Honour School of Mathematics and Statistics

Supplement to the Mathematics and Statistics Undergraduate Handbook
Syllabus and Synopses for Part A 2006–07
for examination in 2007

Contents

Notice of misprints or errors of any kind, and suggestions for improvements in this booklet should be addressed to the Academic Administrator in the Department of Statistics.

1. Honour School of Mathematics and Statistics

Please see the current edition of the Examination Regulations for the full regulations governing these examinations. In Part A each candidate shall be required to offer the 4 written papers from the schedule of papers for Part A (given below).

Part A shall be taken on one occasion only (there will be no resits). At the end of the Part A examination, a candidate will be awarded four 'University Standardised Marks' (USMs) for their performance in Part A – one USM for each paper taken in Part A. These USMs will be carried forward into the classification awarded at the end of the third year. In this classification, the each paper mark in Part A will be given a 'weighting' of 2, and each unit paper mark in Part B will be given a 'weighting' of 3. All students who complete the first three years of the course will be classified, and those wishing to graduate at this point may supplicate for a BA.

Students wishing to take the four-year course should register to do so at the beginning of their third year. They will take Part C in their fourth year, awarded a separate classification and allowed to supplicate for an MMath.

The schedule of papers

Paper AC1 Algebra, Analysis and Differential Equations
This paper will contain 9 short questions, worth 10 marks each, set on the CORE material in Algebra, Analysis and Differential Equations, all of which should be answered.

Paper AC2 Algebra, Analysis and Differential Equations
This paper will contain 9 longer questions, worth 25 marks each, set on the CORE material in Algebra, Analysis and Differential Equations. At most 5 answers may be submitted and the best 4 will be counted.

Paper AS1 Probability, Statistics and Options
This paper will contain short questions, worth 10 marks each, set on the CORE material in Probability and Statistics, and on the OPTIONAL material. There will be 3 questions on each of Probability and Statistics, and on the options there will be 1 question for each 8 lecture course and 2 questions for each 16 lecture course. At most 10 questions should be answered: the best 5 answers on Probability and Statistics, and the best 4 answers on options will be counted.

Paper AS2 Probability, Statistics and Options
This paper will contain longer questions, worth 25 marks each, set on the CORE material in Probability and Statistics, and on the OPTIONAL material. On all subjects, there will be 1 question for each 8 lecture course and 2 questions for each 16 lecture course. At most 5 answers may be submitted, at least 2 of which should be on Probability and Statistics: the best 2 answers on Probability and Statistics, and the best 2 other answers (which may include further Probability and Statistics questions) will be counted.

Papers AC1 and AC2 are identical to those taken by candidates in Mathematics. Papers AS1 and AS2 are similar but not identical to the options papers (AO1 and AO2) taken by candidates in Mathematics.

Syllabus and Synopses

The syllabus details in this booklet are those referred to in the Examination Regulations and have been approved by the Statistics Academic Committee for examination in Trinity Term 2007. The synopses in this booklet give some additional detail, and show how the material is split between the different lecture courses. They also include details of recommended reading.

Please note that for the first time in 2006–07, the 16 Part A lectures on Statistics will all be given in Hilary Term, at the usual rate of 2 lectures per week.

2. CORE MATERIAL

2.1 Syllabus

2.1.1 Algebra
2.1.2 Analysis
2.1.3 Differential Equations

These three subjects (Algebra, Analysis and Differential Equations) are also core subjects for Part A of the Honour School of Mathematics. For the syllabuses and synopses, see those for Part A of the Honour School of Mathematics, which are available on the web at http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/maths.shtml

2.1.4 Probability

Random variables and their distribution; joint distribution, conditional distribution; functions of one or more random variables. Generating functions and applications. Characteristic functions, definition only. Statements of the continuity and uniqueness theorems for moment generating functions. Chebychev and Markov inequalities.
The weak law of large numbers and central limit theorem for independent identically distributed variables with a second moment. Discrete-time Markov chains: definition, transition matrix, n-step transition probabilities, communicating classes, absorption, irreducibility, calculation of hitting probabilities and mean hitting times, recurrence and transience. Invariant distributions, mean return time, positive recurrence, convergence to equilibrium (proof not examinable).

