

When is a defined-benefit pension scheme too small for self-insurance?

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14 July 2011

Motivation

Mortality risk components:

- Systematic risk - distribution of deaths.
- Idiosyncratic risk - random fluctuations.

Outline

- 1 Homogeneous pension scheme
- 2 Executive section
- 3 Risk capital allocation

Homogeneous pension scheme

- N members all age 40.
- Benefit: £1 p.a. from age 65.

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- For example,

$$\mathbb{E}(L_N) = N v^{65-40} {}_{25}p_{40} \bar{a}_{65}.$$

Risk measure

$$\text{Coefficient of variation} = \frac{\text{standard deviation of total liability}}{\text{expectation of total liability}}$$

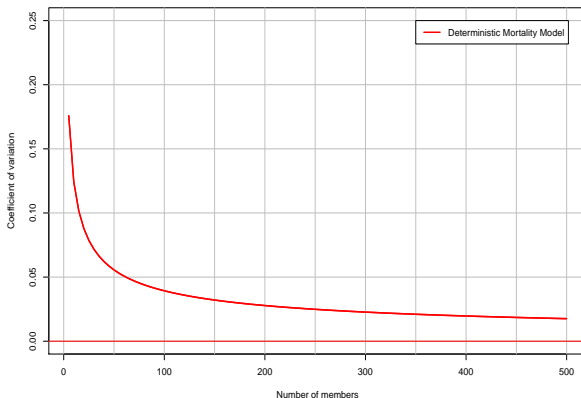
Risk measure

$$\text{Coefficient of variation} = \frac{\text{standard deviation of total liability}}{\text{expectation of total liability}}$$

If Y_1, Y_2, \dots, Y_N are independent, then

$$\text{VCo}(L_N) = \frac{\text{sd}(L_N)}{\mathbb{E}(L_N)} \rightarrow 0 \quad \text{as } N \rightarrow \infty.$$

Numerical results: deterministic mortality model (PMA92C10) and $\delta = 4\%$ p.a.



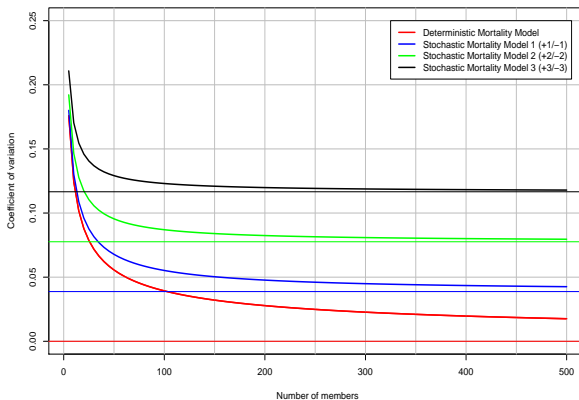
Stochastic mortality model

Stochastic mortality model r

$$\text{age rating} = \begin{cases} r & \text{with probability 0.5,} \\ -r & \text{with probability 0.5.} \end{cases}$$

based on PMA92C10.

Numerical results: $\delta = 4\%$ p.a.



Stochastic mortality model r

Here Y_1, Y_2, \dots, Y_N are not independent, and

$$\text{VCo}_r(L_N) = \frac{\text{sd}_r(L_N)}{\mathbb{E}_r(L_N)} \rightarrow \frac{\sqrt{\text{Cov}_r(Y_1, Y_2)}}{\mathbb{E}_r(Y_1)} \text{ as } N \rightarrow \infty.$$

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Interpretation:

- Systematic risk measure:

$$\frac{\sqrt{\text{Cov}_r(Y_1, Y_2)}}{\mathbb{E}_r(Y_1)}.$$

- Idiosyncratic risk measure:

