods*, Cambridge University Press, 2005.

Further reading
D. R. Cox, *Principles of Statistical Inference*, Cambridge University Press, 2006.
D. Sorensen and D. Gianola, *Likelihood, Bayesian and MCMC Methods in Quantitative Genetics*, Springer, New York, 2002.
Y. Pawitan, *In All Likelihood: Statistical Modelling and Inference using Likelihood*, OUP, 2001.

2.3 **BS3 Stochastic Modelling**

Level: H-level
Method of Assessment: 3-hour or 1¾-hour examination

Prerequisites: Part A Probability for the first 16 lectures. Part A Statistics in addition for the second 16 lectures.

Weight: One unit, or the first 16 lectures can be taken as a half-unit in Applied Probability. The second 16 lectures cannot be taken as a half-unit.

Aims*:* This unit has been designed so that a student obtaining at least an upper second class mark on the whole unit can expect to gain exemption from the Institute of Actuaries' paper CT4, which is a compulsory paper in their cycle of professional actuarial examinations. The first half of the unit, clearly, and also the second half of the unit, apply much more widely than just to insurance models.

2.3.1 *Applied Probability* – 16 MT [Option BS3a if taken as half-unit]

Aims

This course is intended to show the power and range of probability by considering real examples in which probabilistic modelling is inescapable and useful. Theory will be developed as required to deal with the examples.

Synopsis

Poisson processes and birth processes. Continuous-time Markov chains. Transition rates, jump chains and holding times. Forward and backward equations. Class structure, hitting times and absorption probabilities. Recurrence and transience. Invariant distributions and limiting behaviour. Time reversal.

Applications of Markov chains in areas such as queues and queueing networks - M/M/s queue, Erlang's formula, queues in tandem and networks of queues, M/G/1 and G/M/1 queues; insurance ruin models; epidemic models; applications in applied sciences.

Renewal theory. Limit theorems: strong law of large numbers, strong law and central limit theorem of renewal theory, elementary renewal theorem, renewal theorem, key renewal theorem. Excess life, inspection paradox. Applications.

Reading

• J.R. Norris: *Markov Chains*. Cambridge University Press (1997)
• G.R. Grimmett and D.R. Stirzaker: *Probability and Random Processes*. 3rd edition, Oxford University Press (2001)
• G.R. Grimmett and D.R. Stirzaker: *One Thousand Exercises in Probability*. Oxford University Press (2001)
• S.M. Ross: I*ntroduction to Probability Models*. 4th edition, Academic Press (1989)
• D.R. Stirzaker: *Elementary Probability*. 2nd edition, Cambridge University Press (2003)

2.3.2 **Statistical Lifetime-Models** – 16 HT

Aims

The second half of the unit follows on from the first half on Applied Probability. Models introduced there are examined more specifically in a life insurance context where transitions typically model the passage from 'alive' to 'dead', possibly with intermediate stages like 'loss of a limb' or 'critically ill'. The aim is to develop statistical methods to estimate transition rates and more specifically to construct life tables that form the basis in the calculation of life insurance premiums. Survival analysis will allow consideration of the effect of covariates.

Synopsis

Survival models: general lifetime distributions, force of mortality (hazard rate), survival function, specific mortality laws, the single decrement model, curtate lifetimes, life tables.

Estimation procedures for lifetime distributions: empirical lifetime distributions, censoring, Kaplan-Meier estimate, Nelson-Aalen estimate. Parametric models, accelerated life models including Weibull, log-normal, log-logistic. Plot-based methods for model selection. Proportional hazards, partial likelihood.

Two-state and multiple-state Markov models, with simplifying assumptions. Estimation of Markovian transition rates: Maximum likelihood estimators, time-varying transition rates, census approximation.

Graduation, including fitting Gompertz-Makeham model, comparison with standard life table: tests including chi-square test and grouping of signs test, serial correlations test; smoothness.

Reading

• Subject 104 [CT4] *Survival models [Modelling] Core Reading,* Faculty & Institute of Actuaries

• D.R. Cox and D. Oakes: *Analysis of Survival Data.* Chapman & Hall (1984)

Further Reading

• J.P. Klein and M.L. Moeschberger: *Survival Analysis*. Springer (1997)

• C.T. Le: *Applied Survival Analysis.* Wiley (1997)

• H.U. Gerber: *Life Insurance Mathematics*. 3rd edition, Springer (1997)

• N.L. Bowers et al.: *Actuarial mathematics*. 2nd edition, Society of Actuaries (1997)

2.4 **BS4 Actuarial Science**

Level: H-level

Method of Assessment: 3-hour examination

Prerequisites: Part A Probability is useful, but not essential. If you have not done Part A Probability, make sure that you are familiar with Mods work on Probability.

Weight: One unit.

2.4.1 *Actuarial Science I* – 16 MT

Aims

This unit is supported by the Institute of Actuaries. It has been designed to give the undergraduate mathematician an introduction to the financial and insurance worlds in which the practising actuary works. Students will cover the basic concepts of risk management models for investment and mortality, and for discounted cash flows. In the examination, a student obtaining at least an upper second class mark on this unit can expect to gain exemption from the Institute of Actuaries' paper CT1, which is a compulsory paper in their cycle of professional actuarial examinations.

Synopsis

Fundamental nature of actuarial work. Use of generalised cash flow model to describe financial transactions. Time value of money using the concepts of compound interest and discounting. Interest rate models. Present values and accumulated values of a stream of equal or unequal payments using specified rates of interest. Interest rates in terms of different time periods. Equation of value, rate of return of a cash flow, existence criteria.

