Honour School of Mathematics and Statistics

Supplement to the Mathematics and Statistics Undergraduate Handbook
Syllabus and Synopses for Part A 2008–2009 for examination in 2009

Contents

Notice of misprints or errors of any kind, and suggestions for improvements in this booklet should be addressed to the Academic Administrator in the Department of Statistics.

1. Honour School of Mathematics and Statistics

Please see the current edition of the Examination Regulations for the full regulations governing these examinations.

In Part A each candidate shall be required to offer the 4 written papers from the schedule of papers for Part A (below).

Part A shall be taken on one occasion only (there will be no resits). At the end of the Part A examination, a candidate will be awarded four 'University Standardised Marks' (USMs) for their performance in Part A – one USM for each paper taken in Part A. These USMs will be carried forward into the classification awarded at the end of the third year. In this classification, each paper mark in Part A will be given a 'weighting' of 2, and each unit paper mark in Part B will be given a 'weighting' of 3. All students who complete the first three years of the course will be classified, and those wishing to graduate at this point may supplicate for a BA.

Students wishing to take the four-year course should register to do so at the beginning of their third year. They will take Part C in their fourth year, be awarded a separate classification and, if successful, be allowed to supplicate for an MMath.

1.1 The schedule of papers

Altogether, these papers will include 1 short question and 1 longer question for each 8 hour lecture course; 2 short questions and 2 longer questions for each 16 hour lecture course; 3 short questions and 3 longer questions for each 24 hour lecture course.

Paper AC1 Algebra, Analysis and Differential Equations
This paper will contain questions set on the CORE material, and will contain 9 short questions (3 for each course), attracting 10 marks each, all of which should be answered.

Paper AC2 Algebra, Analysis and Differential Equations
This paper will contain questions set on the CORE material, and will contain 9 longer questions (3 for each course), attracting 25 marks each. Candidates may answer at most 5 questions with at least one from each section on Algebra, Analysis and Differential Equations. The best answer from each section plus the next best answer will count for the total mark for this paper. This is a change with effect from October 2008.

Paper AS1 Probability, Statistics and Options
This paper will contain short questions, worth 10 marks each, set on the CORE material in Probability and Statistics, and on the OPTIONAL material. There will be 3 questions on each of Probability and Statistics, and on the options there will be 1 question for each 8 lecture course and 2 questions for each 16 lecture course. At most 10 questions should be answered: the best 5 answers on Probability and Statistics, and the best 4 answers on options will be counted.

__Paper AS2 Probability, Statistics and Options__
This paper will contain longer questions, worth 25 marks each, set on the CORE material in Probability and Statistics, and on the OPTIONAL material. At most 5 answers may be submitted, at least 2 of which should be on Probability and Statistics: the best 2 answers on Probability and Statistics, and the best 2 other answers (which may include further Probability and Statistics questions) will be counted.

Papers AC1 and AC2 are identical to those taken by candidates in Mathematics. Papers AS1 and AS2 are similar but not identical to the options papers (AO1 and AO2) taken by candidates in Mathematics.

1.2 **Marking of Papers**

Mark schemes for questions out of 10 will aim to ensure that the following qualitative criteria hold:

9-10 marks: a completely or almost completely correct answer, showing good understanding of the concepts and skill in carrying through arguments and calculations; minor slips or omissions only.

5-8 marks: a good though not complete answer, showing understanding of the concepts and competence in handling the arguments and calculations.

Mark schemes for questions out of 25 will aim to ensure that the following qualitative criteria hold:

20-25 marks: a completely or almost completely correct answer, showing excellent understanding of the concepts and skill in carrying through arguments and/or calculations; minor slips or omissions only.

13-19 marks: a good though not complete answer, showing understanding of the concepts and competence in handling the arguments and/or calculations. In this range, an answer might consist of an excellent answer to a substantial part of the question, or a good answer to the whole question which nevertheless shows some flaws in calculation or in understanding or in both.

This should be regarded only as a guide conveying the intentions of the examiners.

1.3 **Parts B and C**

__Examination papers__
Examinations for whole unit papers are of 3 hour duration and half unit papers are of 1 ½ hour duration. The rubrics have been revised for 2009.

__Rubrics on 3-hour examination papers__
The rubric on 3-hour examination papers will usually be: “candidates may submit as many questions as they wish; the best two from each section will count for the total mark”.

