Examiners' Report: FHS Mathematics and Statistics Part A, Trinity Term 2010

September 2010

Part I

A. Statistics

- Numbers and percentages in each range

  Candidates are not classified in this examination, rather the marks awarded are carried forward for the use of next year's Part B examiners. In order to summarize performance in Part A we tabulate the distribution of candidates, by rounded average USM, in the ranges associated with the different classes. For comparison, the corresponding distributions for Part A in 2007-2009 are also given.

Range	Part A 2010	Part A 2009	Part A 2008	Part A 2007
70-100	13 (37.14%)	9 (21.43%)	9 (27.3%)	9 (25.7%)
60-69	14 (40.00%)	23 (57.14%)	16 (48.5%)	16 (45.7%)
50-59	4 (11.43%)	9 (19.05%)	5 (15.1%)	9 (25.7%)
40-49	4 (11.43%)	1 (2.38%)	2 (6.1%)	1 (2.86%)
30-39	0 (0%)	0 (0%)	1 (3%)	0 (0%)
0-29	0 (0%)	0 (0%)	0 (0%)	0 (0%)
Exam incomplete	0 (0%)	0 (0%)	0 (0%)	0 (0%)
Total	35	42	33	35

• There were no vivas and no double-marking. The same system of checking was used as in all parts of FHS Mathematics/Mathematics and Statistics.

B. New examining methods and procedures

- A new lecture course on Statistical Programming had one question on AC1 and one on AC2. Only one candidate attempted the AC1 question.
- This was the seventh occasion on which this exam was set. The same procedure was followed in this examination as in the previous year. Details were available to candidates in the Mathematics and Statistics Exam conventions and first and second notices to candidates (attached).
- The Examiners calculate four USMs for each candidate, one for each of the papers AC1, AC2, AS1, AS2. The calculation of a USM is based linearly on the raw marks on that paper. The range in which a particular USM falls is required to have a definite meaning in terms of quality. Thus the range 60–69 must mean

work of upper-second quality, while the much larger range 70–100 must mean work of first class quality. As in previous years, the Examiners adopted an algorithm based on a piecewise linear graph to convert raw marks to a USM for each paper.

- To assist them in arriving at the conversion for each paper, the Examiners have a range of material to take into account. As well as the scripts themselves, and the Examiners' own recent experience as internal examiners, they have statistical information on overall performance on the papers. They also have tables giving the distribution of candidates among classes in FHS Mathematics and related schools for past years. These are made available to the Examiners by the Division of Mathematical and Physical Sciences.
- When constructing the conversion algorithm, the Examiners took particular care to follow the effect of adjustments to the algorithm on the USMs of candidates at the bottom end of the distribution. While it is desirable to have a simple rule, uniformly applied, for converting raw marks to USMs, this care is the counterpart of the individual consideration formerly given to classifying candidates at the lower end. Since the algorithms established for candidates in mathematics are subsequently used to convert raw marks to USMs in the related joint schools, the Examiners paid attention to the effect of adjustments on candidates at the lower end of all four schools (Mathematics and its three joint partners).
- The Final Examiners' Meeting was a joint meeting of the Examiners in Mathematics, Mathematics and Philosophy, and in Mathematics and Statistics, both External Examiners being present. The algorithm was applied in the same way to the Mathematics and Statistics options papers (AS1 and AS2) and the Mathematics options papers (AO1 and AO2).

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

D. Notice of examination conventions for candidates

The candidates were given details of the examination conventions both in a supplement to their handbooks and in the notices sent to them by the Examiners.

Part II

A. General Comments on the Examination

1. The papers were taken on Monday morning (AC1), Tuesday morning (AC2), Wednesday morning (AS1) and Thursday morning AS2) of 9th week of Trinity

Term, June 21st-24th June, at Ewert House in Summertown.

