Mathematics and Statistics Undergraduate Handbook

Supplement to the Handbook

Honour School of Mathematics and Statistics
Syllabus and Synopses for Part A 2012–2013
for examination in 2013

Contents

Notice of misprints or errors of any kind, and suggestions for improvements in this booklet should be addressed to the Academic Administrator in the Department of Statistics.

September 2012

1. Honour School of Mathematics and Statistics

Please see the current edition of the Examination Regulations for the full regulations governing these examinations.

In Part A each candidate shall be required to offer the 4 written papers from the schedule of papers for Part A (below).

Part A shall be taken on one occasion only (there will be no resits). At the end of the Part A examination, a candidate will be awarded four 'University Standardised Marks' (USMs) for their performance in Part A – one USM for each paper taken in Part A. These USMs will be carried forward into the classification awarded at the end of the third year. In this classification, each paper mark in Part A will be given a 'weighting' of 2, and each unit paper mark in Part B will be given a 'weighting' of 3. All students who complete the first three years of the course will be classified, and those wishing to graduate at this point may supplicate for a BA.

Students wishing to take the four-year course should register to do so at the beginning of their third year. They will take Part C in their fourth year, be awarded a separate classification and, if successful, be allowed to supplicate for an MMath.

The schedule of papers

Paper AC1 Algebra, Analysis and Differential Equations
This paper will contain questions set on the CORE material, and will contain 9 short questions (3 for each course), attracting 10 marks each, all of which should be answered.

Paper AC2 Algebra, Analysis and Differential Equations
The paper AC2 contains 9 questions in total with 3 questions in each section, namely 3 on Algebra, 3 on Analysis and 3 on Differential Equations. Candidates may submit answers to as many questions as they wish. The best 4 questions will count for the total mark for this paper, with at least 1 from each section. That is, the best answer from each section together with the next best answer will count for the total mark for this paper.

Paper AS1 Probability, Statistics and Options
This paper will contain short questions, worth 10 marks each, set on the CORE material in Probability and Statistics, and on the OPTIONAL material. There will be 3 questions on each of Probability and Statistics, and on the options there will be 1 question for each 8 lecture course and 2 questions for each 16 lecture course. Candidates may submit answers to as many questions as they wish: the best 5 answers on Probability and Statistics, and the best 4 answers on options will be counted.

Paper AS2 Probability, Statistics and Options
This paper will contain longer questions, worth 25 marks each, set on the CORE material in Probability and Statistics, and on the OPTIONAL material. There will be 2 questions on each 16-lecture course and 1 question on each 8-lecture course. Candidates may submit answers to as many questions as they wish, at least 2 of which should be on Probability and Statistics. The best 2 answers on Probability and Statistics,

and the best 2 other answers (which may include further Probability and Statistics questions) will be counted.

On papers AS1 and AS2, *Statistical Programming* counts as an 8-lecture course: on each of AS1 and AS2, there will be 1 question on *Statistical Programming*.

Papers AC1 and AC2 are identical to those taken by candidates in Mathematics. Papers AS1 and AS2 are similar but not identical to the options papers (AO1 and AO2) taken by candidates in Mathematics.

Syllabus and Synopses

The syllabus details in this booklet are those referred to in the Examination Regulations and have been approved by the Statistics Academic Committee for examination in Trinity Term 2013. The synopses in this booklet give some additional detail, and show how the material is split between the different lecture courses. They also include details of recommended reading.The Part A examination **syllabus** is the mathematical material of the synopses, as separately detailed by paper below.

2. CORE MATERIAL

2.1 **Syllabi**

The examination syllabi of the two core papers AC1 and AC2 shall be the mathematical content of the synopses for the courses

Algebra
Analysis
Differential Equations

For the synopses for these courses, see those for Part A of the Honour School of Mathematics, which are available on the web at
http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/maths

The examination syllabi of the papers AS1 and AS2 include the core courses

Probability
Statistics.

2.2. **Synopses of lectures**

2.2.1 **Probability – 16 lectures HT**

Aims and objectives
The first half of the course takes further the probability theory that was developed in the first year. The aim is to build up a range of techniques that will be useful in dealing with mathematical models involving uncertainty. The second half of the course is concerned with Markov chains in discrete time and Poisson processes in one dimension, both with developing the relevant theory and giving examples of applications.

Synopsis
Continuous random variables. Jointly continuous random variables, independence, conditioning, bivariate distributions, functions of one or more random variables. Moment generating functions and applications. Characteristic functions, definition only. Examples to include some of those which may have later applications in Statistics.

Basic ideas of what it means for a sequence of random variables to converge in probability, in distribution and in mean square. Chebychev and Markov inequalities. The weak law of large numbers and central limit theorem for independent identically distributed variables with a second moment. Statements of the continuity and uniqueness theorems for moment generating functions.

Discrete-time Markov chains: definition, transition matrix, n-step transition probabilities, communicating classes, absorption, irreducibility, calculation of hitting probabilities and mean hitting times, recurrence and transience. Invariant distributions, mean return time, positive recurrence, convergence to equilibrium (proof not examinable). Examples of applications in areas such as: genetics, branching processes, Markov chain Monte Carlo. Poisson processes in one dimension: exponential spacings, Poisson counts, thinning and superposition.

