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Mathematics and Statistics Undergraduate Handbook

Supplement to the Handbook

Honour School of Mathematics and Statistics
Syllabus and Synopses for Part B 2011–2012
for examination in 2012

Contents

BS4 syllabus updated September 2011

1. Honour School of Mathematics and Statistics

See the current edition of the Examination Regulations at
http://www.admin.ox.ac.uk/examregs/
for the full regulations governing these examinations. The examination conventions can be found at http://www.stats.ox.ac.uk/current_students/bammath/examinations

In Part B each candidate shall offer a total of four units from the schedule of units and half-units.

(a) Each candidate shall offer the unit BS1 (Applied Statistics).
(b) Each candidate shall offer a total of at least one unit from the units and half-units available under BS2 (Statistical Inference) and BS3 (Stochastic Modelling).
(c) Each candidate may offer a total of at most one unit from BS4 (Actuarial Science) and the schedule of 'Other units and half units'.
(d) Each candidate may offer at most one unit which is an Extended Essay or Structured Project.

Note: Units from the schedule of 'Mathematics Department units and half-units' for Part B of the Honour School of Mathematics are also available – see Section 3.

[Note regarding (b) above: Since there is only the half unit BS2a available under BS2, the following combinations (and only these) are permissible:
(i) BS2a and BS3a;
(ii) BS2a and BS3;
(iii) BS3.]

We ask that you register by the end of week 10 Trinity Term 2011 for classes for the Mathematics/ Statistics courses that you wish to take. A registration form is attached to these synopses. Some combinations of subjects are not advised and lectures in these subjects may clash. However, when timetabling lectures we will aim to keep clashes to a minimum.

Language Classes: If there are places available, Mathematics and Statistics students will be invited to apply to take classes in a foreign language. In 2011-2012 classes are offered in French and German. Students' performance in these classes will not contribute to the degree classification in Mathematics and Statistics. However successful completion of the course may be recorded on student transcripts. See http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/maths for further information.

BS4 Actuarial Science
In 2011-12 it is possible to take the half-unit BS4a without taking BS4b (this is a change from 2010-11). However, it is **not possible** to gain an exemption from the Institute of Actuaries paper CT1 if you do BS4a only. (It is not possible to do BS4b without BS4a.)

Part C projects 2012/2013
The Part C project allocation process may run differently to that in previous years. Further information will be available in Hilary Term 2012.

2 Statistics units and half units

2.1 BS1 Applied Statistics

Level: H-level
Method of Assessment: 2-hour examination plus assessed practical assignments. The practical assignments contribute 1/3 of the marks for BS1. Please see below for the hand-in deadlines for practical assignments.
Weight: One unit.

Prerequisites: Part A *Probability* and *Statistics*.

Aims
The course aims to develop the theory of statistical methods, and also to introduce students to the analysis of data using a statistical package. The main topics are: simulation based inference, practical aspects of linear models, logistic regression and generalized linear models, and robust and computer-intensive methods.

2.1.1 *Applied Statistics* I – 16 MT

Synopsis
Practical aspects of linear models and analysis of variance: review of multiple regression, model selection, fit criteria, use of residuals, outliers, leverage, Box-Cox transformation, model interpretation.
Logistic regression. Linear exponential families and generalized linear models, scale parameter, link functions, canonical link. Maximum likelihood fitting and iterated weighted least squares. Asymptotic theory: statement and applications to inference, analysis of deviance, model checking, residuals. Inference using simulation methods.

2.1.2 *Applied Statistics II* – 10 HT

Synopsis
Nonparametric inference. Permutation tests. Rank statistics. Robust estimation. Breakdown point. Robust and resistant regression. Smoothing methods (kernels, splines, local polynomials). Bootstrapping.