2. All questions on AC1 and AC2 were composed and marked by the Internal Examiners. Questions, mark schemes and model answers were discussed initially with a second Internal Examiner, then with the whole panel of Internal Examiners and finally were sent for comments to the External Examiner. In arriving at the final form of these questions, the Examiners paid close attention to the published synopses and problem sheets, to the written guidelines on length and style of the short and long questions, and to last year's questions and the statistics of performance on them.
3. Draft questions on AS1 and AS2 were provided by the lecturers, following the usual pattern in other parts of Mathematics/Mathematics and Statistics finals. The drafts, again with mark schemes and model answers, and also with lecture synopses and problem sheets, were first discussed by the setter with one of the Internal Examiners, then by the whole panel of Internal Examiners, and then sent to the External Examiner. Again, close attention was paid to the guidelines, particularly as these covered the desired character of the short questions, and to performance on last year's questions. In all cases lecturers also acted as assessors, marking the questions on their courses.
4. No errors were found in the Mathematics or Mathematics and Statistics examinations.
5. The following points are made in relation to the examining process:

(a) As is often the case with FHS Mathematics, the process of collecting in scripts was quite intricate. It is important for the Chairman of Examiners' to make his wishes for this process clear to the Senior Invigilator, who under the current rules is responsible for the conduct of this and all other aspects of the examination.

(b) It is very important that we continue to collect scripts broken up into sections by the candidates themselves, before handing-in. If the Examiners themselves were obliged to break up the scripts after collection, a great deal more time would be needed, and avoidable errors may be introduced.

(c) The checking of marks, in the sense of the detailed checking of scripts against printed marks lists produced from the database, is a crucial ingredient of the examining process.

(d) Production of the Options papers relies on the cooperation of the lecturers. The timetable is awkward, in that questions must be produced before the course has finished for Hilary Term lecturers, and before it has even started for Trinity Term lecturers. There were some minor delays, but overall this part of the process went very smoothly and our thanks are due to the lecturers for their assistance.

(e) No medical certificates were submitted by Maths and Stats students.

6. There is a single External Examiner for Mathematics and Statistics Part A who was asked to comment on papers AS1 and AS2 only. (The single External Examiner for Mathematics commented on papers AC1 and AC2.) We would like to thank Philip O'Neil, our External, and we are grateful to him for his input this year. He made helpful comments on the draft papers, and contributed in a helpful and constructive way at the Examiners' Meeting.
7. The database used in past years was used. The database functioned well at the Examiners' Meeting and the Examiners are grateful to Waldemar Schlackow for

his work over the year.

8. The Examiners are also grateful for administrative and secretarial support, and especially to Margaret Sloper, Sandy Patel and Jan Boylan.

B. Equal opportunities issues and breakdown of the results by gender

The table below shows numbers of male and female candidates in the different classification ranges in Part A Mathematics and Statistics.

Class	Female	Male	Total
First	6	7	13
Upper Second	6	5	11
Lower Second	2	5	7
Third	2	2	4
Pass	0	0	0
Fail	0	0	0
Exam incomplete	0	0	0
Total	16	19	35

C. Detailed numbers on candidates performance in each part of the exam

(a) It may be helpful to summarize the different papers.

Paper AC1 is taken by Mathematics candidates (152 this year) and Mathematics and Statistics candidates (35 this year). Questions are worth 10 marks each, candidates are instructed to attempt all nine questions.

Paper AC2 is also taken by Mathematics, and Mathematics and Statistics candidates. Questions are worth 25 marks each, candidates are instructed to attempt no more than five questions, the best four of which are counted.
Instructions were also to attempt at least one question from each section.

Paper AS1 is taken by Mathematics and Statistics candidates only (though is very similar to Paper AO1 taken by Mathematics candidates). Questions are worth 10 marks each. Candidates are instructed that the best five questions on Probability and Statistics and the best four questions on options are counted.