Examples of applications in areas such as: genetics, branching processes, Markov chain Monte Carlo. Poisson processes in one dimension: exponential spacings, Poisson counts, thinning and superposition.

2.1.5 Statistics

Estimation: observed and expected information, statement of large sample properties of maximum likelihood estimators in the regular case, methods for calculating maximum likelihood estimates, large sample distribution of sample estimators using the delta method.

Hypothesis testing: simple and composite hypotheses, size, power and p-values, Neyman-Pearson lemma, distribution theory for testing means and variances in the normal model, generalized likelihood ratio, statement of its large sample distribution under the null hypothesis, analysis of count data.

Confidence intervals: exact intervals, approximate intervals using large sample theory, relationship to hypothesis testing.

Regression: correlation, least squares and maximum likelihood estimation, use of matrices, distribution theory for the normal model, hypothesis tests and confidence intervals for linear regression problems, examining assumptions by plotting residuals.

2.2 Synopses of Lectures

2.2.1 Algebra — 24 lectures MT
2.2.2 Analysis — 24 lectures MT
2.2.3 Differential Equations — 24 lectures MT

These three subjects (Algebra, Analysis and Differential Equations) are also core subjects for Part A of the Honour School of Mathematics. For the syllabuses and synopses, see those for Part A of the Honour School of Mathematics, which are available on the web at http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/maths.shtml

2.2.4 **Probability** — 16 lectures HT

Aims and Objectives
The first half of the course takes further the probability theory that was developed in the first year. The aim is to build up a range of techniques that will be useful in dealing with mathematical models involving uncertainty. The second half of the course is concerned with Markov chains in discrete time and Poisson processes in one dimension, both with developing the relevant theory and giving examples of applications.

Synopsis
Continuous random variables. Jointly continuous random variables, independence, conditioning, bivariate distributions, functions of one or more random variables. Moment generating functions and applications. Characteristic functions, definition only. Examples to include some of those which may have later applications in Statistics.
Basic ideas of what it means for a sequence of random variables to converge in probability, in distribution and in mean square. Chebychev and Markov inequalities. The weak law of large numbers and central limit theorem for independent identically distributed variables with a second moment. Statements of the continuity and uniqueness theorems for moment generating functions.

Discrete-time Markov chains: definition, transition matrix, n-step transition probabilities, communicating classes, absorption, irreducibility, calculation of hitting probabilities and mean hitting times, recurrence and transience. Invariant distributions, mean return time, positive recurrence, convergence to equilibrium (proof not examinable). Examples of applications in areas such as: genetics, branching processes, Markov chain Monte Carlo. Poisson processes in one dimension: exponential spacings, Poisson counts, thinning and superposition.

Reading
G R GRIMMETT and D R STIRZAKER, Probability and Random Processes, 3rd edition, OUP (2001) Chapters 4, 6.1-6.5, 6.8
G R GRIMMETT and D R STIRZAKER, One Thousand Exercises in Probability, OUP (2001)
G R GRIMMETT and D J A WELSH, Probability: An Introduction, OUP (1986) Chapters 6, 7.4, 8, 11.1-11.3
J R NORRIS, Markov Chains, CUP (1997) Chapter 1
D R STIRZAKER, Elementary Probability, CUP 2nd Edition (2003) Chapters 7-9 excluding 9.9

2.2.5 **Statistics** — 16 lectures HT

Aims and Objectives
Building on the first year course, this course develops statistics for mathematicians, emphasising both its underlying mathematical structure and its application to the logical interpretation of scientific data. Advances in theoretical statistics are generally driven by the need to analyse new and interesting data which come from all walks of life.

Synopsis
Estimation: observed and expected information, statement of large sample properties of maximum likelihood estimators in the regular case, methods for calculating maximum likelihood estimates, large sample distribution of sample estimators using the delta method.

Hypothesis testing: simple and composite hypotheses, size, power and p-values, Neyman-Pearson lemma, distribution theory for testing means and variances in the normal model, generalized likelihood ratio, statement of its large sample distribution under the null hypothesis, analysis of count data.

Confidence intervals: exact intervals, approximate intervals using large sample theory, relationship to hypothesis testing.