Reading (Michaelmas Term)
A. C. Davison, *Statistical Models*, CUP, 2003
A. J. Dobson, *An Introduction to Generalized Linear Models*, Chapman and Hall (1990)
D. Lunn, *Notes* (2003)

Reading (Hilary Term)
P. J. Rousseeuw and A. M. Leroy, *Robust Regression and Outlier Detection*, Wiley (1987), pp 1-194.
J. D. Gibbons, *Nonparametric Statistical Inference*, Marcel Dekker (1985), pp 1-193, 273- 290.
R. H. Randles and D. A. Wolfe, *Introduction to the Theory of Nonparametric Statistics*, Wiley (1979), pp 1-322.

Further Reading
F. L. Ramsey and D. W. Schafer, *The Statistical Sleuth: A Course in Methods of Data Analysis*, 2nd edition, Duxbury (2002)
W. N. Venables and B. D. Ripley, *Modern Applied Statistics with S*, Springer (2002)
L. Wasserman, *All of Nonparametric Statistics*, Springer (2004)

Practicals
In addition to the lectures there will be five supervised practicals. Four of these contain problems whose written solutions will be assessed as part of the unit examination.

The hand-in deadlines for the four assessed practicals are:
1st practical: 12 noon Tuesday week 8, Michaelmas Term 2011
2nd practical: 12 noon Tuesday week 2, Hilary Term 2012
3rd practical: 12 noon Tuesday week 7, Hilary Term 2012
4th practical: 12 noon Friday week 1, Trinity Term 2012.

Candidates who miss the above deadlines may ask their college to apply to the Head of the Department of Statistics for permission to submit late. Where there is a valid reason, the Head of Department would normally approve the late submission without penalty. Where it is deemed that there is no valid reason, the Head of Department will advise the Examiners to apply a penalty of at least 5% of the marks available for that practical.

2.2 **BS2a Foundations of Statistical Inference – 16 MT**

Level: H-level
Method of Assessment: 1 ½ -hour examination
Weight: Half-unit

Prerequisites: Part A *Statistics* course.

Learning outcomes
Understanding how data can be interpreted in the context of a statistical model. Working knowledge and understanding of key-elements of model-based statistical inference, including awareness of similarities, relationships and differences between Bayesian and frequentist approaches.

Synopsis
Exponential families: Curved and linear exponential families; canonical parametrization; likelihood equations. Sufficiency: Factorization theorem; sufficiency in exponential families.

Frequentist estimation: unbiasedness; method of moments; the Cramer-Rao information inequality; statement of the large sample distribution of the MLE; proof for curved exponential families assuming consistency.

The Bayesian paradigm: subjective probability; prior to posterior analysis; conjugacy; examples from exponential families. Choice of prior distribution: proper and improper priors; Jeffreys' and maximum entropy priors. Hierarchical Bayes models, graphical representation.

Computational techniques: Markov chain Monte Carlo methods; sampling importance resampling; data examples.

Decision theory: risk function; randomized decision rules; admissibility. Rao-Blackwell theorem: Rao-Blackwellization; illustration with squared error loss. Minimax rules, Bayes rules and admissibility. Hypothesis testing as decision problem.

Reading

P. H. Garthwaite, I. T. Jolliffe and Byron Jones, *Statistical Inference,* Second ed. Oxford University Press, 2002

G.A.Young and R.L. Smith, *Essentials of Statistical Inference*, Cambridge University Press, 2005.

T. Leonard and J.S.J. Hsu, *Bayesian Methods*, Cambridge University Press, 2005.

Further reading

D. R. Cox, *Principles of Statistical Inference*, Cambridge University Press, 2006

2.3 BS3 Stochastic Modelling

Level: H-level

Method of Assessment: 3-hour or 1 ½ -hour examination

Weight: One unit, or the first 16 lectures can be taken as a half-unit in Applied Probability. The second 16 lectures cannot be taken as a half-unit.

Prerequisites: Part A *Probability* for the first 16 lectures. Part A *Statistics* in addition for the second 16 lectures.

Aims*:* This unit has been designed so that a student obtaining at least an upper second class mark on the whole unit BS3 can expect to gain exemption from the Institute of Actuaries' paper CT4, which is a compulsory paper in their cycle of professional actuarial examinations. An Independent Examiner approved by the Institute of Actuaries will inspect examination papers and scripts and may adjust the pass requirements for exemptions. The first half of the unit, clearly, and also the second half of the unit, apply much more widely than just to insurance models.

