Fixation probabilities of beneficial mutations in organisms with sweepstakes recruitment

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A key result of theoretical population genetics is that the fixation probability of a mutation with relative fitness $1 + s$ present at frequency p in an population of size N is approximately

$$u(p) = \frac{1 - e^{-2Nsp}}{1 - e^{-2Ns}}.$$

This result depends on a diffusion approximation, which is valid when the population size is large, the absolute value of the selection coefficient is small, and the distribution of the number of offspring produced by any one individual has finite moments.

The aim of this project will be to study the effectiveness of natural selection in a population in which the reproductive output of individuals is so heavily skewed that the third of these assumptions is no longer appropriate. For example, this may be true of many marine organisms in which individual females are capable of laying millions of eggs; although most broods will be lost to predation or be swept into unsuitable habitat, individuals may occasionally win the reproductive 'sweepstakes' and contribute thousands or more offspring to the next generation.

Although there are many ways of modeling this scenario, we will consider a vanilla model in which, at some constant rate, a fraction x of the population dies and is replaced by the descendants of a single individual alive in the preceding generation. Our goals will be to: (1) determine how selection can be incorporated into this model (e.g., fecundity vs. viability selection), (2) to derive approximations for the fixation probability of a selected allele, and (3) to compare these approximations with values obtained by simulating the model directly.

This project would best suit a student with a strong background in stochastic processes and differential equations and an interest in population genetics.

References:

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