Mathematics and Statistics Undergraduate Handbook
Supplement to the Handbook

Honour School of Mathematics and Statistics
Syllabus and Synopses for Part B 2016–2017
for examination in 2017

Contents

Every effort is made to ensure that the list of courses offered is accurate at the time of going online. However, students are advised to check the up-to-date version of this document on the Department of Statistics website.

Notice of misprints or errors of any kind, and suggestions for improvements in this booklet should be addressed to the Academic Administrator in the Department of Statistics.

Updated June 2016

1. Honour School of Mathematics and Statistics

1.1 Units and double-units and methods of examination

See the current edition of the Examination Regulations at
http://www.admin.ox.ac.uk/examregs/
for the full regulations governing these examinations. The examination conventions can be found at http://www.stats.ox.ac.uk/current_students/bammath/examinations

In Part B each candidate shall offer a total of **eight units** from the schedule of units and double units.

(a) Each candidate shall offer the double unit SB1.
(b) Each candidate shall offer a total of at least two units from SB2 and SB3.
(c) Each candidate may offer a total of at most two units from SB4 and the schedule of 'Other units'.
(d) Each candidate may offer at most one double unit which is an Extended Essay or Structured Project.

Note: Units from the schedule of 'Mathematics Department units' for Part B of the Honour School of Mathematics are also available – see Section 3.

We ask that you register by the end of week 10 Trinity Term 2016 for classes for the Mathematics/ Statistics courses that you wish to take. A registration form is attached to these synopses. Some combinations of subjects are not advised and lectures in these subjects may clash. However, when timetabling lectures we will aim to keep clashes to a minimum.

1.2 Language Classes

If spaces are available, Mathematics and Statistics students are also invited to apply to take classes in a foreign language. In 2016-2017, French and German language classes will be offered. Students' performance in these classes will not contribute to the degree classification in Mathematics and Statistics. However, successful completion of the course, may be recorded on a student's transcript. See http://www.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/handbooks-synopses for further information.

1.3 Part B courses in future years

In any year, most courses available in Part B that year will normally also be available in Part B the following year. However, sometimes new options will be added or existing options may cease to run. The list of courses that will be available in Part B in any year will be published by the end of the preceding Trinity Term.

Details of Part C units, examinable in 2018, will be published before Michaelmas Term 2017.

1.4 **Course list by term**

The list of 2016-2017 Part B courses by term is:

Michaelmas Term

SB1a Applied Statistics [double unit with SB1b]
SB2a Foundations of Statistical Inference
SB3a Applied Probability
SB4a Actuarial Science I

Hilary Term

SB1b Computational Statistics [double unit with SB1a]
SB2b Statistical Machine Learning
SB3b Statistical Lifetime Models
SB4b Actuarial Science II

2 Statistics units and double units

2.1 SB1 Applied and Computational Statistics

Level: H-level
Method of Assessment: written examination plus assessed practical assignments. The practical assignments contribute 1/3 of the marks for SB1. Please see below for the hand-in deadlines for practical assignments.
Weight: Double unit.

Prerequisites: A8 *Probability* and A9 *Statistics*.

Aims
The course aims to develop the theory of statistical methods, and also to introduce students to the analysis of data using a statistical package. The main topics are: simulation based inference, practical aspects of linear models, logistic regression and generalized linear models, and computer-intensive methods.

2.1.1 SB1a Applied Statistics – 13 MT

Synopsis
The normal linear model: use of matrices, least squares and maximum likelihood estimation, normal equations, distribution theory for the normal model, hypothesis tests and confidence intervals.

Practical aspects of linear models and analysis of variance: multiple regression, categorical variables and interactions, blocks and treatments, orthogonality, model selection (including AIC, but not the derivation of AIC), fit criteria, use of residuals, outliers, leverage, model interpretation.

Normal linear mixed models, hierarchical models.

Generalised Linear Models: logistic regression, linear exponential families and generalized linear models, scale parameter, link functions, canonical link. Maximum likelihood fitting. Iteratively reweighted least squares. Asymptotic theory: statement and applications to inference, analysis of deviance, model checking, residuals.

