Examiners' Report: FHS Mathematics and Statistics Part A, Trinity Term 2011

October 2011

Part I

A. Statistics

- Numbers and percentages in each range

  Candidates are not classified in this examination, rather the marks awarded are carried forward for the use of next year's Part B examiners. In order to summarize performance in Part A we tabulate the distribution of candidates, by rounded average USM, in the ranges associated with the different classes. For comparison, the corresponding distributions for Part A in 2007-2010 are also given.

Range	Part A 2011	Part A 2010	Part A 2009	Part A 2008	Part A 2007
70-100	5 (16.7%)	13 (37.2%)	9 (21.4%)	9 (27.3%)	9 (25.7%)
60-69	18 (60%)	14 (40.0%)	23 (57.1%)	16 (48.5%)	16 (45.7%)
50-59	6 (20%)	4 (11.4%)	9 (19.1%)	5 (15.1%)	9 (25.7%)
40-49	1 (3.3%)	4 (11.4%)	1 (2.4%)	2 (6.1%)	1 (2.86%)
30-39	0	0 (0%)	0 (0%)	1 (3%)	0 (0%)
0-29	0	0 (0%)	0 (0%)	0 (0%)	0 (0%)
Exam incomplete	0	0 (0%)	0 (0%)	0 (0%)	0 (0%)
Total	30	35	42	33	35

There were no vivas and no double-marking. The same system of checking was used as in all parts of FHS Mathematics/Mathematics and Statistics.

B. Examining methods and procedures

- This was the eighth occasion on which this exam was set. The same procedure was followed in this examination as in the previous year. Details were available to candidates in the Mathematics and Statistics Exam conventions and first and second notices to candidates (attached).

- The Examiners calculate four USMs for each candidate, one for each of the papers AC1, AC2, AS1, AS2. The calculation of a USM is based linearly on the raw marks on that paper. The range in which a particular USM falls is required to have a definite meaning in terms of quality. Thus the range 60–69 must mean work of upper-second quality, while the much larger range 70–100 must mean work of first class quality. As in previous years, the Examiners adopted an algorithm based on a piecewise linear graph to convert raw marks to a USM for each paper.

- To assist them in arriving at the conversion for each paper, the Examiners have a range of material to take into account. As well as the scripts themselves, and the Examiners' own recent experience as internal examiners, they have statistical information on overall performance on the papers. They also have tables giving the distribution of candidates among classes in FHS Mathematics and related schools for past years. These are made available to the Examiners by the Division of Mathematical and Physical Sciences.

- When constructing the conversion algorithm, the Examiners took particular care to follow the effect of adjustments to the algorithm on the USMs of candidates at the bottom end of the distribution. While it is desirable to have a simple rule, uniformly applied, for converting raw marks to USMs, this care is the counterpart of the individual consideration formerly given to classifying candidates at the lower end. Since the algorithms established for candidates in mathematics are subsequently used to convert raw marks to USMs in the related joint schools, the Examiners paid attention to the effect of adjustments on candidates at the lower end of all four schools (Mathematics and its three joint partners).

- The Final Examiners' Meeting was a joint meeting of the Examiners in Mathematics, Mathematics and Philosophy, and in Mathematics and Statistics, both External Examiners being present. The same general algorithm was applied in the same way to the Mathematics and Statistics options papers (AS1 and AS2) and the Mathematics options papers (AO1 and AO2). However, it was decided in this case to scale the AS1 paper slightly differently from the AO1 paper. The students writing AS1 must answer 5 out of 6 probability and statistics questions, more than half of the 9 questions total that they answer for the paper; the mathematics students only have 4 probability and statistics questions, and none of the questions are compulsory. In the event, several of the probability and statistics questions were among the hardest, as judged by the average score of those who turned in solutions. It seemed plausible to suppose that this might have cost the average AS1 student about 2 points raw marks relative to the average AO1 student. In the end, we shifted the middle and bottom break-points for AS1 -- the ones that determine USM 57 and 37 -- by 2 points, but the top break-point for AS1 -- the one that determines USM 72 -- by just 1.5 points, based on a careful consideration of the particular papers that would be pushed above or below the first-class cutoff.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

D. Notice of examination conventions for candidates

The candidates were given details of the examination conventions both in a supplement to their handbooks and in the notices sent to them by the Examiners.

