

Combinatorial Optimisation
HT 2010

Chapter 0: What is Combinatorial Optimisation?

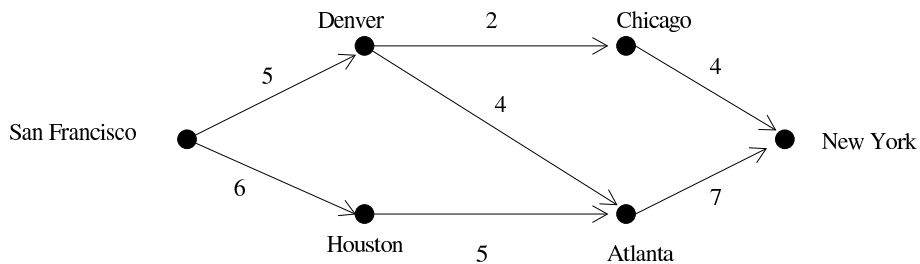
Example 1: the assignment problem

You manage a racing car team with five drivers I - V and five cars A - E. You are entered for a race where the winning team is the one with the lowest total time for all five cars. The table gives the expected time in which a driver will complete the course when paired with each car. Minimise the total expected time.

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
<i>I</i>	3	5	6	9	10
<i>II</i>	4	8	8	11	13
<i>III</i>	6	9	10	12	14
<i>IV</i>	8	10	10	15	16
<i>V</i>	13	13	17	18	20

Example 2: the maximum flow problem

A group of travellers wants to fly from San Francisco to New York City at short notice. The seat availabilities on possible flights are as shown. How many people can make it?



Example 3: minimum spanning trees

You wish to construct a cable-network that connects the sites *A, B, C, D, E*. The cost of building a direct connection between two sites are given in the table below. Connect all the sites whilst minimising the total cost.

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
<i>A</i>	0	1	3	5	8
<i>B</i>	1	0	2	4	7
<i>C</i>	3	2	0	5	9
<i>D</i>	5	4	5	0	6
<i>E</i>	8	7	9	6	0

So, what is combinatorial optimisation? In order to minimise $f(x)$ over $x \in [0, 1]$ we are used to setting $f'(x) = 0$. This approach will not work for us here. Typically, we will be given a large but finite domain S , for example the set of all spanning trees in a graph, and we are to find an $x \in S$ minimising $f(x)$. The set S may be huge but will be described in some compact way, and we need to understand its structure in order to find good ways to solve the problem.

Plan of the course

After this introduction, the plan is to break the lectures into the following chapters, with a rough indication of the corresponding lectures.

- 1 Minimum spanning trees (L 1-2)
- 2 Shortest paths (L 3-4)
- 3 Dynamic programming (L 5-6)
- 4 Scheduling (L 6-7)
- 5 Matchings in bipartite graphs (L 8-9)
- 6 Assignment problem (L 9-10)
- 7 Flows in networks (L 11-12)

Housekeeping

Lectures will be on Mondays and Wednesdays at 10am in weeks 1-2 and 4-7. There will be 2 problem sheets, with corresponding problem classes in weeks 5 and 8. Lecture notes (and perhaps a few slides) and the problem sets will be available soon from my web page, under 'teaching'. The lecture notes are adapted from the lecture notes from last year, still on the web.

Further reading:

- R.K. Ahuja, T.L. Magnanti and J.B. Orlin, *Network Flows*, Prentice Hall, 1993.
- V. Chvátal, *Linear Programming*, Freeman, 1983.
- T.H. Cormen, C.E. Leiserson and R.L. Rivest, *Introduction to Algorithms*, MIT Press, 1990.
- W.J. Cook, W.H. Cunningham, W.R. Pulleyblank and A. Schrijver, *Combinatorial Optimization*, Wiley, 1998.
- M. Grötschel, L. Lovász and A. Schrijver, *Geometric Algorithms and Combinatorial Optimization*, Springer-Verlag, 1988.
- A. Schrijver, *Combinatorial Optimisation*, Springer 2003.
- R.J. Wilson, *Introduction to Graph Theory*, 4th ed., Longman, 1996.