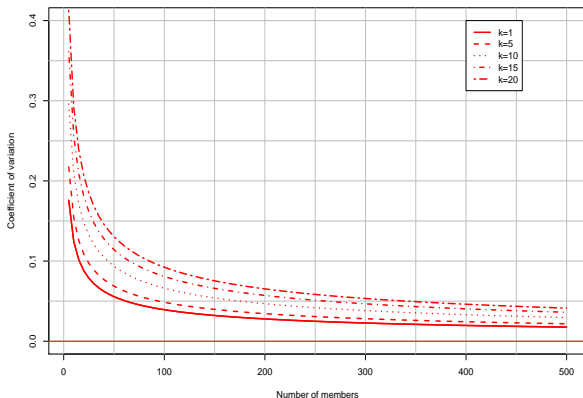
$$\text{VCo}_r(L_N) - \frac{\sqrt{\text{Cov}_r(Y_1, Y_2)}}{\mathbb{E}_r(Y_1)}.$$

Pension scheme with executive section

As before except

- αN are executives.
- Executive benefit: $\pounds k$ p.a. from age 65.

Numerical results: deterministic mortality model (PMA92C10), $\delta = 4\%$ p.a. and $\alpha = 5\%$.



Pension scheme with executive section

- Setting

$$f(\alpha, k) = \frac{\alpha k^2 + 1 - \alpha}{(\alpha k + 1 - \alpha)^2},$$

we find

$$\begin{aligned} & \text{VCo}_r(L_N) \\ &= \frac{1}{\mathbb{E}_r(Y_1)} \\ & \cdot \left(\frac{1}{N} f(\alpha, k) (\text{Var}_r(Y_1) - \text{Cov}_r(Y_1, Y_2)) + \text{Cov}_r(Y_1, Y_2) \right)^{1/2}. \end{aligned}$$

Risk capital allocation

$$\text{Risk capital} := \text{sd}_r(L_N)$$

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Find amounts $\pi_1, \pi_2, \dots, \pi_N$ such that

$$\sum_{n=1}^N \pi_n = \text{sd}_r(L_N).$$

Euler capital allocation principle

If X_n is P.V. r.v. of benefit due to member n then

$$\pi_n = \frac{\text{Cov}_r(X_n, L_N)}{\text{sd}_r(L_N)}$$

is the risk capital allocated to member n .

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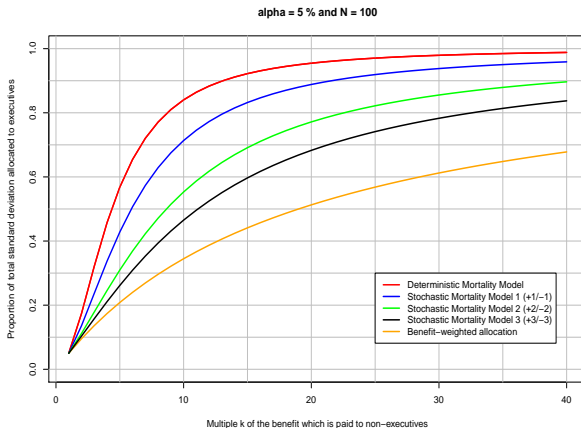
is the risk capital allocated to member n .

Consider

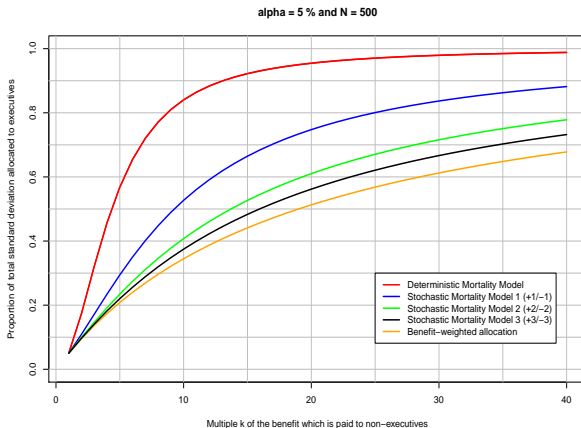
$$\frac{\sum_{\text{execs}} \pi_n}{\text{sd}_r(L_N)},$$

the proportion of risk capital allocated to executive section.

Numerical results: $\alpha = 5\%$, $N = 100$ and $\delta = 4\%$ p.a.



