Examiners' Report: FHS Mathematics and Statistics Part A, Trinity Term 2005

Part I

A Statistics

- Numbers and percentages in each range

  Candidates are not classified in this examination, rather the marks awarded are carried forward for the consideration of Examiners in subsequent years. What is tabulated in this section, in order to summarise performance in Part A, is the distribution of candidates by rounded average USM in the ranges associated with the different classes. For comparison, the final column is a pooled average of performance in 2004 FHS Mathematics Part 2 and 2003 FHS Mathematics Part 1 and Mathematical Sciences (the most recent complete cohort).

Range	Number	Percentages %	2004 cohort %
70-100	9	33.3	37.3
60-69	10	37.0	40.8
50-59	6	22.2	15.4
40-49	0	0	4.1
30-39	2	7.4	1.2
0-29	0	0	1.2
Total	27	99.9	100

- There were no vivas and no double-marking. The same system of checking was used as in all parts of FHS Mathematics.
- All candidates take all four papers, namely AC1, AC2, AS1 and AS2. Papers AC1 and AC2 contain respectively short and long questions on the compulsory core Mathematics courses, while AS1 and AS2 respectively contain short and long questions on Probabilty and Statistics and options.

B. New examining methods and procedures

- This was just the second occasion on which this exam was set. Where possible the same or similar procedures were followed in this examination as in other parts of FHS Mathematics.
- Initially, the Examiners calculate four USMs for each candidate, one for each of the papers AC1, AC2, AS1 and AS2. These USMs are calculated from the raw marks obtained on each paper using the same algorithms as used by the Examiners in FHS Mathematics Part A, and described in the Chairman's report for that school. The piecewise linear rule for converting raw marks to USMs is the same as that used for converting the marks of Mathematics Part A candidates.
- On paper AC1, ten marks were available for 'the assessment of mathematical presentation'. Since AC1 contains three sections, each marked by one of the Examiners, a presentation mark out of ten was assigned for each section and these were averaged to arrive at the actual presentation mark. These marks were regarded as USMs rather than raw marks and so were not subject to later rescaling.
- Papers AC2 and AS2 are marked out of 100. Because of the marks available for presentation, paper AC1 is marked out of 90. This year for the first time the raw mark for paper AS1 was a mark out of 90, that is to say the best nine answers contribute to the total mark.
- Candidates are permitted to proceed to Part B provided they pass Part A. Thus the Examiners are required to produce a Pass List. (Note that this is not a list of those deserving of honours.)
- The pass list for Part A Mathematics and Statistics is based on 4 USMs: the USMs for AC1 and AC2, and $\mathrm{USM}_{\mathrm{prob}}$ and $\mathrm{USM}_{\mathrm{other}}$ which relate to the combined performance on papers AS1 and AS2 on Probability and Statistics and on other options respectively. These USMs are calculated as weighted averages

$$\mathrm{USM}_{\mathrm{prob}} = p_1 \mathrm{USM}_{\mathrm{AS1}} + p_2 \mathrm{USM}_{\mathrm{AS2}}$$

and

$$\mathrm{USM}_{\mathrm{other}} = (1 - p_1)\mathrm{USM}_{\mathrm{AS1}} + (1 - p_2)\mathrm{USM}_{\mathrm{AS2}}$$

where $\mathrm{USM}_{\mathrm{AS1}}$ and $\mathrm{USM}_{\mathrm{AS2}}$ are the USMs for papers AS1 and AS2 and p_1 and p_2 are the proportions of raw marks that the candidate in question is awarded for Probability and Statistics questions on papers AS1 and AS2 respectively.