Reading
A. C. Davison, *Statistical Models*, CUP, 2003
J.J. Faraway, *Linear Models with R*, Chapman and Hall, 2005
A. J. Dobson and A.G Barnett, *An Introduction to Generalized Linear Models*, Chapman and Hall, 2008
J.J. Faraway, *Extending the Linear Model with R : Generalized Linear, Mixed Effects and Nonparametric Regression Models*, Chapman and Hall, 2006

Further Reading
F. L. Ramsey and D. W. Schafer, *The Statistical Sleuth: A Course in Methods of Data Analysis*, 2nd edition, Duxbury, 2002.

2.1.2 SB1b Computational Statistics – 13 HT

Synopsis
Smoothing methods (local polynomials). Nonparametric inference (density estimation and bandwidth).

Inference using simulation methods. Monte-Carlo Tests. Permutation tests. Rank statistics.

Bootstrapping.

Hidden Markov Models: specification. Forward-backward algorithm. Kalman filter.

Reading
J. D. Gibbons, *Nonparametric Statistical Inference*, Marcel Dekker, 1985, pp 1-193, 273- 290.
G.H. Givens and J.A. Hoeting, *Computational Statistics*, 2nd edition, Wiley, 2012.
G. James, D. Witten, T. Hastie, R. Tibshirani, *An Introduction to Statistical Learning*, Springer , 2013. This book is freely available online: http://www-bcf.usc.edu/~gareth/ISL/
R. H. Randles and D. A. Wolfe, *Introduction to the Theory of Nonparametric Statistics*, Wiley 1979, pp 1-322.
L. Wasserman, *All of Nonparametric Statistics*, Springer, 2005.
L. Wasserman, *All of Statistics*, Springer, 2004.

Further Reading
A.C. Davison and D.V. Hinkley, *Bootstrap Methods and their Application,* CUP, 1997.
C.R. Shalizi, *Advanced Data Analysis from an Elementary Point of View*, http://www.stat.cmu.edu/~cshalizi/ADAfaEPoV/.

Practicals
In addition to the lectures there will be five supervised practicals. Four of these contain problems whose written solutions will be assessed as part of the unit examination.

The hand-in deadlines for the four assessed practicals are:
1st practical: 12 noon Monday week 8, Michaelmas Term 2016
2nd practical: 12 noon Monday week 2, Hilary Term 2017
3rd practical: 12 noon Monday week 8, Hilary Term 2017
4th practical: 12 noon Monday week 2, Trinity Term 2017.

Candidates who miss the above deadlines may ask their college to apply to the Head of the Department of Statistics for permission to submit late. Where there is a valid reason, the Head of Department would normally approve the late submission without penalty. Where it is deemed that there is no valid reason, the Head of Department will advise the Examiners to apply a penalty in accordance with the late penalty tariff found in the Mathematics and Statistics Examination Conventions.

2.2.1 SB2a Foundations of Statistical Inference – 16 MT

Level: H-level
Method of Assessment: written examination
Weight: Unit

Prerequisites: A9 *Statistics*. A8 *Probability* would be useful.

Learning outcomes
Understanding how data can be interpreted in the context of a statistical model. Working knowledge and understanding of key-elements of model-based statistical inference, including awareness of similarities, relationships and differences between Bayesian and frequentist approaches.

Synopsis
Exponential families: Curved and linear exponential families; canonical parametrization; likelihood equations. Sufficiency: Factorization theorem; sufficiency in exponential families.

Frequentist estimation: unbiasedness; method of moments; the Cramer-Rao information inequality; Rao-Blackwell theorem: Lehmann-Scheffé Theorem and Rao-Blackwellization; Statement of complete sufficiency for Exponential families.

The Bayesian paradigm: likelihood principal; subjective probability; prior to posterior analysis; asymptotic normality; conjugacy; examples from exponential families. Choice of prior distribution: proper and improper priors; Jeffreys' and maximum entropy priors. Hierarchical Bayes models.