Paper AS2 is also taken by Mathematics and Statistics candidates only (and is similar to Paper AO2 taken by Mathematics candidates). Questions are worth 25 marks each. Candidates are instructed that the best two Probability and Statistics questions and the best two other questions (which may include further Probability and Statistics questions) are counted.

(b) Means and standard deviations are given below in raw marks and USMs for each of the four papers and, for comparison purposes, for papers AO1 and AO2 (taken by Mathematics candidates only).

Paper	avRaw	sdRaw	avUSM	sdUSM
AC1	54.31	17.24	64.4	11.69
AC2	49.94	14.82	62.6	10.07
AS1	54.69	18.17	65.6	13.68
AS2	68.94	20.21	68.97	14.8
AO1	54.82	15.15	65.77	10.83
AO2	70.6	17.43	69.95	12.73

(c) Here we give means, standard deviations and number of attempts on individual questions for Mathematics and Students students. The data for all mathematics and other students is included In the Mathematics examiners report.

School	Paper	QuestionName	Mean	MeanUsed	Std Dev	Used	Unused
M&S3	AC1	Q1	5.714286	5.714286	3.073333	35	0
M&S3	AC1	Q2	6.085714	6.085714	2.417722	35	0
M&S3	AC1	Q3	4.742857	4.742857	3.791188	35	0
M&S3	AC1	Q4	6.428571	6.428571	2.118347	35	0
M&S3	AC1	Q5	5.885714	5.885714	2.259248	35	0
M&S3	AC1	Q6	6.121212	6.121212	2.607173	33	0
M&S3	AC1	Q7	8.114286	8.114286	1.811263	35	0
M&S3	AC1	Q8	6.235294	6.235294	3.798395	34	0
M&S3	AC1	Q9	5.514286	5.514286	3.184257	35	0
M&S3	AC2	Q1	10.66667	12.6	6.831301	5	1
M&S3	AC2	Q2	13.38235	13.38235	5.045338	34	0
M&S3	AC2	Q3	14	14	5.09902	5	0
M&S3	AC2	Q4	13.03448	13.44444	5.341768	27	2
M&S3	AC2	Q5	8.882353	10	6.34313	15	2
M&S3	AC2	Q6	14.5	14.5	6.276057	10	0
M&S3	AC2	Q7	13.36667	13.75862	6.697829	29	1
M&S3	AC2	Q8	4.866667	5	2.065591	11	4
M&S3	AC2	Q9	9.6	12	5.412947	4	1
M&S3	AS1	A1	5	5		1	0
M&S3	AS1	B1	10	10		1	0

M&S3	AS1	C1	8	8	2	3	0
M&S3	AS1	D1	5.214286	5.214286	3.190818	14	0
M&S3	AS1	D2	4.1	4.1	1.969207	10	0
M&S3	AS1	E1	5.4	5.4	3.04959	5	0
M&S3	AS1	E2	5.5	5.5	2.12132	2	0
M&S3	AS1	G1	6	6	2.581989	7	0
M&S3	AS1	H1	3	3	2.828427	2	0
M&S3	AS1	K1	0.5	1	0.707107	1	1
M&S3	AS1	K2	5	5		1	0
M&S3	AS1	M1	5.6	5.588235	1.818209	34	1
M&S3	AS1	M2	6.266667	6.392857	2.303421	28	2
M&S3	AS1	M3	6.735294	6.735294	2.440924	34	0
M&S3	AS1	O1	5.71875	5.833333	2.865078	30	2
M&S3	AS1	O2	6.789474	6.789474	2.572913	19	0
M&S3	AS1	O3	6.884615	6.884615	2.688151	26	0
M&S3	AS1	P1	7.222222	7.222222	2.016274	18	0
M&S3	AS1	P2	7.75	7.75	1.912875	12	0
M&S3	AS1	R1	8.666667	8.75	1.527525	20	1
M&S3	AS1	S1	8.5	8.5	0.83666	6	0
M&S3	AS1	T1	5.869565	5.869565	2.436547	23	0
M&S3	AS1	U1	4	4		1	0
M&S3	AS2	D3	16.22222	17.375	6.64789	8	1
M&S3	AS2	D4	17	17		1	0
M&S3	AS2	E3	12.33333	12.33333	4.50925	3	0
M&S3	AS2	E4	11.5	11.5	4.949747	2	0
M&S3	AS2	G2	17.85714	17.85714	2.91139	7	0
M&S3	AS2	M4	16.58621	16.78571	6.213564	28	1
M&S3	AS2	M5	17.96296	18.11538	6.022276	26	1
M&S3	AS2	O4	8	11.66667	7.668116	3	3
M&S3	AS2	O5	20.75	20.75	4.45509	24	0
M&S3	AS2	P3	14	14		1	0
M&S3	AS2	P4	11	11	8.831761	6	0
M&S3	AS2	R2	17.13333	17.78571	6.717426	14	1
M&S3	AS2	S2	19.66667	24	7.505553	2	1
M&S3	AS2	T2	14.94118	14.73333	3.171565	15	2