In the exam, on each half-unit there will be 3 questions set.

BS1 Applied Statistics
This will be examined via a 2-hour examination paper plus assessed practical assignments. BS1 will have 3 questions on the Michaelmas term material and 2 questions on the Hilary term material. For the 2-hour BS1 paper, a candidate's Hilary term section will count for the total mark.

Rubrics on 1 ½ hour examination papers
The rubric on 1 ½ -hour examination papers will usually be: "candidates may submit as many questions as they wish; the best two will count for the total mark".
Number of questions set per half-unit
In the exam, on each half-unit there will be 3 questions set.

1.3.1 **Analysis of marks**

Part A
At the end of the Part A examination, a candidate will be awarded a University standardised mark (USM) for each of the four papers. The Examiners will recalibrate the raw marks to arrive at the USMs reported to candidates. In arriving at this recalibration, the examiners will principally take into account the total sum over all four papers of the marks for each question, subject to the rubric on each paper.

The Examiners aim to ensure that all papers and all subjects within a paper are fairly and equally rewarded, but if in any case a paper, or a subject within a paper, appears to have been problematical, then the Examiners may take account of this in calculating USMs.

The USMs awarded to a candidate for papers in Part A will be carried forward into a classification as described below.

Part B
The Board of Examiners in Part B will assign USMs for full unit and half unit papers taken in Part B and may recalibrate the raw marks to arrive at university standardised marks reported to candidates. The full unit papers are designed so that the raw marks sum to 100, however, Examiners will take into account the relative difficulty of papers when assigning USMs. In order to achieve this, Examiners may use information on candidates' performances on the Part A examination when recalibrating the raw marks. They may also use other statistics to check that the USMs assigned fairly reflect the students' performances on a paper.

The USMs awarded to a candidate for papers in Part B will be aggregated with the USMs from Part A to arrive at a classification.

Part C
The Board of Examiners in Part C will assign USMs for full unit and half unit papers taken in Part C and may recalibrate the raw marks to arrive at university standardised marks reported to candidates. The full unit papers are designed so that the raw marks sum to 100, however, Examiners will take into account the relative difficulty of papers when assigning USMs. In order to achieve this, Examiners may use information on

candidates' performances on the earlier Parts of the examination when recalibrating the raw marks. They may also use other statistics to check that the USMs assigned fairly reflect the students' performances on a paper.
The USMs awarded to a candidate for papers in Part C will be aggregated to arrive at a classification for Year 4.

1.3.2 **Aggregation of marks**

For Mathematics and Statistics Examinations in Parts B and C from 2009 onwards the following apply:

A *strong paper rule* will be adopted for classification in 2009 onwards.

By the *nth class strong paper rule* we means that for a candidate to be classified at the nth class standard, at least 3 papers from Parts A and B must lie in the nth class (or above) and at least one of these must be at Part B. For example, for a First class award, a candidate would need at least 3 of their whole unit paper USMs to be first class marks (with at least 1 first class whole unit at Part B) together with a weighted average score of parts A and B over 70.

Let AvUSM-Part A&B = Average weighted USM in Parts A and B together;

The Part A USMs are given a weighting of 2 and the Part B USMs a weighting of 3 for a full unit and 1.5 for a half unit.

First Class		*Av USM* -Part A&B≥ 70 and the first class strong paper rule
Upper Second Class		*Av USM -Part A&B* ≥ 70 and not satisfying the first class strong paper rule
	or	70 > *Av USM -Part A&B* ≥ 60 and the upper second strong paper rule satisfied
Lower Second Class		70 > *Av USM -Part A&B* ≥ 60 and not satisfying the upper second strong paper rule
	or	60 > *Av USM* ≥ 50 and the lower second strong paper rule satisfied
Third Class		50 > *Av USM -Part A&B* ≥ 40
	or	60> *Av USM -Part A&B* ≥ 50 and not satisfying the lower second strong paper rule
Pass		40 > *Av USM -Part A&B* ≥ 30
Fail		*Av USM -Part A&B* < 30.

Half unit papers count as half a paper when determining the average USM or determining the number of strong papers.

BA in Mathematics
All candidates who wish to leave at the end of their third year and who satisfy the Examiners will be awarded a classified BA in Mathematics and Statistics at the end of Part B based on the above classification.