Computational techniques: Variational Bayesian methods; the EM algorithm; approximations to marginal likelihood – Laplace approximation and BIC.

Decision theory: risk function; Minimax rules, Bayes rules. Point estimators and admissibility of Bayes rules. The James-Stein estimator, shrinkage estimators and Empirical Bayes. Hypothesis testing as decision problem.

Reading
P. H. Garthwaite, I. T. Jolliffe and Byron Jones, *Statistical Inference,* 2nd edition, Oxford University Press, 2002.
G.A.Young and R.L. Smith, *Essentials of Statistical Inference*, Cambridge University Press, 2005.
T. Leonard and J.S.J. Hsu, *Bayesian Methods*, Cambridge University Press, 2005.

Further reading
D. Barber, *Bayes Reasoning and Machine Learning*, Cambridge University Press, 2012.
D. R. Cox, *Principles of Statistical Inference*, Cambridge University Press, 2006.
H. Liero and S Zwanzig, *Introduction to the Theory of Statistical Inference*, CRC Press, 2012.

2.2.2 SB2b Statistical Machine Learning – 16 HT

Level: H-level
Method of Assessment: Written examination
Weight: Unit

Recommended prerequisites: Part A A9 Statistics and A8 Probability. SB2a Foundations of Statistical Inference useful by not essential.

Aims and Objectives:
Machine learning studies methods that can automatically detect patterns in data, and then use these patterns to predict future data or other outcomes of interest. It is widely used across many scientific and engineering disciplines.

This course covers statistical fundamentals of machine learning, with a focus on supervised learning and empirical risk minimisation. Both generative and discriminative learning frameworks are discussed and a variety of widely used classification algorithms are overviewed.

Synopsis
Fundamentals: Statistical learning theory, bias/variance trade-off, generalization and overfitting, regularization. Evaluating learning methods with training/test sets and cross-validation.

Supervised learning : K-nearest neighbours. Generative methods: naive Bayes, linear discriminant analysis, quadratic discriminant analysis. Discriminative methods: logistic regression, neural networks, support vector machines, decision trees, bagging, random forests.

Reading
C. Bishop, *Pattern Recognition and Machine Learning*, Springer, 2007.
T. Hastie, R. Tibshirani, J Friedman, *Elements of Statistical Learning*, Springer, 2009.
K. Murphy, *Machine Learning: a Probabilistic Perspective*, MIT Press, 2012.

Further Reading
B. D. Ripley, *Pattern Recognition and Neural Networks*, Cambridge University Press, 1996.
G. James, D. Witten, T. Hastie, R. Tibshirani, *An Introduction to Statistical Learning*, Springer, 2013.

2.3 SB3

A student obtaining at least an upper second class mark on the **units SB3a and SB3b together** can expect to gain exemption from the Institute of Actuaries' paper CT4, which is a compulsory paper in their cycle of professional actuarial examinations. An Independent Examiner approved by the Institute of Actuaries will inspect examination papers and scripts and **may adjust the pass requirements for exemptions**.

Please note that it is **not possible** to gain an exemption from the Institute of Actuaries paper CT4 if you do SB3a only. SB3a is a required prerequisite for SB3b.

2.3.1 **SB3a Applied Probability** – 16 MT

Level: H-level
Method of Assessment: written examination
Weight: Unit.

Prerequisite: A8 *Probability*.

Aims
This course is intended to show the power and range of probability by considering real examples in which probabilistic modelling is inescapable and useful. Theory will be developed as required to deal with the examples.

Synopsis
Poisson processes and birth processes. Continuous-time Markov chains. Transition rates, jump chains and holding times. Forward and backward equations. Class structure, hitting times and absorption probabilities. Recurrence and transience. Invariant distributions and limiting behaviour. Time reversal.

Renewal theory. Limit theorems: strong law of large numbers, strong law and central limit theorem of renewal theory, elementary renewal theorem, renewal theorem, key renewal theorem. Excess life, inspection paradox.