2.3.1 *Applied Probability* – 16 MT [Option BS3a if taken as half-unit]

Aims

This course is intended to show the power and range of probability by considering real examples in which probabilistic modelling is inescapable and useful. Theory will be developed as required to deal with the examples.

Synopsis

Poisson processes and birth processes. Continuous-time Markov chains. Transition rates, jump chains and holding times. Forward and backward equations. Class structure, hitting times and absorption probabilities. Recurrence and transience. Invariant distributions and limiting behaviour. Time reversal.

Renewal theory. Limit theorems: strong law of large numbers, strong law and central limit theorem of renewal theory, elementary renewal theorem, renewal theorem, key renewal theorem. Excess life, inspection paradox.

Applications in areas such as: queues and queueing networks - M/M/s queue, Erlang's formula, queues in tandem and networks of queues, M/G/1 and G/M/1 queues;insurance ruin models; applications in applied sciences.

Reading

• J.R. Norris: *Markov Chains*. Cambridge University Press (1997)
• G.R. Grimmett and D.R. Stirzaker: *Probability and Random Processes*. 3rd edition, Oxford University Press (2001)
• G.R. Grimmett and D.R. Stirzaker: *One Thousand Exercises in Probability*. Oxford University Press (2001)
• S.M. Ross: I*ntroduction to Probability Models*. 4th edition, Academic Press (1989)
• D.R. Stirzaker: *Elementary Probability*. 2nd edition, Cambridge University Press (2003)

2.3.2 *Statistical Lifetime-Models* – 16 HT

Aims

The second half of the unit follows on from the first half on Applied Probability. Models introduced there are examined more specifically in the context of measuring 'lifetimes' in the broad sense. In a life insurance context Markov transitions may model the passage from 'alive' to 'dead', possibly with intermediate stages like 'loss of a limb' or 'critically ill'. The same models are used to model fertility transitions, the progress of a disease, and the reliability of a mechanical device. The aim is to develop statistical methods to estimate transition rates, and to use these transition rates to construct life tables that form the basis in the calculation of life insurance premiums and pension projections. We will also cover the basics of survival analysis, to model the influence of covariates (eg weight, smoking, use of a medication) on lifespans.

Synopsis

Life tables: Basic notation, life expectancy and remaining life expectancy, curtate lifetimes.
Census approximation, Lexis diagrams.
Survival models: general lifetime distributions, force of mortality (hazard rate), survival function, , the single decrement model and mortality in mixed populations.

Estimation procedures for lifetime distributions: empirical lifetime distributions, censoring and truncation, Kaplan-Meier estimate, Nelson-Aalen estimate. Parametric models, accelerated life models including Gompertz Weibull, log-normal, log-logistic. Plot-based methods for model selection. Cox regression. Proportional hazards, partial likelihood, semiparametric estimation of survival functions, use and overuse of proportional hazards in insurance calculations and epidemiology.
Two-state and multiple-state Markov models, with simplifying assumptions. Estimation of Markovian transition rates: Maximum likelihood estimators, time-varying transition rates, census approximation. Life expectancy and occupation times in Markov models: eigenvector formalism. Applications to reliability, medical statistics, ecology.

Graduation, including fitting Gompertz-Makeham model, comparison with standard life table: tests including chi-square test and grouping of signs test, serial correlations test; smoothness. Application to pension plans.

Reading
• Subject 104 [CT4] *Survival models [Modelling] Core Reading,* Faculty & Institute of Actuaries
• D.R. Cox and D. Oakes: *Analysis of Survival Data.* Chapman & Hall (1984)
Further Reading
• J.P. Klein and M.L. Moeschberger: *Survival Analysis*. Springer (1997)
• C.T. Le: *Applied Survival Analysis*. 2nd edition Wiley (2003)
• H.U. Gerber: *Life Insurance Mathematics*. 3rd edition, Springer (1997)
• N.L. Bowers et al.: *Actuarial mathematics*. 2nd edition, Society of Actuaries (1997)

2.4 **BS4 Actuarial Science**

BS4 syllabus updated September 2011

Level: H-level
Method of Assessment: 3-hour (BS4) or 1 ½ -hour examination for BS4a only.
Weight: One unit, or the first 16 lectures can be taken as a half-unit BS4a.
BS4a is a required prerequisite for BS4b.

