

Title: Queues, reversibility, random growth,

Proposer: James Martin

Prerequisite courses: Part B Applied Probability

Theoretical project in applied probability.

Brief Description:

Burke's Theorem states that the output process of an M/M/1 queue in equilibrium has the same distribution as the input process: both are Poisson processes (see Part B Applied Probability). This can be proved very nicely using reversibility arguments.

These arguments can be extended to much more general queues and to networks of queues. A particular case is a system of queues in tandem.

Using related arguments, one can say a lot about the way that a sequence of customers, all starting at the first queue at time 0, spreads through the system. This can be related to certain models of random growth of a cluster in two dimensions; the speed of the customers flowing through the system relates to the speed of growth of the growing cluster.

The project could start by reviewing the fundamental models and reversibility arguments. There are then extensions in many possible directions, which the project could explore.

One example, which could be an interesting direction for someone who had done Part B Communication Theory, would be the idea of a queue, say an M/M/1 queue, as a communication channel. The transmitter controls the arrival process to the queue, and the receiver sees only the departure process. At what rate can information be passed through such a channel?

How can this be extended to more general networks? (even the case of two such queues in tandem is not well understood.)

The project is a theoretical project in applied probability. It would be suitable for someone with a strong probability background. Ability to run simulations would be an advantage.