- The requirements for a candidate to pass Part A are that (a) the average of the USMs for AC1 and AC2, (b) the average of $\mathrm{USM}_{\mathrm{prob}}$ and $\mathrm{USM}_{\mathrm{other}}$, and (c) $\mathrm{USM}_{\mathrm{prob}}$ are all greater than or equal to 30.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

This year, as last year, the parameters of the raw mark to USM conversion algorithm were determined at the Final Meeting of the Mathematics Part A examiners. In both years some Mathematics and Statistics Part A examiners were also Mathematics Part A examiners, and so present when the conversion was determined. In future we recommend that the Mathematics and Statistics examiners who are not Mathematics examiners are also represented when the conversion is determined, in particular the Mathematics and Statistics external examiner. We suggest a joint meeting at which the parameters of the conversion algorithm are determined, attended by the Mathematics examiners, the Mathematics and Statistics external examiner and one further Mathematics and Statistics examiner.

This year, the calculation of $\mathrm{USM}_{\mathrm{prob}}$ and $\mathrm{USM}_{\mathrm{other}}$, via the method described above, produced a small number of $\mathrm{USM}_{\mathrm{prob}}$ values of greater than 100 (this is possible for candidates who score very highly overall and very highly on probability and statistics questions in particular). We dealt with this, in the only way we thought sensible, by assigning such candidates a $\mathrm{USM}_{\mathrm{prob}}$ value of 100 and adding the excess over 100 to their $\mathrm{USM}_{\mathrm{other}}$. We also noticed that the $\mathrm{USM}_{\mathrm{prob}}$ and $\mathrm{USM}_{\mathrm{other}}$ values seemed to give a less accurate reflection of a candidate's performance than simply reporting $\mathrm{USM}_{\mathrm{AS1}}$ and $\mathrm{USM}_{\mathrm{AS2}}$. To cope with both of these issues, we recommend that future examiners report $\mathrm{USM}_{\mathrm{AS1}}$ and $\mathrm{USM}_{\mathrm{AS2}}$ to each candidate, instead of $\mathrm{USM}_{\mathrm{prob}}$ and $\mathrm{USM}_{\mathrm{other}}$. That is, each candidate should receive four USMs: $\mathrm{USM}_{\mathrm{AC1}}$, $\mathrm{USM}_{\mathrm{AC2}}$, $\mathrm{USM}_{\mathrm{AS1}}$ and $\mathrm{USM}_{\mathrm{AS2}}$.

D. Notice of examination conventions for candidates

The candidates were given details of the examination conventions both in a supplement to their handbooks and in the notices sent to them by the Examiners and attached with this report.

Part II

Section A. General Comments on the Examination

1. The papers were taken on Thursday morning (AC1) and afternoon (AC2), and Friday morning (AS1) and afternoon (AS2) of 9th week of Trinity Term, June 24th and 25th, at Ewert House in Summertown.
2. All questions on the Compulsory Core papers, AC1 and AC2, were composed and marked by the Internal Examiners and one Assessor. Questions, mark schemes and model answers were discussed initially with a second Internal Examiner (and, in the case of one section, with the lecturer), then with the whole panel of Internal Examiners and finally were sent for comments to the External Examiner. In arriving at the final form of these questions,

the Examiners paid close attention to the published synopses and problem sheets, to the written guide-lines on length and style of the short and long questions, and to last year's questions as set and the statistics of performance on them.

3. Draft questions on the Options papers, AS1 and AS2, were provided by the lecturers, following the usual pattern in other parts of maths finals. The drafts, again with mark schemes and model answers, and also with lecture synopses and problem sheets, were first discussed by the setter with one of the Internal Examiners, then by the whole panel of Internal Examiners, and then sent to the External Examiner. Again, close attention was paid to the guide-lines, particularly as these covered the desired character of the short questions, and to performance on last year's questions. Setters were reminded of performance on last year's questions, with the intention that this would reduce variation between options.

   The lecturers also acted as assessors, marking the questions on their courses.

4. No points on the conduct of the examination separate from those in the FHS Mathematics Part A report need to be made.

5. There is a single External Examiner for Part A who was asked to comment on papers AS1 and AS2 only. We were well-served by Professor Taylor, our External, and are grateful to him. He found errors and ambiguities in the draft papers, and contributed in a helpful and constructive way at the Examiners' Meeting.