Applications in areas such as: queues and queueing networks - M/M/s queue, Erlang's formula, queues in tandem and networks of queues, M/G/1 and G/M/1 queues; insurance ruin models; applications in applied sciences.

Reading
J.R. Norris: *Markov Chains*. Cambridge University Press, 1997.
G.R. Grimmett and D.R. Stirzaker: *Probability and Random Processes*, 3rd edition, Oxford University Press, 2001.
G.R. Grimmett and D.R. Stirzaker: *One Thousand Exercises in Probability*. Oxford University Press, 2001.
S.M. Ross: I*ntroduction to Probability Models*, 4th edition, Academic Press, 1989.
D.R. Stirzaker: *Elementary Probability*, 2nd edition, Cambridge University Press, 2003.

2.3.2 **SB3b Statistical Lifetime-Models** – 16 HT

Level: H-level
Method of Assessment: written examination
Weight: Unit.

Prerequisites: A8 *Statistics,* SB3a *Applied Probability*

Aims

SB3b *Statistical Lifetime Models* follows on from SB3a Applied *Probability*. Models introduced there are examined more specifically in the context of measuring 'lifetimes' in the broad sense. In a life insurance context Markov transitions may model the passage from 'alive' to 'dead', possibly with intermediate stages like 'loss of a limb' or 'critically ill'. The same models are used to model fertility transitions, the progress of a disease, and the reliability of a mechanical device. The aim is to develop statistical methods to estimate transition rates, and to use these transition rates to construct life tables that form the basis in the calculation of life insurance premiums and pension projections. We will also cover the basics of survival analysis, to model the influence of covariates (eg weight, smoking, use of a medication) on lifespans.

Synopsis

Life tables: Basic notation, life expectancy and remaining life expectancy, curtate lifetimes.
Census approximation, Lexis diagrams. Survival models: general lifetime distributions, force of mortality (hazard rate), survival function, the single decrement model and mortality in mixed populations.

Estimation procedures for lifetime distributions: empirical lifetime distributions, censoring and truncation, Kaplan-Meier estimate, Nelson-Aalen estimate. Parametric models, accelerated life models including Gompertz Weibull, log-normal, log-logistic. Plot-based methods for model selection. Cox regression. Proportional hazards, partial likelihood, semiparametric estimation of survival functions, use and overuse of proportional hazards in insurance calculations and epidemiology. Two-state and multiple-state Markov models, with simplifying assumptions. Estimation of Markovian transition rates: Maximum likelihood estimators, time-varying transition rates, census approximation. Life expectancy and occupation times in Markov models: eigenvector formalism. Applications to reliability, medical statistics, ecology.

Graduation, including fitting Gompertz-Makeham model, comparison with standard life table: tests including chi-square test and grouping of signs test, serial correlations test; smoothness. Application to pension plans.

Reading

Subject CT4 Models Core Reading, Faculty & Institute of Actuaries.
D.R. Cox and D. Oakes: *Analysis of Survival Data.* Chapman & Hall, 1984.

Further Reading

J.P. Klein and M.L. Moeschberger, *Survival Analysis,* Springer, 1997.
C.T. Le, *Applied Survival Analysis,* 2nd edition, Wiley, 2003.
H.U. Gerber, *Life Insurance Mathematics,* 3rd edition, Springer, 1997.
N.L. Bowers et al, *Actuarial mathematics*, 2nd edition, Society of Actuaries, 1997.

2.4 SB4 Actuarial Science

A student obtaining at least an upper second class mark on the **units SB4a and SB4b together** can expect to gain exemption from the Institute of Actuaries' paper CT1, which is a compulsory paper in their cycle of professional actuarial examinations. An Independent Examiner approved

by the Institute of Actuaries will inspect examination papers and scripts and **may adjust the pass requirements for exemptions**.

Please note that it is **not possible** to gain an exemption from the Institute of Actuaries paper CT1 if you do SB4a only. SB4a is a required prerequisite for SB4b.

2.4.1 **SB4a Actuarial Science I** – 16 MT

Level: H-level
Method of Assessment: written examination
Weight: Unit.