Part II

A. General Comments on the Examination

1. The papers were taken on Monday morning (AC1), Tuesday morning (AC2), Wednesday morning (AS1) and Thursday morning AS2) of 9th week of Trinity Term, June 21st-24th June, at Ewert House in Summertown.
2. All questions on AC1 and AC2 were composed and marked by the Internal Examiners. Questions, mark schemes and model answers were discussed initially with a second Internal Examiner, then with the whole panel of Internal Examiners and finally were sent for comments to the External Examiner. In arriving at the final form of these questions, the Examiners paid close attention to the published synopses and problem sheets, to the written guidelines on length and style of the short and long questions, and to last year's questions and the statistics of performance on them.
3. Draft questions on AS1 and AS2 were provided by the lecturers, following the usual pattern in other parts of Mathematics/Mathematics and Statistics finals. The drafts, again with mark schemes and model answers, and also with lecture synopses and problem sheets, were first discussed by the setter with one of the Internal Examiners, then by the whole panel of Internal Examiners, and then sent to the External Examiner. Again, close attention was paid to the guidelines, particularly as these covered the desired character of the short questions, and to performance on last year's questions. In all cases lecturers also acted as assessors, marking the questions on their courses.
4. No errors were found in the Mathematics or Mathematics and Statistics examinations.
5. The following points are made in relation to the examining process:

(a) As is often the case with FHS Mathematics, the process of collecting in scripts was quite intricate. It is important for the Chairman of Examiners to make his wishes for this process clear to the Senior Invigilator, who under the current rules is responsible for the conduct of this and all other aspects of the examination.

(b) The checking of marks, in the sense of the detailed checking of scripts against printed marks lists produced from the database, is a crucial ingredient of the examining process.

(c) Production of the Options papers relies on the cooperation of the lecturers. The

6. There is a single External Examiner for Mathematics and Statistics Part A who was asked to comment on papers AS1 and AS2 only. (The single External Examiner for Mathematics commented on papers AC1 and AC2.) We would like to thank Philip O'Neil, our External, and we are grateful to him for his input this year. He made helpful comments on the draft papers, and contributed in a helpful and constructive way at the Examiners' Meeting. He was impressively diligent in evaluating the borderline papers, to help guarantee our marking standards.
7. The database used in past years was used. The database functioned well at the Examiners' Meeting and the Examiners are grateful to Waldemar Schlackow for his work over the year.
8. The Examiners are also grateful for administrative and secretarial support, and especially to Sandy Patel and Jan Boylan.

B. Equal opportunities issues and breakdown of the results by gender

The table below shows numbers of male and female candidates in the different classification ranges in Part A Mathematics and Statistics.

Class	Female	Male	Total
First	2	3	5
Upper Second	10	8	18
Lower Second	4	2	6
Third	0	1	1
Pass	0	0	0
Fail	0	0	0
Exam incomplete	0	0	0
Total	16	14	30

C. Detailed numbers on candidates performance in each part of the exam

(a) It may be helpful to summarize the different papers.

Paper AC1 is taken by Mathematics candidates (165 this year) and Mathematics and Statistics candidates (30 this year). Questions are worth 10 marks each, candidates are instructed to attempt all nine questions.

Paper AC2 is also taken by Mathematics, and Mathematics and Statistics candidates. Questions are worth 25 marks each, candidates are instructed to attempt any number of questions, including at least one from each of the three sections. The best score from each section and then the best remaining question are counted.

Paper AS1 is taken by Mathematics and Statistics candidates only (though is very similar to Paper AO1 taken by Mathematics candidates). Questions are worth 10 marks each. Candidates are instructed that the best five questions on Probability and Statistics and the best four questions on options are counted.

Paper AS2 is also taken by Mathematics and Statistics candidates only (and is similar to Paper AO2 taken by Mathematics candidates). Questions are worth 25 marks each. Candidates are instructed that the best two Probability and Statistics questions and the best two other questions (which may include further Probability and Statistics questions) are counted.

(b) Means and standard deviations are given below in raw marks and USMs for each of the four papers and, for comparison purposes, for papers AO1 and AO2 (taken by Mathematics candidates only).