6. The data-base for the Part A examination was revised for this year by David Mayers. The data-base functioned well at the Examiners' Meeting and the Examiners are grateful to David for his work over the year.

7. The Examiners are also grateful for administrative and secretarial support to Jan Boylan, Ana Fraser, Keith Gillow, Nick Iles, Maria Moreno, Christine Stone and Sue Wood.

B. Equal opportunities issues and breakdown of the results by gender

The table below shows number and percentages of male and female candidates in the different ranges of average USM in Part A Mathematics and Statistics.

	Male	Female	Total
70-100	4 (31%)	5 (36%)	9 (33%)
60-69	4 (31%)	6 (43%)	10 (37%)
50-59	4 (31%)	2 (14%)	6 (22%)
40-49	0 (0%)	0 (0%)	0 (0%)
30-39	1 (7%)	1 (7%)	2 (7%)
0-29	0 (0%)	0 (0%)	0 (0%)
Total	13	14	27

C. Detailed numbers on candidates performance in each part of the exam

1. It may be helpful to summarise what the rubrics for the different papers call for, and to note how far candidates followed these rubrics.

   For paper AC1, candidates are instructed to attempt all nine questions. All but ten did so. Of these three attempted eight questions, five attempted seven questions, one attempted six questions and one attempted five questions.

   For paper AC2, candidates are instructed to attempt no more than five questions, of which the best four are credited. The average number attempted was 4.5, and all candidates except one received credit for four answers.

   For paper AS1, candidates are instructed to attempt no more than ten questions, of which the best nine are credited. The average number attempted was 8.7. Last year, the rubric for this paper called for ten attempts but the average was 9.5. Lowering the requirement to nine has made the target easier to reach and, as we shall see, the average raw mark on the paper has actually increased.

   For paper AS2, as for AC2, candidates are instructed to attempt no more than five questions, of which the best four are credited. All candidates except received credit for four answers.

   In summary, the rubrics were followed quite closely. A few candidates hand in attempts to more questions than the upper limit specified in the rubric.

2. Means and standard deviations are given below in raw marks and USMs for each of the four papers.

Paper	aveRaw	sdRaw	aveUSM	sdUSM
AC1	49.1	21.1	60.3	17.8
AC2	56.7	20.8	62.5	16.1
AS1 Prob	36.5	10.1		
AS2 Prob	40.6	14.4	80.9	14.0
AS1 Other	24.7	10.2		
AS2 Other	28.7	12.4	59.3	20.2
AS1 Total			70.1	15.8
AS2 Total			70.0	12.8

   As noted above, the raw mark on AC1 is a mark out of 90, as there were ten marks available for presentation. The average presentation mark was 5.9 and the aveUSM on AC1 includes the presentation mark. Taking account of that, the average raw marks on all papers are close to where the Examiners were aiming. Last year, aveRaw on AS1 was 56.3, this year it is 61.2. This year it is higher as a mark out of 90 than last year as a mark out of 100. Candidates have been able to do more and this is a good vindication of the decision to drop the number of questions contributing to the total mark from 10 to 9.

In comparison with Mathematics candidates, Mathematics and Statistics candidates averaged slightly lower USMs on AC1/AC2, and slightly higher USMs on AS1/AS2 than Mathematics candidates on the corresponding Mathematics options papers. This is not too surprising given that Mathematics and Statistics have probably chosen Mathematics and Statistics rather than Mathematics because of their preference for subjects such as probability and statistics which appear on AS1/AS2.

3. The algorithms converting raw marks to USMs for each paper are given in the Chairman's report for FHS Mathematics Part A.

4. Here we give means, standard deviations and number of attempts on individual problems. On papers AC2 and AS2 both the number of attempts for which credit was given and the number of 'unused' attempts is presented (recall that only the best 4 answers are credited to the total).