MMath in Mathematics and Statistics in 2009 onwards
In order to proceed to Part C, a candidate must minimally achieve lower second class Honours standard in Part A and Part B together.

Candidates successfully studying for a fourth year will receive a separate classification based on their University standardised marks in Part C papers according to the following rules:

The classification conventions for part C are:

- First class 70 ≤ *Av USM Part C*
- Upper Second Class 60 ≤*Av USM Part C* < 70
- Lower Second Class 50 ≤ *Av USM Part C*< 60
- Third class 40 ≤*Av USM Part C* < 50

A pass degree will not be awarded for Year 4. Candidates achieving

AvUSM Part C< 40 may supplicate for a BA.

Half unit papers count as a half a paper when determining the average USM.

Candidates leaving after four years who satisfy the Examiners will be awarded an MMath in Mathematics and Statistics with two associated classifications. Candidates are not awarded a BA degree and an MMath degree.

2. SYLLABUS AND SYNOPSES – CORE MATERIAL

The syllabus details in this booklet are those referred to in the Examination Regulations and have been approved by the Statistics Academic Committee for examination in Trinity Term 2009. The synopses in this booklet give some additional detail, and show how the material is split between the different lecture courses. They also include details of recommended reading.

2.1 Syllabus

2.1.1 Algebra
2.1.2 Analysis
2.1.3 Differential Equations

These three subjects (Algebra, Analysis and Differential Equations) are also core subjects for Part A of the Honour School of Mathematics. For the syllabuses and synopses, see those for Part A of the Honour School of Mathematics, which are available on the web at http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/maths

2.1.4 **Probability**

Random variables and their distribution; joint distribution, conditional distribution; functions of one or more random variables. Generating functions and applications. Characteristic functions, definition only. Statements of the continuity and uniqueness theorems for moment generating functions. Chebychev and Markov inequalities. The weak law of large numbers and central limit theorem for independent identically distributed variables with a second moment. Discrete-time Markov chains: definition, transition matrix, n-step transition probabilities, communicating classes, absorption, irreducibility, calculation of hitting probabilities and mean hitting times, recurrence and transience. Invariant distributions, mean return time, positive recurrence, convergence to equilibrium (proof not examinable).

Examples of applications in areas such as: genetics, branching processes, Markov chain Monte Carlo. Poisson processes in one dimension: exponential spacings, Poisson counts, thinning and superposition.

2.1.5 **Statistics**

Estimation: observed and expected information, statement of large sample properties of maximum likelihood estimators in the regular case, methods for calculating maximum likelihood estimates, large sample distribution of sample estimators using the delta method.

Hypothesis testing: simple and composite hypotheses, size, power and p-values, Neyman-Pearson lemma, distribution theory for testing means and variances in the normal model, generalized likelihood ratio, statement of its large sample distribution under the null hypothesis, analysis of count data.

Confidence intervals: exact intervals, approximate intervals using large sample theory, relationship to hypothesis testing.

Regression: correlation, least squares and maximum likelihood estimation, use of matrices, distribution theory for the normal model, hypothesis tests and confidence intervals for linear regression problems, examining assumptions by plotting residuals.

2.2 Synopses of Lectures

2.2.1 Algebra — 24 lectures MT
2.2.2 Analysis — 24 lectures MT
2.2.3 Differential Equations — 24 lectures MT

These three subjects (Algebra, Analysis and Differential Equations) are also core subjects for Part A of the Honour School of Mathematics. For the syllabuses and synopses, see those for Part A of the Honour School of Mathematics, which are available on the web at http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/maths

2.2.4 **Probability** — 16 lectures HT

Aims and Objectives
The first half of the course takes further the probability theory that was developed in the first year. The aim is to build up a range of techniques that will be useful in dealing with mathematical models involving uncertainty. The second half of the course is concerned with Markov chains in discrete time and Poisson processes in one dimension, both with developing the relevant theory and giving examples of applications.

Synopsis
Continuous random variables. Jointly continuous random variables, independence, conditioning, bivariate distributions, functions of one or more random variables. Moment generating functions and applications. Characteristic functions, definition only. Examples to include some of those which may have later applications in Statistics.
Basic ideas of what it means for a sequence of random variables to converge in probability, in distribution and in mean square. Chebychev and Markov inequalities. The weak law of large numbers and central limit theorem for independent identically distributed variables with a second moment. Statements of the continuity and uniqueness theorems for moment generating functions.