Prerequisites: A8 *Probability* is useful, but not essential. If you have not done A8 *Probability*, make sure that you are familiar with Prelims work on Probability.

Synopsis
Fundamental nature of actuarial work. Use of generalised cash flow model to describe financial transactions. Time value of money using the concepts of compound interest and discounting. Interest rate models. Present values and accumulated values of a stream of equal or unequal payments using specified rates of interest. Interest rates in terms of different time periods. Equation of value, rate of return of a cash flow, existence criteria.

Loan repayment schemes. Investment project appraisal, funds and weighted rates of return. Inflation modelling, inflation indices, real rates of return, inflation-adjustments. Valuation of fixed-interest securities, taxation and index-linked bonds.

Single decrement model. Present values and accumulated values of a stream of payments taking into account the probability of the payments being made according to a single decrement model. Annuity functions and assurance functions for a single decrement model. Risk and premium calculation.

Liabilities under a simple assurance contract or annuity contract. Premium reserves, Thiele's differential equation. Expenses and office premiums.

Reading
All of the following are available from the Faculty and Institute of Actuaries, 1st Floor, Park Central, 40/41 Park End Street, Oxford, OX1 1JD and https://www.actuaries.org.uk/shop

Subject CT1 Financial Mathematics Core Reading Faculty & Institute of Actuaries.
J.J. McCutcheon and W.F. Scott, *An Introduction to the Mathematics of Finance*. Heinemann, 1986.
P. Zima and R.P. Brown: *Mathematics of Finance*. McGraw-Hill Ryerson, 1993.
N.L. Bowers et al, *Actuarial mathematics,* 2nd edition, Society of Actuaries, 1997.
J. Danthine and J. Donaldson: *Intermediate Financial Theory*. 2nd edition, Academic Press Advanced Finance, 2005.

2.4.2 **SB4b Actuarial Science II** – 16 HT

Level: H-level
Method of Assessment: written examination
Weight: Unit.

Prerequisites: SB4a *Actuarial Science I*

Synopsis
Price and value of forward contracts. Term structure of interest rates, spot rates, forward rates and yield curves. Duration, convexity and immunisation. Simple stochastic interest rate models. Investment and risk characteristics of investments. Risk pooling.

Theories of value, St Petersburg Paradox, statement of Expected Utility Theory (EUT) and Subjective Expected Utility (SEU) representation theorems

Risk aversion, the Arrow-Pratt approximation, comparative risk aversion, classical utility functions.

First and second order stochastic dominance, the Rothschild-Stiglitz Proposition.
EUT justification for insurance, Mossin's Theorem, Arrow's Theorem on the optimality of deductibles, static portfolio choice.

Desynchronisation and financial systems, Static exchange economy and Pareto efficiency, the mutuality principle, efficient allocation of aggregate risk.

Reading
All of the following are available from the Faculty and Institute of Actuaries, 1st Floor, Park Central, 40/41 Park End Street, Oxford, OX1 1JD and https://www.actuaries.org.uk/shop

Subject CT5 *Contingencies Core Reading* Faculty & Institute of Actuaries.
H.U. Gerber: *Life Insurance Mathematics*. 3rd edition, Springer , 1997.
L. Eeckhoudt, C. Gollier and H.Schlesinger, *Economic and Financial Decisions under Risk*, Princeton University Press Princeton and Oxford, 2005, Chapters 1-4, 10.
C. Gollier, *The Economics of Risk and Time*, MIT Press, 2001, Topics in chapters 1-4, 20.
Subject CT8: Financial Economics Core reading, Faculty & Institute of Actuaries, Units (i), (iii), (v)-(vi).
N.L. Bowers et al, *Actuarial mathematics,* 2nd edition, Society of Actuaries, 1997.