Paper	Num	Mean raw score	SD raw score	Mean USM	SD USM
AC1	195	64.8	12.2	65.4	9.4
AC2	195	71.7	16.0	66.0	10.0
AS1	30	60.3	11.9	64.1	9.1
AS2	30	69.1	14.2	65.5	8.6
AO1	165	63.2	13.8	64.9	10.6
AO2	165	69.0	16.2	65.7	10.5

(c) Here we give means, standard deviations and number of attempts on individual questions for Mathematics and Students students. The data for all mathematics and other students is included In the Mathematics examiners report.

Exam	Question	Mean	Mean Used	Stand. Dev.	# Used	Unused Attempts
AC1	Q1	7.4	7.4	1.96	30	0
AC1	Q2	6.03	6.03	2.34	30	0
AC1	Q3	5.17	5.17	2.87	24	0
AC1	Q4	6.93	6.93	1.80	30	0
AC1	Q5	6.63	6.63	2.01	30	0
AC1	Q6	5.9	5.9	1.49	30	0
AC1	Q7	7.41	7.41	2.21	29	0
AC1	Q8	8.47	8.47	1.43	30	0
AC1	Q9	7.87	7.87	1.63	30	0

Exam	Question	Mean	Mean Used	Stand. Dev.	# Used	Unused Attempts
AC2	Q1	16.1	16.4	4.95	19	1
AC2	Q2	14.8	16.2	6.61	10	3
AC2	Q3	13.3	22.0	10.2	2	2
AC2	Q4	11.5	11.5	8.23	4	0
AC2	Q5	15.6	15.8	5.33	19	1
AC2	Q6	12.6	14.3	5.78	10	2
AC2	Q7	16.9	16.9	4.0	22	1
AC2	Q8	18.9	18.9	3.1	14	0
AC2	Q9	17.2	17.5	4.42	20	1
AS1	C1	6.14	6.14	2.9	7	0
AS1	D1	6.4	6.4	1.43	10	0
AS1	D2	5.4	5.33	1.35	9	1
AS1	G1	8.77	8.77	1.34	22	0
AS1	H1	4.5	4.5	2.12	2	0
AS1	J1	10.0	10.0		1	0
AS1	K1	10.0	10.0		1	0
AS1	M1	7.52	7.81	2.49	26	1
AS1	M2	6.38	6.58	1.36	19	2
AS1	M3	5.92	6.1	1.85	24	2
AS1	O1	5.21	5.33	2.4	27	1
AS1	O2	7.7	7.7	2.02	29	0
AS1	O3	6.27	6.4	2.16	25	1
AS1	P1	8.69	8.69	2.06	13	0
AS1	P2	3.2	3.2	1.30	5	0
AS1	R1	7.39	7.39	2.17	23	0
AS1	S1	7.0	7.0	1.4	2	0
AS1	T1	5.3	5.5	2.96	18	2
AS1	U1	6.33	6.33	2.52	3	0
AS2	D3	1.0			0	1
AS2	D4	16.8	17.6	4.66	9	1
AS2	G2	12.7	13.6	5.76	8	2
AS2	J2	17.0	17.0		1	0
AS2	K4	16.0	16.0		1	0
AS2	M4	8.7	9.9	5.58	8	2
AS2	M5	15.2	16.3	4.54	15	2
AS2	O4	19.1	19.7	6.16	21	1
AS2	O5	17.9	18.2	4.79	24	2
AS2	P3	14.0	16.3	6.83	3	1
AS2	P4	21.7	21.7	4.39	7	0
AS2	R2	17.9	17.9	3.92	9	0
AS2	S2	10.0			0	1
AS2	T2	17.3	17.1	5.61	14	1

D. Comments on papers and individual questions

(i) AC1 and AC2

Comments on the Analysis, Algebra and Differential Equations questions are included in the FHS Mathematics Part A report.

(ii) AS1 and AS2

Comments on the questions in sections A–K and P are included in the FHS Mathematics Part A report.

O: Statistics

O1: Most candidates who did this question made good attempts, though some did nothing for (c). The most common source of error was the calculation of the expected information in (b): a number of candidates went wrong doing this – some were not sure which quantities were random variables when taking the expectation, others calculated the expectation incorrectly. (Some used the observed information, which was fine, and which avoids calculating an expectation.)

O2. Some candidates did not say what was being tested then stating the Neyman-Pearson lemma. This is important since the lemma applies to null and alternative hypotheses that are both simple. Part (b) was done well and (c) seemed to be a good final part to test who could apply (b). In (c) several candidates' conclusions depended on the value of 'α' rather than them concluding how much (or how little) evidence there was to reject/not reject H_0 without anyone telling them the value of 'α'.