Prerequisites: Part A Probability is useful, but not essential. If you have not done Part A Probability, make sure that you are familiar with Mods work on Probability.

Aims
This unit is supported by the Institute of Actuaries. It has been designed to give the undergraduate mathematician an introduction to the financial and insurance worlds in which the practising actuary works. Students will cover the basic concepts of discounted cash flows and applications, as well as models of financial decision making under uncertainty.

In the examination, a student obtaining at least an upper second class mark on the whole unit BS4 can expect to gain exemption from the Institute of Actuaries' paper CT1, which is a compulsory paper in their cycle of professional actuarial examinations. An Independent Examiner approved by the Institute of Actuaries will inspect examination papers and scripts and may adjust the pass requirements for exemptions.

Please note that it is **not possible** to gain an exemption from the Institute of Actuaries paper CT1 if you do BS4a only.

2.4.1 *Actuarial Science I* – 16 MT [Option BS4a if taken as half-unit]

Synopsis
Fundamental nature of actuarial work. Use of generalised cash flow model to describe financial transactions. Time value of money using the concepts of compound interest and discounting.

Interest rate models. Present values and accumulated values of a stream of equal or unequal payments using specified rates of interest. Interest rates in terms of different time periods. Equation of value, rate of return of a cash flow, existence criteria.
Loan repayment schemes. Investment project appraisal, funds and weighted rates of return. Inflation modelling, inflation indices, real rates of return, inflation-adjustments. Valuation of fixed-interest securities, taxation and index-linked bonds.
Price and value of forward contracts. Term structure of interest rates, spot rates, forward rates and yield curves. Duration, convexity and immunisation. Simple stochastic interest rate models. Investment and risk characteristics of investments.

Reading
All of the following are available from the Publications Unit, Institute of Actuaries, 4 Worcester Street, Oxford OX1 2AW

• Subject 102 [CT1] *Financial Mathematics Core Reading* Faculty & Institute of Actuaries.
• J.J. McCutcheon and W.F. Scott: *An Introduction to the Mathematics of Finance*. Heinemann (1986)
• P. Zima and R.P. Brown: *Mathematics of Finance*. McGraw-Hill Ryerson (1993)
• N.L. Bowers et al: *Actuarial mathematics*. 2nd edition, Society of Actuaries (1997)
• J. Danthine and J. Donaldson: *Intermediate Financial Theory*. 2nd edition, Academic Press Advanced Finance (2005)

2.4.2 *Actuarial Science II* – 16 HT

Synopsis
Theories of value, expected present value with/without asset sale, revealed preference, market value, no arbitrage value.
Expected Utility Theory (EUT), EUT axioms, St Petersburg Paradox, risk aversion, risk premium, certainty equivalent, EUT justification for insurance, changes in risk, first and second order stochastic dominance.
Desynchronisation of income and consumption streams and Edgeworth box. Portfolio theory. Mutuality principle, social welfare function and Wilson's Theorem. Consumption under certainty. The Capital Asset Pricing Model. Behavioural economics.
Departures from full risk sharing. Financial institutions as intermediaries, capital requirements, ruin models and ruin probabilities. The efficient markets hypothesis, index funds. Moral hazard, adverse selection.

Reading
• C. Gollier: *The Economics of Risk and Time*, MIT Press (2001), Chapters 1-4, 13, 15, 21-22
• Subject CT8: Financial Economics Core reading. Faculty & Institute of Actuaries, Units (i), (iii), (v)-(vi)
L. Eeckhoudt, C. Gollier and H.Schlesinger, *Economic and Financial Decisions under Risk*, Princeton University Press Princeton and Oxford, (2005)

Further reading
• Y. Lengwiler: Microfoundations of financial economics. Princeton University Press (2006), Chapters 4-5.

