

**18. Title: Random forests on weighted vertices.
Proposer: Colin McDiarmid**

Brief Description: Consider the forests F on the vertex set $V = \{1, \dots, n\}$. Given positive weights $w(v)$ on the vertices v in V , let $\mu(F)$ be the product over the edges uv of F of $w(u)w(v)$.

This can also be written as the product over the vertices v in V of $w(v)$ raised to the power $\deg(v)$, where $\deg(v)$ denotes the degree of v in F . Thus

$$\mu(F) = \prod_{\text{edges } uv \text{ of } F} w(u)w(v) = \prod_{v \in V} w(v)^{\deg(v)}.$$

The aim is to investigate the properties of the random forest R , where the probability that $R=F$ is proportional to $\mu(F)$, in particular in relation to the uniform case when each weight $w(v)=1$. A natural focus is on the probability that R is connected.

Prerequisite courses: none, though graph theory and combinatorial optimisation would help.

theoretical project/simulation project

References:

Colin McDiarmid, Angelika Steger and Dominic Welsh, Random planar graphs, *J. Combinatorial Theory B* 93(2005) 187 -- 206.

Colin McDiarmid, Angelika Steger and Dominic Welsh, Random graphs from planar and other addable classes. in *Topics in Discrete Mathematics* (M. Klazar, J. Kratochvíl, M. Loebl, J. Matousek, R. Thomas, P. Valtr eds), *Algorithms and Combinatorics* 26, Springer, 2006, 231 -- 246.