3 Mathematical and Other units

The other units that students in Part B Mathematics and Statistics may take are drawn from Part B of the Honour School of Mathematics.
For full details of these units, see the Syllabus and Synopses for Part B of the Honour School of Mathematics, which are available on the web at
https://www1.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/handbooks-synopses

3.1 **Mathematics units**

The Mathematics units that are available are as follows:

B1.1: Logic	16 MT
B1.2: Set Theory	16 HT
B2.1: Introduction to Representation Theory	16 MT
B2.2: Commutative Algebra	16 MT
B3.1: Galois Theory	16 MT
B3.2: Geometry of Surfaces	16 MT
B3.3: Algebraic Curves	16 HT
B3.4: Algebraic Number Theory	16 HT
B3.5: Topology and Groups	16 MT
B4.1: Banach Spaces	16 MT
B4.2: Hilbert Spaces	16 HT
B5.1:Stochastic Modelling of Biological Processes	16 MT
B5.2: Applied Partial Differential Equations	16 HT
B5.3: Viscous Flow	16 MT
B5.4: Waves and Compressible Flow	16 HT
B5.5: Further Mathematical Biology	16 MT
B5.6: Nonlinear Systems	16 HT
B6.1 Numerical Solution of Differential Equations I	16 MT
B6.2 Numerical Solution of Differential Equations II	16 HT
B6.3 Integer Progamming	16 MT
B7.1 Classical Mechanics	16 HT
B7.2 Electromagnetism	16 MT
B7.3 Further Quantum Theory	16 HT
B8.1: Martingales Through Measure Theory	16 MT
B8.2: Continuous Martingales and Stochastic Calculus	16 HT
B8.3: Mathematical Models of Financial Derivatives	16 HT
B8.4: Communication Theory	16 MT
B8.5: Graph Theory	16 HT
BEE Mathematical Extended Essay	MT & HT [double unit]
or	
BSP: Structured Projects	MT & HT [double unit]
[Note: Students **cannot take both** BEE and BSP]	

Other units:

BN1: Undergraduate Ambassadors' Scheme	MT & HT [double unit]
or	
BN1.1: Mathematics Education	MT

(These are the units referred to in Section 1 as 'Mathematics Department units for Part B of the Honour School of Mathematics.)

See the "Projects Guidance Notes" on the web at https://www1.maths.ox.ac.uk/members/students/undergraduate-courses/teaching-and-learning/projects for more information on the Extended Essay option and an application form.

Please note that the following **are not permitted options** in Part B of the Honour School of Mathematics and Statistics:

BO1.1 History of Mathematics
BOE ''Other Mathematical'' Extended Essay

3.2 Other units

The other units available are as follows:

BN1: Undergraduate Ambassadors' Scheme	MT & HT [double unit]
or	
BN1.1: Mathematics Education	MT

4. Registration for Part B courses 2016-2017

We ask that students register in advance for the classes they wish to take, by Friday of week 10 Trinity Term 2016, using the form overleaf.

Because of the large number of options which are available in Part B, some lectures will clash.

REGISTRATION FORM: PART B CLASSES 2016-2017

SURNAME . . . . . FIRST NAME . . . . . .

EMAIL ADDRESS . . . . . .

COLLEGE . . . . . .

Note: As described in Section 1, you need to do a total of 8 units in Part B: all Mathematics and Statistics students do the double unit SB1, and also at least two units from the units available under SB2 and SB3.

For the Statistics units SB1–SB4, and the Mathematics or Other units, please give details of subjects in which you wish to take classes.

I wish to take classes in the following subjects: [Please Tick]

✓	SB1 Applied Statistics (MT and HT, double unit, compulsory for Mathematics and Statistics students)
	SB2a Foundations of Statistical Inference (MT, unit)
	SB2b Statistical Machine Learning (HT, unit)
	SB3a Applied Probability (MT, unit)
	SB3b Statistical Lifetime Models (HT, unit, SB3a is required pre-requisite)
	SB4a Actuarial Science I (MT unit)
	SB4b Actuarial Science II (HT, unit, SB4a is a required pre-requisite)

For Mathematics or Other units, please list the unit code and name:
Unit code Unit name

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Please return this form to the Academic Administrator, Department of Statistics, 1 South Parks Road, by Friday of week 10 Trinity Term 2016.