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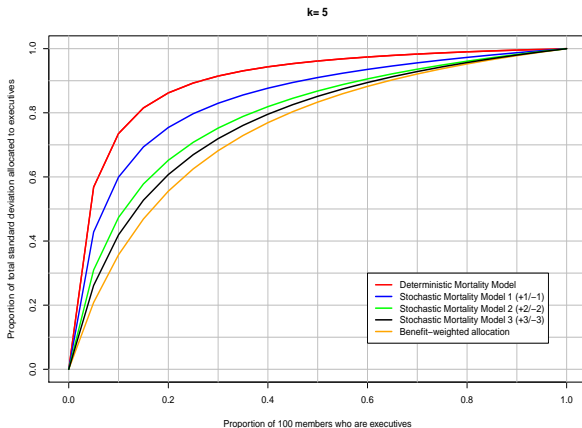
Euler capital allocation principle

We find

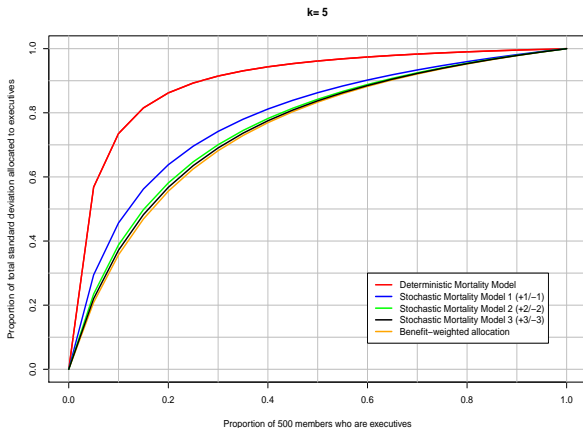
- systematic risk: execs contribute k times non-execs.
- idiosyncratic risk: execs contribute k times non-execs *plus*

$$k(k-1) \frac{\text{Var}_r(Y_1) - \text{Cov}_r(Y_1, Y_2)}{\text{sd}_r(L_N)}.$$

Numerical results: $k = 5$, $N = 100$ and $\delta = 4\%$ p.a.



Numerical results: $k = 5$, $N = 500$ and $\delta = 4\%$ p.a.



Future work

- More complex examples.
- Incorporate financial risks.
- Risk mitigation strategies.

Acknowledgements and references

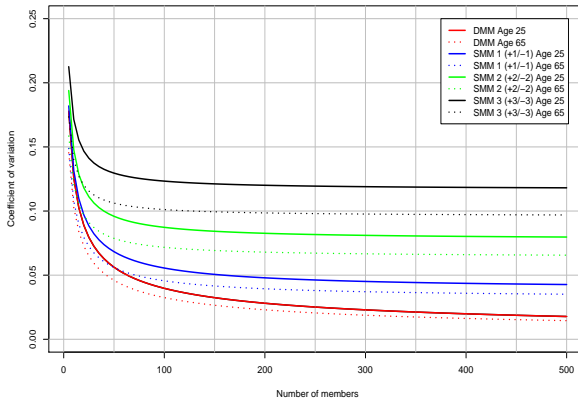
Thanks to The Actuarial Professional for financial support to attend this conference.

More details:

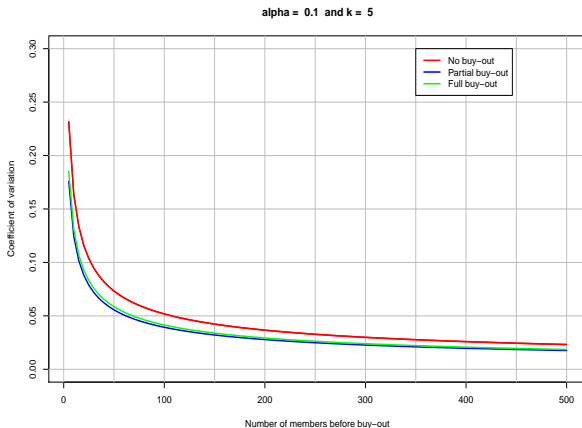
2011 C. Donnelly. *Quantifying mortality risk in a small defined-benefit pension schemes.*

<http://arxiv.org/abs/1107.1380>.

Effect on VCo of change in age: $\delta = 4\%$ p.a.



Effect on VCo of executive buyout: $\delta = 4\%$ p.a.



Effect on standard deviation of executive buyout: $\delta = 4\%$ p.a.