D. Comments on papers and individual questions

(i) AC1 and AC2

Comments on the Analysis, Algebra and Differential Equations questions are included in the FHS Mathematics Part A report.

(ii) AS1 and AS2

Comments on the questions in sections A–K and P are included in the FHS Mathematics Part A report.

M: Probability

M1: Parts (a), (b)(i) and (b)(ii) are absolutely standard and most candidates did these well. Parts (b)(iii) and (b)(iv) are slightly more difficult and many candidates were unable to do these -- however (b)(iii) and an example similar but slightly different to (b)(iv) are in the lecture notes. The most popular answer to (b)(iii) was $2X_1$ which is wrong; some candidates wrote $X_1 + X_1'$ (which follows the notation in the notes) but failed to define X_1'. Few candidates got very far with the mixture of two exponential distributions in (c): there is an example in the notes of finding a random variable with mgf $\sum_j p_j M_j(t)$ (where $p_j > 0$, $\sum_j p_j = 1$), but few spotted that the given mgf is $(m_1(t) + m_2(t))/2$ where $m_i(t)$ is the mgf of an exponential with parameter i, for i=1,2. Of those who did spot this, only a few were able to get much further.

M2: Most candidates did (a), (b) and (c)(i) well. In particular, many took care in (b) to check that (Y_n) is Markov and in finding its transition matrix. A common error in (c)(ii) was to calculate the expected number of steps until reaching state 2, rather than determining the distribution of the number of steps until reaching state 2. The question was done well, though there were few marks of 9 or 10 out of 10.

M3: This question produced a fairly wide range of marks. Parts (a)-(c) are standard and were generally done well. Part (d) will probably have been new to candidates and as expected was found the hardest part of the question, though there were a good number of excellent answers to (d).

M4: This was a familiar type of question on change of variables and most candidates were able to make a good attempt at most of it. In (a)(i) a common minor error was to give the answer as $f(u)g(uv)u$ instead of $f(u)g(uv)|u|$. Part (b) was harder: many candidates could find the form of the density of U and V, but many went wrong when trying to determine the region on which the density is non-zero and when trying to find the marginal density of V.

M5: This is a standard question on Markov chains and was done well. Bookwork on Markov chains has not always been done well in past exams, but the bookwork here was generally done very well. Further, in (e)(i) many candidates chose to use induction to obtain the form of the stationary distribution and took care to prove inductively their claimed form for the stationary distribution. For those who got to the end of the question (e)(ii) proved to be surprisingly difficult: the answer is simply $1/\pi_0$ (after checking that the conditions of the relevant general result do hold here), but few candidates spotted this.