Discrete-time Markov chains: definition, transition matrix, n-step transition probabilities, communicating classes, absorption, irreducibility, calculation of hitting probabilities and mean hitting times, recurrence and transience. Invariant distributions, mean return time, positive recurrence, convergence to equilibrium (proof not examinable). Examples of applications in areas such as: genetics, branching processes, Markov chain Monte Carlo. Poisson processes in one dimension: exponential spacings, Poisson counts, thinning and superposition.

Reading
G R GRIMMETT and D R STIRZAKER, Probability and Random Processes, 3rd edition, OUP (2001) Chapters 4, 6.1-6.5, 6.8
G R GRIMMETT and D R STIRZAKER, One Thousand Exercises in Probability, OUP

(2001)
G R GRIMMETT and D J A WELSH, Probability: An Introduction, OUP (1986) Chapters 6, 7.4, 8, 11.1-11.3
J R NORRIS, Markov Chains, CUP (1997) Chapter 1
D R STIRZAKER, Elementary Probability, CUP 2nd Edition (2003) Chapters 7-9 excluding 9.9

2.2.5 **Statistics** — 16 lectures HT

Aims and Objectives
Building on the first year course, this course develops statistics for mathematicians, emphasising both its underlying mathematical structure and its application to the logical interpretation of scientific data. Advances in theoretical statistics are generally driven by the need to analyse new and interesting data which come from all walks of life.

Synopsis
Estimation: observed and expected information, statement of large sample properties of maximum likelihood estimators in the regular case, methods for calculating maximum likelihood estimates, large sample distribution of sample estimators using the delta method.

Hypothesis testing: simple and composite hypotheses, size, power and p-values, Neyman-Pearson lemma, distribution theory for testing means and variances in the normal model, generalized likelihood ratio, statement of its large sample distribution under the null hypothesis, analysis of count data.

Confidence intervals: exact intervals, approximate intervals using large sample theory, relationship to hypothesis testing.

Regression: correlation, least squares and maximum likelihood estimation, use of matrices, distribution theory for the normal model, hypothesis tests and confidence intervals for linear regression problems, examining assumptions by plotting residuals.
Examples: statistical techniques will be illustrated with relevant datasets in the lectures.

Reading
F DALY, D J HAND, M C JONES, A D LUNN and K J McCONWAY, Elements of Statistics, Addison Wesley (1995) Chapters 7-10 (and Chapters 1-6 for background)
J A RICE, Mathematical Statistics and Data Analysis, 2nd edition, Wadsworth (1995) Sections 8.5, 8.6, 9.1-9.7, 9.9, 10.3-10.6, 11.2, 11.3, 12.2.1, 13.3, 13.4.

Further Reading
G CASELLA and R L BERGER, Statistical Inference, 2nd edition, Wadsworth (2001)

3 OPTIONS

3.1 Syllabus

3.1.1 Graph Theory

Finite graphs and digraphs. Eulerian and Hamiltonian graphs. Trees and their characterisation, Cayley's theorem. Planar graphs, Euler's formula, dual graphs. Vertex and face colourings, Brooks' theorem. Matchings, Hall's theorem, Menger's theorem.

3.1.2 Simulation

Simulation by inversion and rejection. Transformation methods. Multivariate Normal. Simulation of stochastic processes: Poisson. Birth-death. Output Analysis. Regeneration. Application to queues, resource management. The Metropolis algorithm (finite space), reversibility, ergodicity. Variance reduction: importance sampling. Comparison with rejection and independence sampler. Applications: likelihood and missing data, Monte Carlo confidence intervals and significance tests, simulation of the Ising model, graphs or graph colorings sampled uniformly at random.

3.1.3 Linear Programming

Standardisation of problems, slack variables. The simplex method, excluding procedures to cope with degeneracy. The dual problem, duality theorem (proof by analysing the simplex method), complementary slackness. Economic interpretation of dual variables, sensitivity analysis. Two person zero-sum games.