   For paper AC1, where each question is marked out of ten, and all questions should be attempted (by 27 candidates):

Subject	Question	rawAve	rawSD	Attempts
Algebra	1	5.22	3.14	27
	2	4.36	2.94	25
	3	6.42	2.46	19
Analysis	4	5.68	2.93	25
	5	7.69	2.45	26
	6	6.38	2.95	24
DiffEqns	7	8.04	2.43	27
	8	4.07	2.89	27
	9	5.78	3.67	23

   For paper AC2, where each question is marked out of 25:

Subject	Question	rawAve	rawSD	Attempts	Unused
Algebra	1	13.45	6.06	11	1
	2	13.68	5.21	19	1
	3	15.20	9.23	5	0
Analysis	4	17.62	4.89	13	0
	5	15.74	6.38	19	1
	6	12.33	5.31	12	0
DiffEqns	7	17.14	7.31	7	5
	8	12.40	8.02	15	2
	9	11.00	3.03	6	5

   For paper AS1, where each question is marked out of ten:

Subject	Question	rawAve	rawSD	Attempts
Groups in action	A1	7.00	3.24	9
Intro to Fields	B1	7.29	3.04	7
Number Theory	C1	6.00	2.00	3
Integration	D1	3.33	3.21	3
	D2	4.50	2.12	2
Topology	E1	9.33	1.15	3
	E2	3.00	1.41	2
Multivble Calc	F1			0
Calc. of Variations	G1	7.11	2.52	9
Class. Mech.	H1	6.50	4.95	2
Quant. Mech.	J1	6.00	1.41	2
Fluids & Waves	K1	7.00	3.61	3
	K2	6.67	4.93	3
Prob. (Laws)	M1	8.37	1.88	27
	M2	6.73	3.51	22
Prob. (Donnelly)	N1	8.57	1.73	23
Statistics	O1	6.81	3.23	21
	O2	6.55	3.46	20
	O3	7.37	2.59	19
Numerical Analysis	P1	8.00	0.00	2
	P2	7.50	2.12	2
Graph Theory	R1	7.17	3.17	18
Comb. Opt.	S1	1.00	0.00	2
	S2	8.60	1.95	5
Lin. Prog.	T1	6.68	2.87	22

A question with an average much above 6.5 can be regarded as high-scoring and much below as low-scoring. Problems on Statistics scored much closer to the average this year than last.

For paper AS2, where each question is marked out of 25:

Subject	Question	rawAve	rawSD	Attempts	Unused
Groups in action	A2	23.50	1.76	6	0
Intro to Fields	B2	20.00		1	0
Number Theory	C2	25.00		1	0
Integration	D3	15.33	7.64	3	0
	D4	14.00		1	0
Topology	E3	11.00	2.83	2	0
	E4			0	0
Multivble Calc	F2			0	0
Calc. of Variations	G2	11.33	6.66	3	0
Class. Mech.	H2	16.00		1	0
Quant. Mech.	J2	7.00		1	0
Fluids & Waves	K3	25.00		1	0
	K4	24.00		1	0
Prob. (Laws)	M3	19.13	5.00	23	2
Prob. (Donnelly)	N2	12.60	6.85	10	0
Statistics	O4	18.50	5.73	12	1
	O5	18.18	4.81	17	1
Numerical Analysis	P3			0	0
	P4			0	0
Graph Theory	R2	16.22	4.94	9	1
Comb. Opt.	S3			0	0
	S4	8.50	3.54	2	0
Lin. Prog.	T2	16.93	6.63	14	0

The averages are better this year than last in Statistics and Probability.

Section D. Comments on papers and individual questions

(i) AC1 and AC2

Comments on the Analysis, Algebra and Differential Equations questions are included in the FHS Mathematics Part A report.

(ii) AS1 and AS2

Comments on the questions in sections A–K and P are included in the FHS Mathematics Part A report.

M: Probability (Laws)

- M1 A standard and straightforward question on which candidates generally got high marks. Calculating the m.g.f. of X_n caused the most problems: the majority of candidates managed it in the end, but many did not use the most efficient method.
- M2 Although slighly less popular, and slightly lower scoring than question M1, this question was answered fairly well.
- M3 This was a familiar type of question involving the transformation of a pair of random variables. The last part seemed to discriminate reasonably well. Many candidates showed good understanding by producing high scoring answers.