O : Statistics

S1. This was a question on the paired t-test, with an end part testing the student understanding of the connection between frequentist hypothesis testing and confidence intervals. The question had a large number of attempts. A few candidates, who perhaps were not familiar with the part A material, made attempts based only on applying theory from Mods statistics, in particular the central limit theorem, and gained little credit since they failed to introduce the t-distribution. Otherwise, the question was generally reasonably well answered, particularly the first bookwork parts, with quite a number of perfect solutions. A number of candidates were unable to derive the 1-sided confidence interval upper bound b, or made mistakes, such as claiming b=1.746. There were quite a number of fudged attempts, e.g with multiple sign switches, in the derivation of b. Those who succeeded at obtaining b usually also correctly described the resulting hypothesis testing. Most candidates applied the correct degrees of freedom and used a 1-sided approach, though there were some mistakes here including algebraic slips and also mistakes with which direction to use for the 1-sided confidence interval and test.

S2. This was a question on simple linear model theory, with a new application to estimating the radioactivity of an isotope. Again it has many attempts and was generally reasonably well answered, perhaps slightly better answered than O1. A few candidates wasted time deriving the least squares estimator (it was asked to be written down), and some lost marks on deriving the variance matrix, usually by giving insufficient detail in the solution, for example not stating or justifying the fact that the data **Y** variance matrix is simply $\sigma^2\mathbf{I}$. Some candidates struggled at a more basic level with the required matrix manipulations, e.g. treating matrices as if scalars. However most gained full marks on this part, and marks were generally instead lost by giving up before the end, or making mistakes in getting to the variance matrix for the example. Quite a few candidates were unable to formulate the linear model in the correct manner, especially by incorrectly thinking they needed to linearise the exponential function, or by simply stopping at this point. The last part was really quite well done by those that attempted it, either by using matrix-based application of results used in the earlier parts, or by direct calculation of the variance of a sum, both of which were valid.

S3. This question involved contingency table testing, applied to data on the number of boys among children in a large set of families. Some candidates got the number of degrees of freedom incorrect for the first part, especially using "d.f. = (2-1)x(3-1)=2" for the first part, rather than (correct) 3. Most, though not all, then still got the correct degrees of freedom for the alternative, and for the chi-squared test. A few candidates wrote down the likelihood ratio test instead of Pearson's statistic, or gave incomplete or wrong definitions: for example not showing the summation over entries in the table. The final part asked whether the data suggested a relationship between family size and the sex of the first two children – most noticed the data support this idea, but some claimed the opposite, arguing the data were not informative or (with more merit) suggesting additional testing would help.

S4. The first long question involved using standard techniques from the course – maximum likelihood theory – to derive a score test for independent binomial samples. Although relatively straightforward, this question saw few attempts, perhaps because there was relatively little bookwork, and O5 was much preferred. A number of the attempts were also of low standard and very incomplete, with candidates only attempting the first few parts, and no candidate scored full marks, with only a minority of candidates scoring over 15 marks. Most candidates were able to write down the likelihood correctly and obtain the m.l.e's of the binomial probabilities p and q. A number made mistakes in obtaining Fisher's information matrix, and did not give the joint normal asymptotic distribution of the m.l.e's correctly – e.g. giving the mean but failing to supply or derive the variance matrix, for which partial credit was awarded. The same issue arose for the distribution of the difference of the m.l.e's. A few candidates were unable to correctly derive the statistic Z of the question, but correctly constructed the test based on Z, gaining partial credit for this, reduced further for those that assumed α=0.05 rather than considering the general case. More credit was given in the case where the answer was correct apart from an error in obtaining the estimate of the variance of Z. Finally, some candidates correctly performed the hypothesis testing parts at the end of the question, even without deriving the test statistics, and gained marks for this, but many did not get this far.