Loan repayment schemes. Investment project appraisal, funds and weighted rates of return. Inflation modelling, inflation indices, real rates of return, inflation-adjustments. Valuation of fixed-interest securities, taxation and index-linked bonds.

Uncertain payments, corporate bonds, fair prices and risk. Single decrement model, present values and accumulated values of a stream of payments taking into account the probability of the payments being made according to a single decrement model. Annuity

functions and assurance functions for a single decrement model. Risk and premium calculation.

Reading
All of the following are available from the Publications Unit, Institute of Actuaries, 4 Worcester Street, Oxford OX1 2AW

• Subject 102[CT1] *Financial Mathematics Core Reading* Faculty & Institute of Actuaries.
• J.J. McCutcheon and W.F. Scott: *An Introduction to the Mathematics of Finance*. Heinemann (1986)
• P. Zima and R.P. Brown: *Mathematics of Finance*. McGraw-Hill Ryerson (1993)
• H.U. Gerber: *Life Insurance Mathematics*. 3rd edition, Springer (1997)
• N.L. Bowers et al: *Actuarial mathematics*. 2nd edition, Society of Actuaries (1997)

2.4.2 *Actuarial Science II* – 16 HT

Synopsis
Liabilities under a simple assurance contract or annuity contract. Premium reserves, Thiele's differential equation. Expenses and office premiums.

The no-arbitrage assumption, arbitrage-free pricing. Price and value of forward contracts, effect of fixed income or fixed dividend yield from the asset. Futures, options and other financial products.

Investment and risk characteristics of investments. Term structure of interest rates, spot rates and forward rates, yield curves. Stability of investment portfolios, analysis of small changes in interest rates, Redington immunisation.
Simple stochastic interest rate models, mean-variance models, log-normal models. Mean, variance and distribution of accumulated values of simple sequences of payments.

Reading
All of the following are available from the Publications Unit, Institute of Actuaries, 4 Worcester Street, Oxford OX1 2AW
• Subject 102[CT1] *Financial Mathematics Core Reading* Faculty & Institute of Actuaries .
• J.J. McCutcheon and W.F. Scott: *An Introduction to the Mathematics of Finance*. Heinemann (1986)
• H.U. Gerber: *Life Insurance Mathematics*. 3rd edition, Springer (1997)
• N.L. Bowers et al: *Actuarial mathematics*. 2nd edition, Society of Actuaries (1997)

3 **Other units and half units**

The other units and half units that students may take are drawn from Part B of the Honour School of Mathematics. For full details of these units and half-units, see the Syllabus and Synopses for Part B of the Honour School of Mathematics, which are available on the web at

http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/maths.shtml

3.1 *Mathematics units and half units*
The Mathematics units and half-units that are available are as follows:

B1 (or B1a or B1b) Logic and Set Theory
B2 (or B2a or B2b) Algebra
B3 (or B3a or B3b) Geometry
B4 (or B4a) Analysis
B5 (or B5a or B5b) Differential Equations and Applications
B6 (or B6a or B6b) Theoretical Mechanics
B7.1 (or B7.1a) Quantum Mechanics; Quantum Theory and Quantum Computers
B7.2 (or B7.2a) Relativity
B8 (or B8a or B8b) Topics in Applied Mathematics
B9 (or B9a) Number Theory
B10 (or B10a or B10b) Martingales and Financial Mathematics
B11 Communication Theory
C3.1 (or C3.1a or C3.1b) Lie Groups and Differentiable Manifolds
(M-Level)
C5.1a Partial Differential Equations for Pure and Applied Mathematicians (M-level)
C9.1a Analytic Number Theory (M-Level)
BE "Mathematical" Extended Essay

(These are the units referred to in Section 1, item (d), as 'Mathematics Department units and half units' for Part B of the Honour School of Mathematics.)

See the "Projects Guidance Notes" on the web at
http://www.maths.ox.ac.uk/current-students/undergraduates/projects/
for more information on the Extended Essay option and an application form.

3.2 *Other units and half units*
N1 Undergraduate Ambassadors' Scheme (a half-unit, details in the Syllabus and Synopses for Part B of the Honour School of Mathematics)

4. Registration for Part B courses 2007–2008

We ask that students register in advance for the classes they wish to take, by Monday of week 9 Trinity Term 2007, using the form overleaf.

Because of the large number of options which are available in Part B, some lectures will clash. See the Syllabus and Synopses for Part B of the Honour School of Mathematics for information on which lectures may clash.

REGISTRATION FORM: PART B CLASSES 2007-2008

SURNAME . . . . . .

FIRST NAME . . . . . .

EMAIL ADDRESS . . . . . .

COLLEGE . . . . . .

Note: As described in Section 1, you need to do a total of 4 units in Part B: all Mathematics and Statistics students do the unit BS1, and also at least one unit from the units and half-units available under BS2 and BS3.

For the Statistics units BS1–BS4, and the Mathematics or Other units, please give details of subjects in which you wish to take classes.

I wish to take classes in the following subjects: [Please Tick]

☑ BS1 Applied Statistics (MT and HT, compulsory for Mathematics and Statistics students)

☐ BS2 Statistical Inference (MT and HT)

☐ BS2a Foundations of Statistical Inference (MT, half-unit only)

☐ BS3 Stochastic Modelling (MT and HT)

☐ BS3a Applied Probability (MT, half-unit only)

☐ BS4 Actuarial Science (MT and HT)

For Mathematics or Other units or half-units, please list the unit or half-unit code and name:
Unit code Unit name

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Please return this form to the Academic Administrator, Department of Statistics, 1 South Parks Road, by Monday of week 9 Trinity Term 2007.