N: Probability (Donnelly)

- N1 Candidates answered this question very well. Across the Maths, and Maths and Stats, papers there were 108 attempts. More than half the candidates obtained full marks, and more than 75% of the candidates received 8 or more marks.
- N2 There were 39 attempts at this question across the papers. Half the candidates achieved 16 or more marks, with 1/4 of the candidates scoring 20 or more.

  The most disappointing aspect of the solutions was the bookwork. Very few candidates successfully answered the second part of the bookwork (that recurrence and transience were class properties), although the proof given in lectures is only a few lines, and good intuition would allow the construction of an *ad hoc* proof. Even the first part of the bookwork (definitions of transience, recurrence, positive and null recurrence) was not particularly well-answered.

  Candidates did well in deriving the transition probabilities for the Markov chain described in words in the question, which was very encouraging (although most omitted to give an argument for the Markov property). Most at least derived the expressions given in the question for the stationary probabilities π_2 and π_3, and many made considerable further progress on the general case.

  A surprising feature was the non-trivial number of candidates who did not seem to know how to sum a geometric progression.

O: Statistics

Both the short and the long questions seemed to be at an appropriate level of difficulty. Almost all candidates who made serious attempts at the questions had a good idea of the material and got good marks.

- O1 This was a straightforward question: many candidates knew the material well and scored highly; most of the others showed some understanding, by doing one or two of the three parts of the question well.
- O2 I think this question was at an appropriate level of difficulty, but I suspect the average mark for the question will reflect a reasonable number of not very serious attempts. About half of the attempts were very good (say 8 or more marks out of 10), but quite a few of the other attempts did very little of the question (e.g. attempted only to define a confidence interval). A common error in the second part was to ignore the fact that the variance σ^2 was unknown.
- O3 Students were generally able to state the Neyman-Pearson lemma and most had a good idea of how to use it to construct the critical region of the test.
- O4 Although the material in this quesiton is not particularly easy, most attempts were good and scored well. Most candidates were able to make a good attempt at almost all of the question. The last part of the question was the hardest and more often than not candidates failed to see that the normal distribution of $\hat{\gamma}$ could be used for the required hypothesis test.
- O5 Many of the attempts at this question were good. Candidates who attempted the question could generally attempt all of it, but tended to lose marks in various places by not giving a sufficiently detailed answer.

R: Graph Theory

- R1 This question was generally well done by those candidates who attempted it seriously.
- R2 Ten attempted the question, with a reasonable understanding of Hall's marriage problem and bipartite graphs. The part requiring calculation in a specific graph was well done. A single candidate correctly answered the unseen part of the question.

S: Combinatorial Optimisation

- S1 This was a standard type of scheduling question. There were only two very weak attempts.
- S2 This question was reasonably popular. Those attempting it knew their way around the required method, and produced good high-scoring answers.
- S3 No attempts.
- S4 Most of this question is standard. However it was not popular, and the attempts made did not cover the definitions/bookwork very accurately.

T: Linear Programming

- T1 A standard question on some basic theory - hopefully about right for a short question. The question was popular and scored resonably high marks.
- T2 Quite a popular long question, and again generally well done.

Section E. Comments on performance of identifiable individuals

There is nothing to report here.

F. Names of members of the Board of Examiners

Dr C N Laws (Chairman), Prof G D Reinert, Dr A Dancer, Prof K P Tod, Prof C Taylor (external).

Assessors for Papers AS1 and AS2:

Dr D J Acheson, Dr T Browning, Dr W A Day, Prof P Donnelly, Prof R C Griffiths, Dr K Hannabuss, Dr G Luke, Prof P K Maini, Dr W A Sutherland, Prof M Vaughan-Lee, Dr G Vincent-Smith, Dr A Wathen, Dr G Wilson.

The Examiners are grateful to all the assessors for their help and cooperation.

Neil Laws

Chairman of Part A Examiners

14.10.2005