O3: Almost all candidates did (a) and (b) well, and the majority made at least, reasonable attempts at (c) and (d).

O4: Most candidates who attempted this question knew how to approach finding MLEs via Lagrange multipliers. When working through the calculations, some were most successful when doing (a) than (b). Most attempts were fairly successful in getting as far as calculating the degrees of freedom correctly in (c) but made some errors or were not clear in what they wrote about the testing of H_0 when A =11.8.

O5: Most candidates who attempted this question got on well with it. Although a few marks were often lost on (d), part (c) was probably the least well answered part, eg several attempts suggested using the sample variance of the observed valeus of Y rather than using the residual sum of squares of the model. When the residual sum of squares was usesd, it was sometimes divided by an incorrect number of degrees of freedom.

M: Probability

M1: Most students who tried this question did well on most of it. This question seems to have been somewhat easier than the other two.

In part c, only a few students tried to do it with moment generating functions instead of the CLT. None of them applied mgf correctly. For those who did apply the CLT, the most common deficiency was to fail to mention that X_n could be represented as a sum of n independent random variables with Poisson distribution with parameter λ.

Part d(i) A few students misunderstood the question to be asking for real- world conditions that would justify modelling the arrivals with a Poisson process (rather than the mathematical definition of the Poisson process). This is not an absurd interpretation. In principle, they might have been given full credit, but none of them gave what could be considered complete conditions.

Part d(iii) A few students wrote a formula for the exact computation from the Poisson distribution, for which they received partial credit. Many students made small errors in the variance, particularly by a factor of $\sqrt{2}$. Otherwise, most hit one of the answers $\Phi(1)$, $\Phi(29.5/30)$, and $\Phi(29/30)$, which were treated as equivalently correct, since the continuity correction is not formally part of the syllabus.

M2: All but part (d) seem to have been fairly straightforward.

In part (a) several students confused the two states, so they computed $P\{X_3 = -1|X_1 = -1\}$. They received full credit if their work was complete enough to make it clear that this was a simple substitution error. Some students computed the third power of the matrix instead of the second, for which they received partial credit. A few seem to have assumed that $p+q$ must be equal to 1. This made several of their answers wrong, but they lost only one point in total if they followed it through consistently.

In part (b), a few students gave the convergence to the stationary distribution as a definition, which was not acceptable. It was taken for granted that the stationary distribution was a probability distribution, so no points were deducted if they wrote that $\sum \pi_x = 1$ but didn't mention that $\pi_x \geq 0$.

Many students made no attempt at part (c). Quite a few supposed that T was geometric with parameter 1/3 or 2/3. Some computed all the probabilities correctly, but then did not compute a probability generating function.

M3: Parts a and b were very straightforward. The main problem in part a was that some gave too-brief answers like the conditional density is $f(x,y)/f(y)$. Some description of what the letters mean would be required. In part b, very few know that $\cos^2$ s averages to 1/2 over a half period, most were able to compute the integral. They lost significant credit only when they came up with absurd answers, like that it was negative. Similarly, in part bii many students didn't notice that the expectation is 0 because the density is symmetric around 0, but most were able to do the integral correctly.

Part c was challenging. It was not an adequate answer to say X is normal because all linear combinations of normal random variables are normal. Some description of how this is proved, at a minimum, was required. Surprisingly few students were able to compute the variance correctly

Part dii was attempted by few, and only a couple of students recognised that they needed to make the covariance 0. And among the few who recognised that $E[Y|X=5]=5a+E[W]$, based on the previous part, all for some reason failed to recognise that $E[W]=0$.

M4: This was a somewhat technical question about generating functions, and less popular than M3. Among those who did answer it, there were few complete answers, but many with substantial numbers of points, and considerable dispersion among which portions of the question were troubling. The one common problem was part ai, which most students didn't even attempt, and few of the attempts were even plausibly in the right direction.

Part aii was problematic. The benefit of the characteristic function was straightforward. The other part was not so clear — reasonably so, since when the mgf exists, it's not substantially different from the cf. The model answer — that the mgf immediately led to the Chernoff bounds – was not obviously better than others, so it seemed reasonable to accept a variety of alternative answers that did not misrepresent the possible applications of the cf. Several students said that we could use the mgf for the same applications as the cf. when it existed, but without the complication of complex integration, which is plausible enough. On the other hand, answers that implied that the mgf but not the cf could be used for proving convergence of distributions were inadequate.