Regression: correlation, least squares and maximum likelihood estimation, use of matrices, distribution theory for the normal model, hypothesis tests and confidence intervals for linear regression problems, examining assumptions by plotting residuals.
Examples: statistical techniques will be illustrated with relevant datasets in the lectures.

Reading
F DALY, D J HAND, M C JONES, A D LUNN and K J McCONWAY, Elements of Statistics, Addison Wesley (1995) Chapters 7-10 (and Chapters 1-6 for background)
J A RICE, Mathematical Statistics and Data Analysis, 2nd edition, Wadsworth (1995) Sections 8.5, 8.6, 9.1-9.7, 9.9, 10.3-10.6, 11.2, 11.3, 12.2.1, 13.3, 13.4.

Further Reading
G CASELLA and R L BERGER, Statistical Inference, 2nd edition, Wadsworth (2001)

3 OPTIONS

3.1 Syllabus

3.1.1 Graph Theory

Finite graphs and digraphs. Eulerian and Hamiltonian graphs. Trees and their characterisation, Cayley's theorem. Planar graphs, Euler's formula, dual graphs. Vertex and face colourings, Brooks' theorem. Matchings, Hall's theorem, Menger's theorem.

3.1.2 Combinatorial Optimisation

Minimum spanning trees. Finding shortest paths in networks, algorithms of Dijkstra and Floyd. Knapsack problem, dynamic programming. Scheduling problems, algorithms of Moore and Johnson, critical path method. Maximum matching in bipartite graphs. Assignment problem. Matchings in general graphs. Finding maximum flows in networks. Max-flow-min-cut theorem. Finding minimum cost flows.

3.1.3 Linear Programming

Standardisation of problems, slack variables. The simplex method, excluding procedures to cope with degeneracy. The dual problem, duality theorem (proof by analysing the simplex method), complementary slackness. Economic interpretation of dual variables, sensitivity analysis. Two person zero-sum games.

3.1.4 Other optional subjects

The other optional subjects are drawn from Part A of the Honour School of Mathematics and are as follows:

- Groups in Action
- Introduction to Fields
- Number Theory
- Integration
- Topology
- Multivariable Calculus
- Calculus of Variations
- Classical Mechanics
- Electromagnetism
- Fluid Dynamics and Waves
- Numerical Analysis

For the syllabuses and synopses, see those for Part A of the Honour School of Mathematics, which are available on the web at http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/maths.shtml

3.2 **Synopses of Lectures**

3.2.1 **Graph Theory** — 8 lectures HT

Aims and Objectives
The aim of the course is to introduce students to this central part of Discrete Mathematics, dating back to Euler and now important in theoretical computer science and the mathematics of operational research. For example, an attractive part of the theory concerns colouring the vertices of a graph so that adjacent vertices get distinct colours. Perhaps the vertices here correspond to examinations and two vertices are adjacent if some student sits both examinations: a colouring then corresponds to a timetable.

Synopsis
We start with basic definitions of graph, path, connected, tree, and so on: many of these definitions should come as no surprise. Trees are an important class of graphs: we consider some characterisations of trees, and how to count trees: in particular, we prove Cayley's theorem that there are n^{n-2} trees on the vertices 1, . . . , n.
Then we consider traversing graphs and digraphs. When can we walk along each edge exactly once? This is where Euler came in, and gave a neat answer. When is there a path passing through each vertex exactly once? This question (associated with Hamilton) is far harder: we find some sufficient conditions.

Some graphs sit naturally in the plane: we meet Euler's formula $v - e + f = 2$ relating the numbers of vertices, edges and faces, and we meet the dual graph. Concerning colouring the vertices of a graph, we prove for example Brooks' Theorem relating the number of colours needed to the maximum degree of a vertex; and the result that any planar graph needs at most 5 colours (we do not prove the 4-colour theorem!).
Finally we consider matchings and disjoint paths in graphs. We prove Hall's 'marriage theorem' and Menger's theorem on disjoint paths, together with various related results. The discussions on trees, matchings and disjoint paths link particularly closely with combinatorial optimisation.

Reading
R J WILSON, Introduction to Graph Theory, 4th edition, Longman (1996).
D B WEST, Introduction to Graph Theory, 2nd edition, Prentice Hall (2001).