3.1.4 Other optional subjects

The other optional subjects are drawn from Part A of the Honour School of Mathematics and are as follows:

- Groups in Action
- Introduction to Fields
- Number Theory
- Integration
- Topology
- Multivariable Calculus
- Calculus of Variations
- Classical Mechanics
- Electromagnetism
- Fluid Dynamics and Waves
- Numerical Analysis

For the syllabuses and synopses, see those for Part A of the Honour School of Mathematics, which are available on the web at
http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/maths.shtml

3.2 Synopses of Lectures

3.2.1 **Graph Theory** — 8 lectures HT

Aims and Objectives
The aim of the course is to introduce students to this central part of Discrete Mathematics, dating back to Euler and now important in theoretical computer science and the mathematics of operational research. For example, an attractive part of the theory concerns colouring the vertices of a graph so that adjacent vertices get distinct colours. Perhaps the vertices here correspond to examinations and two vertices are adjacent if some student sits both examinations: a colouring then corresponds to a timetable.

Synopsis
We start with basic definitions of graph, path, connected, tree, and so on: many of these definitions should come as no surprise. Trees are an important class of graphs: we consider some characterisations of trees, and how to count trees: in particular, we prove Cayley's theorem that there are n^{n-2} trees on the vertices 1, . . . , n.
Then we consider traversing graphs and digraphs. When can we walk along each edge exactly once? This is where Euler came in, and gave a neat answer. When is there a path passing through each vertex exactly once? This question (associated with Hamilton) is far harder: we find some sufficient conditions.

Some graphs sit naturally in the plane: we meet Euler's formula $v - e + f = 2$ relating the numbers of vertices, edges and faces, and we meet the dual graph. Concerning colouring the vertices of a graph, we prove for example Brooks' Theorem relating the number of colours needed to the maximum degree of a vertex; and the result that any planar graph needs at most 5 colours (we do not prove the 4-colour theorem!). Finally we consider matchings and disjoint paths in graphs. We prove Hall's 'marriage theorem' and Menger's theorem on disjoint paths, together with various related results. The discussions on trees, matchings and disjoint paths link particularly closely with combinatorial optimisation.

Reading
R J WILSON, Introduction to Graph Theory, 4th edition, Longman (1996).
D B WEST, Introduction to Graph Theory, 2nd edition, Prentice Hall (2001).

3.2.2 **Simulation** — 8 lectures TT

Aims and Objectives
Building on Part A probability and Mods statistics, this course introduces Monte Carlo methods, collectively one of the most important analytical tools of modern statistical inference.

Synopsis
1-2) Motivation. Inversion and rejection. Transformation methods.
Multivariate Normal.
3) Simulation of stochastic processes: Poisson. Birth-death.
4) Output Analysis. Regeneration. Application to queues, resource management.
5) The Metropolis algorithm (finite space), reversibility, ergodicity.

6) Variance reduction: importance sampling. Comparison with rejection and independence sampler.
7-8) Applications: likelihood and missing data, Monte Carlo confidence intervals and significance tests, simulation of the Ising model, graphs or graph colorings sampled uniformly at random.

Classes.
Three classes, the first a computer lab-based class to include an introduction to using R. About one third of problem sheet problems to require R coding.

Recommended Reading.
S.M. Ross, Simulation, Elsevier, 4th ed., 2006

Reference.
J.R. Norris, Markov Chains, CUP, 1997
B.D. Ripley, Stochastic Simulation, Wiley, 1987

3.2.3 **Linear Programming** — 8 lectures TT

Aims and Objectives
Linear programming is about making the most of limited resources. Specifically, it deals with maximising a linear function of variables subject to linear constraints. Applications range from economic planning and environmental management to the diet problem. The aim is to provide a simple introduction to the subject.

Synopsis
Linear programming problems, examples. Standardisation of problems, slack variables. Sufficiency of basic feasible solutions; equivalence of basic feasible solutions and extreme points. The simplex method, excluding procedures to cope with degeneracy. The dual problem, duality theorem (proof by analysing the simplex method), complementary slackness. Economic interpretation of dual variables, sensitivity analysis. Two person zero-sum games.

Reading
V CHVATAL, Linear Programming, Freeman (1983), Chapters 1–5, 15.
K TRUSTRUM, Linear Programming, RKP (1971) – out of print, but available in college libraries, Chapters 1–5.
D G LUENBERGER, Linear and Nonlinear Programming, Addison-Wesley (1984), Chapters 2–4.

3.2.4 **Other optional subjects**
For synopses of the other optional subjects listed in Section 3.1.4, including details of recommended reading, see those for Part A of the Honour School of Mathematics, which are available on the web at http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/maths