S5. The second long format question was based on stating and proving the Neyman-Pearson Lemma (NPL) – a long bookwork proof not examined for some years, followed by a standard application to the exponential distribution, and a novel question on power for the derived test. This question proved popular and was often very well answered, with several perfect solutions and a majority of candidates attempting this question scoring 13 or higher. The statement of the NPL was generally accurate though some marks were lost through for example not clearly stating the null/alternative hypotheses, or not defining the likelihood. The proof was also usually well performed though mistakes were made throughout, some candidates becoming lost or confused in the algebra, taking short cuts resulting in omitting important steps, or failing to justify statements made (e.g. those reliant on the type I error of two potential tests both being equal to α). In the application to the exponential case, almost all wrote down the likelihood correctly. Some candidates did not begin by assuming point null and alternative hypotheses, instead confusing the NPL with the likelihood ratio statistic and incorrectly basing the deriviation on the m.l.e of λ. This gained at most two of five marks for this part. Other mistakes included incorrectly cancelling terms in the likelihood ratio for the null and alternative, that are not equal. A number of candidates failed to prove the test was uniformly most powerful (UMP) by moving from the point alternative hypothesis to the generalised one-sided alternative, and lost marks for this. Many candidates answered the last, new, parts based on power very well, which was pleasing. However some did not get beyond noticing the link with the Gamma distribution and others incorrectly applied this link, or confused α and $1-\alpha$ in obtaining c_α, the critical region boundary. Most who could correctly derive c_α then correctly derived the power function for the test.

R : Graph Theory

R1: This question was answered well by most candidates that attempted it. R1 (a) and (b) were not always done correctly, but they usually were.

R2: The bookwork part of the question in (i) was well done by candidates, but many candidates did not answer (v) (a) and (b) correctly.

S : Simulation

S1: The topic examined here (Markov chain Monte Carlo) is the last and most difficult topic of the topics taught in this 8 lecture Trinity term course. A student with a high mark on this question can design a MCMC algorithm for a scalar random variable. However, this question was similar to problems students had encountered in problem sheets (different target distribution, state space). The marks are high. I believe (from the real quality of the answers) that this is because the question has been chosen by a few very able students (of the approx. 10 that attended the final revision class).

S2: This question examines the rejection algorithm. Part (a) is bookwork, part (b) is new. I received two near-perfect answers and one incomplete. The quality of the two very high marking answers (careful and clearly laid out calculations of bounds etc) leads me to believe that these are two very able students, so that these marks, though high, are not out of proportion.

T : Linear Programming

T1: There were plenty of good answers to this question, but there were no marks of 10/10 and very few of 9/10. In (b) when candidates were asked to \explain carefully" how the given problem relates to the game, the explanations of several candidates were not careful enough. Part (c) was found to be the hardest part of the question and few attempts were close to being convincing.

T2: Part (a) was a combination of bookwork and a standard application and was done very well: candidates knew this material well and were able to score high marks on it. Part (b) was harder, with (b)(ii) being a fairly substantial unseen problem that candidates generally struggled with. So most candidates were able to score at least reasonable marks on the question via (a), but there were no really high marks on the question in view of (b).

U: Statistical Programming

U1: Only one candidate attempted this question on AC1 and obtained 4/10. The question on AC2 was not attempted.

E. Comments on performance of identifiable individuals

There are at present no prizes available in Part A, and nothing to report here.

F. Names of members of the Board of Examiners

Prof Robert Griffiths, Dr David Steinsaltz, Prof N M J Woodhouse, Prof B Niethammer, Dr Anne Henke, Dr A Day, Prof Philip O'Neill (Statistics External), Prof E Winstanley (Mathematics External).

Assessors
Dr A Day
Dr R Griffiths
Dr K Kremnizer
Dr. D Testa
Prof A Etheridge
Dr J Marchini
Dr P Neumann
Dr G Nicholls
Prof K Tod
Dr P Howell
Dr N Laws
Dr S Myers

The Examiners are grateful to all the assessors for their help and cooperation.

Robert Griffiths
Chairman of Part A Examiners
FHS Mathematics and Statistics
21/09/2010

Attached: Mathematics and Statistics Exam conventions 2009-2010. Part A First and Second notices to candidates.