3.2.2 **Combinatorial Optimisation** — 16 lectures HT

Aims and Objectives
Combinatorial Optimisation (CO) is concerned with methods for finding an optimal object in a finite set of objects. Typically the set may be very large but has a concise representation, for example the set of all spanning trees in a large network, and we need to use the structure of the set in order to design efficient algorithms.
Many CO problems arise from networks. The network might be of airports, or oil pipelines, or class-rooms and teachers, or telecommunications links, or simply represent the possible sequences in which a set of tasks may be carried out. The task is to devise schedules, time-tables, or priority rules which in some sense optimise the system.

Economic models from manufacturing industry and elsewhere present many such problems, and computers have also been a strong stimulus to developments in this field, both by presenting design problems for hardware and software, and as the means whereby CO algorithms may be implemented.

The aim of the course is to introduce some of the fundamental ideas in this attractive branch of modern applied mathematics, both for their own sake and because of their practical value. There are no mathematical prerequisites, though some basic graph theory, and the ideas of duality and complementary slackness from linear programming, would be helpful.

Synopsis
Finding minimal cost spanning trees, ‘greedy’ algorithms of Kruskal and Prim.
Finding shortest paths in networks, in the acyclic case, the case of non-negative edgelengths, and more generally when there are no negative cycles, algorithms of Dijkstra and Floyd. Brief introduction to the critical path method.
Knapsack problem, dynamic programming, idea of Lagrangean relaxation.
Scheduling problems, earliest due-date schedules, algorithms of Moore and Johnson.
Maximum matching in bipartite graphs. Assignment problem, the Hungarian method.
Maximum matchings in general graphs, algorithm of Edmonds (non-examinable).
Finding maximum flows in networks. The maximum-flow-minimum-cut theorem and the integrality theorem. Finding minimum cost flows.
Brief introduction to matroids and the greedy algorithm (non-examinable).

Reading
The course is based on:
C J H McDIARMID, Lecture notes on Combinatorial Optimisation, Mathematical Institute Notes (updated 2003).

For useful background on graphs and matroids:
R J WILSON, Introduction to Graph Theory, 4th edition, Longman (1996).
E L LAWLER, Combinatorial Optimisation, Networks and Matroids, Dover (2001), originally published by Holt, Rinehart and Winston (1976), pp 15–19, 59–73, 82–91, 109–115, 129–135, 182–195, 201–206, 217–238, 264–279, 300–314.

Further Reading
W J COOK, W H CUNNINGHAM, W R PULLEYBLANK and A SCHRIJVER, Combinatorial Optimisation, Wiley (1998).
S FRENCH, Sequencing and Scheduling, Ellis Horwood (1982), Chapter 3 and Sections 5.1–5.3 – out of print but in college libraries.
C H PAPADIMITRIOU and K STEIGLITZ, Combinatorial Optimisation: Algorithms and Complexity, Prentice-Hall (1982).

3.2.3 **Linear Programming** — 8 lectures TT

Aims and Objectives
Linear programming is about making the most of limited resources. Specifically, it deals with maximising a linear function of variables subject to linear constraints. Applications range from economic planning and environmental management to the diet problem. The aim is to provide a simple introduction to the subject.

Synopsis
Linear programming problems, examples. Standardisation of problems, slack variables. Sufficiency of basic feasible solutions; equivalence of basic feasible solutions and extreme points. The simplex method, excluding procedures to cope with degeneracy. The dual problem, duality theorem (proof by analysing the simplex method), complementary slackness. Economic interpretation of dual variables, sensitivity analysis. Two person zero-sum games.

Reading
V CHVATAL, Linear Programming, Freeman (1983), Chapters 1–5, 15.
K TRUSTRUM, Linear Programming, RKP (1971) – out of print, but available in college libraries, Chapters 1–5.
D G LUENBERGER, Linear and Nonlinear Programming, Addison-Wesley (1984), Chapters 2–4.

3.2.4 **Other optional subjects**
For synopses of the other optional subjects listed in Section 3.1.4, including details of recommended reading, see those for Part A of the Honour School of Mathematics, which are available on the web at
http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/maths.shtml