The variance in bii stymied quite a few. The most common error was to write $g''(1)-g'(1)^2$, forgetting that $g''(1)=E[T^2-T]$. A number of students confused g with a moment generating function, and so tried to derive expectations and variances from derivatives at 0, although it should have been clear that $g(0)\neq 1$.

Part bv was intended to lead on from iv, using the expectation of 2^T to derive the bound, but in fact it is possible to derive even better bounds for $P\{T \geq 6\}$ applying Markov's inequality directly to the expectation or $E[T^6]$. All were equally acceptable.

M5: A standard "story-based" Markov chain problem. Most students did well on most of the question.

Most students explained in some plausible way why X_n is a Markov chain and Y_n is not. A few merely restated the definition of a Markov chain without any specific application to the current circumstances.

Recurrence-transience was well defined by most. A few erroneously wrote something like: i is recurrent if $P\{X_n = i | X_0 = i\} = 1$ for some n (thus putting the quantifier in the wrong place.)

A number of people didn't try av and/or avi. Among those who did try av, most did well.

The new transition matrix in part b was correctly identified by most. Those who didn't most commonly made the mistake of keeping the state 0 and having the process first move into state 0, and only after that leave 0 for the positive states, which is not consistent with the description in the problem. (It is acceptable to include the state 0 but have no transitions into it.)

In part bii, hardly anyone seems to have noticed that it is necessary to explain why the chain is irreducible and aperiodic. Also, while most wrote down the equation that the stationary distribution needs to satisfy, relatively few did the necessary algebra to show that the given distribution does satisfy the condition.

R: Graph theory

R1: 23 students attempted the question that was generally well answered apart from a very few cases. The part of the question where students obtained lower marks was part (c) and the proof by induction.

R2: Only 9 students attempted this question and 5 of them obtained very good marks. The part of the question about bookwork was generally well answered whereas part (c) and (d), that presented new problems, obtained lower marks.

S: Simulation

S1: Only two students attempted this question, both with good answers losing a few marks here and there. Between them they scored all marks except for the mark for the simplification of the rejection condition.

S2: There was only one incomplete answer with a reasonable attempt at (a), but considerable problems, even with the bookwork parts of (b) and (c), quite possibly combined with lack of time.

T: Linear Programming

T1: This concerned a maximising LP problem with equality constraints, duality and complementary slackness.
Some perfect answers but there was the full range of possibilities down to little understanding.

T2: (Activity analysis, simplex method, duality and sensitivity) The LP had 2 constraints, and the simplex method required 2 pivots (if you followed the method suggested in lectures of picking the entering variable as one with maximum reduced profit) and 3 pivots otherwise. Mostly the question was straightforward and very well done, but disappointingly no one got the very last part (about the possible shadow prices).

U: Statistical Programming

Three students attempted the short question. Parts (a) and (c) were done reasonably well. Parts (b) and (d) less well. This question was very similar to one on the question sheets.

No students attempted the long question.

I suspect that students may still be wary of attempting questions from this course as there is no stockpile of past paper questions. Also, the type of question is rather different i.e. writing code, and this may put them off attempting the questions.

E. Comments on performance of identifiable individuals

[removed in web version]

F. Names of members of the Board of Examiners

Dr David Steinsaltz (Chairman), Dr Anne Henke, Professor Frances Kirwan, Dr Neil Laws, Professor Barbara Niethammer, Prof Philip O'Neill (External), Professor Paul Tod, Professor Elizabeth Winstanley (External)

Assessors

Prof C. Drutu, Prof A. Etheridge, Dr P. Howell, Prof F. Kirwan, Dr K. Kremnizer, Prof M. Lackenby, Dr J. Marchini, Dr S. Massa, Prof C. McDiarmid, Dr R. Norton, Dr J. Sparks, Prof E. Süli, Dr B. Szendroi, Prof U. Tillmann, Dr M. Winkel, Dr A. Zarnescu

The Examiners are grateful to all the assessors for their help and cooperation.

David Steinsaltz
Chairman of Part A Examiners
FHS Mathematics and Statistics
04/10/2011

Attached: Mathematics and Statistics Exam conventions 2010-2011. Part A First and Second notices to candidates.