Reading
G. R. Grimmett and D. R. Stirzaker, Probability and Random Processes (3rd edition, OUP, (2001). Chapters 4, 6.1-6.5, 6.8.
G.R. Grimmett and D. R. Stirzaker, One Thousand Exercises in Probability (OUP, 2001).
G. R. Grimmett and D J A Welsh, Probability: An Introduction (OUP, 1986). Chapters 6, 7.4, 8, 11.1-11.3.
J. R. Norris, Markov Chains (CUP, 1997). Chapter 1.
D. R. Stirzaker, Elementary Probability (Second edition, CUP, 2003). Chapters 7-9 excluding 9.9.

2.2.2 **Statistics – 16 lectures HT**

Synopsis
Order statistics, probability plots.

Estimation: observed and expected information, statement of large sample properties of maximum likelihood estimators in the regular case, methods for calculating maximum likelihood estimates, large sample distribution of sample estimators using the delta method.

Hypothesis testing: simple and composite hypotheses, size, power and p-values, Neyman-Pearson lemma, distribution theory for testing means and variances in the normal model, generalized likelihood ratio, statement of its large sample distribution under the null hypothesis, analysis of count data.

Confidence intervals: exact intervals, approximate intervals using large sample theory, relationship to hypothesis testing.

Probability and Bayesian Inference. Posterior and prior probability densities. Constructing priors including conjugate priors, subjective priors, Jeffreys priors. Bayes estimators and credible intervals. Statement of asymptotic normality of the posterior. Model choice via posterior probabilities and Bayes factors.
Examples: statistical techniques will be illustrated with relevant datasets in the lectures.

3 OPTIONS

3.1 **Syllabi**

The examination syllabi of the two papers of options, AS1 and AS2, also include the content of the synopses for the optional courses

Simulation
Linear Programming
Statistical Programming

The other optional subjects are drawn from Part A of the Honour School of Mathematics and are as follows:

Introduction to Fields
Group Theory
Number Theory
Integration
Topology
Multivariable Calculus
Calculus of Variations
Classical Mechanics
Quantum Theory
Fluid Dynamics and Waves
Numerical Analysis

For the synopses, see those for Part A of the Honour School of Mathematics, which are available on the web at
http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses

3.2 **Synopses of Lectures**

3.2.1 **Simulation** — 8 lectures TT

Aims and Objectives
Building on Part A probability and Mods statistics, this course introduces Monte Carlo methods, collectively one of the most important analytical tools of modern statistical inference.

Synopsis
Motivation. Inversion. Transformation methods.
Rejection.

Variance reduction via Importance sampling.

The Metropolis algorithm (finite space), reversibility, ergodicity.

Applications: conditioned and extreme events, likelihood and missing data, sampling uniformly at random.

Problem sheets in classes will include a separate section with some examples of simulation using R.

Reading
S.M Ross, *Simulation,* Elsevier, 4th edition, 2006
J.R Norris, *Markov Chains*, CUP, 1997
C.P. Robert and G Casella, *Monte Carlo Statistical Methods*, Springer, 2004
B.D Ripley, *Stochastic Simulation*, Wiley, 1987

3.2.2 **Linear Programming** — 8 lectures TT

Aims and Objectives
Linear programming is about making the most of limited resources. Specifically, it deals with maximising a linear function of variables subject to linear constraints. Applications range from economic planning and environmental management to the diet problem. The aim is to provide a simple introduction to the subject.

Synopsis
Linear programming problems, examples. Standardisation of problems, slack variables.
Sufficiency of basic feasible solutions; equivalence of basic feasible solutions and extreme points. The simplex method, excluding procedures to cope with degeneracy.

The dual problem, duality theorem (proof by analysing the simplex method), complementary slackness. Economic interpretation of dual variables, sensitivity analysis. Two person zero-sum games.

Reading
V CHVATAL, Linear Programming, Freeman (1983), Chapters 1–5, 15.
K TRUSTRUM, Linear Programming, RKP (1971) – out of print, but available in college libraries, Chapters 1–5.
D G LUENBERGER, Linear and Nonlinear Programming, Addison-Wesley (1984), Chapters 2–4.
D G LUENBERGER, Linear and Nonlinear Programming, Addison-Wesley (1984), Chapters 2–4.

3.2.3 **Statistical Programming – 6 lectures and associated practicals HT**

The workload of this course is equivalent to an 8-lecture course.

Aims and Objectives
The aim of the course is to introduce students to how to carry out statistical analysis using a computer and the theory of statistical programming and its related techniques. The course will be based on the R programming language which is now widely used in all branches of applied statistics.

Synopsis

(1) A basic introduction to using R to carry out basic numerical calculations, how numbers are represented and different data storage types used in R.

(2) Reading data into R and how to plot and summarize data.

(3) Writing functions and programs in R. Program control and debugging.

(4) Solving equations and numerical optimization.

(5) Solving equations. Runtime analysis.

(6) Simulation of stochastic processes.

Course Structure
The course will consist of 6 lectures, each followed by an associated practical session. There will also be up to 2 classes covering the theory of statistical programming

Reading
W.J. Braun and D.J. Murdoch, *A First Course in Statistical Programming with R*. CUP 2007