3 Mathematical and Other units and half units

The other units and half units that students in Part B Mathematics and Statistics may take are drawn from Part B of the Honour School of Mathematics.
For full details of these units and half-units, see the Syllabus and Synopses for Part B of the Honour School of Mathematics, which are available on the web at
http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/maths

3.1 *Mathematics units and half units*

The Mathematics units and half-units that are available are as follows:

B1a	Logic
B1b	Set Theory
B2a	Introduction to representation theory
B2b	Group Theory and an Introduction to Character Theory
B3a	Geometry of surfaces
BS3b	Algebraic curves
B3.1a	Topology and groups
B4a	Banach spaces
B4	Analysis (Banach spaces and Hilbert spaces) (full unit)
B5a	Techniques of applied mathematics
B5b	Applied partial differential equations
B5.1a	Dynamics and energy minimization
B6a	Viscous flow
B6b	Waves and compression flow
B7.1a	Quantum mechanics
or	
B7.1a/ C7.1b	Quantum mechanics and Quantum theory and quantum computers (full unit)
or	
B7.1a/B7.2b	Quantum mechanics and Special relativity and electromagnetism (full unit)
B8a	Mathematical ecology and biology
B8b	Nonlinear systems
B9a	Galois theory
B9	Number theory (Galois theory and algebraic number theory) (full unit)
B10a	Martingales through measure theory
B10b	Mathematical models of financial derivatives
B11a	Communication Theory
B21a	Numerical Solution of differential equations I
B21b	Numerical Solution of differential equations II

BE	"Mathematical" Extended Essay
or	
BSP	Mathematical Modelling and Numerical Computation Structured Projects

[Note: Students **cannot take both** BE and BSP]

(These are the units referred to in Section 1 as 'Mathematics Department units and half units' for Part B of the Honour School of Mathematics.)

See the "Projects Guidance Notes" on the web at
http://www.maths.ox.ac.uk/current-students/undergraduates/projects/
for more information on the Extended Essay option and an application form.

Please note that the following **are not permitted options** in Part B of the Honour School of Mathematics and Statistics:

O1	History of Mathematics
OE	''Other Mathematical'' Extended Essay
B22a	Integer Programming.

3.2 *Other units and half units*

The other unit or half unit available is as follows:

N1:	Mathematics Education and Undergraduate Ambassadors Scheme
or	N1a Mathematics Education (half-unit)

4. Registration for Part B courses 2011-2012

We ask that students register in advance for the classes they wish to take, by Friday of week 10 TrinityTerm 2011, using the form overleaf.

Because of the large number of options which are available in Part B, some lectures will clash. See the Syllabus and Synopses for Part B of the Honour School of Mathematics for information on which lectures may clash http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/maths .

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REGISTRATION FORM: PART B CLASSES 2011-2012

SURNAME . . . . . FIRST NAME . . . . . .

EMAIL ADDRESS . . . . . .

COLLEGE . . . . . .

Note: As described in Section 1, you need to do a total of 4 units in Part B: all Mathematics and Statistics students do the unit BS1, and also at least one unit from the units and half-units available under BS2 and BS3.

For the Statistics units BS1–BS4, and the Mathematics or Other units, please give details of subjects in which you wish to take classes.

I wish to take classes in the following subjects: [Please Tick]

☑ BS1 Applied Statistics (MT and HT, compulsory for Mathematics and Statistics students)

☐ BS2a Foundations of Statistical Inference (MT, half-unit only)

☐ BS3 Stochastic Modelling (MT and HT) (full unit)

- ☐ BS3a Applied Probability (MT, half-unit only)

☐ BS4 Actuarial Science (MT and HT) (full unit)

- ☐ BS4a Actuarial Science I (MT) (half-unit only)

For Mathematics or Other units or half-units, please list the unit or half-unit code and name:
Unit code Unit name

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Please return this form to the Academic Administrator, Department of Statistics, 1 South Parks Road, by Friday of week 10 Trinity Term 2011.