18. Title **Graphs with few disjoint cycles**

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Brief Description:
Graphs with few disjoint cycles (that is, pairwise vertex-disjoint cycles) are well understood in some ways. Call a set of vertices in a graph a {\em blocker} if the graph obtained by deleting these vertices has no cycles. The classical theorem of Erd\H{o}s and P\'{o}sa from 1965 states that for each positive integer k there is a positive integer f(k) such that the following holds:
each graph which does not have k+1 disjoint cycles, has a blocker of size at most f(k). The least value we may take for f(k) here is of order k \log k.

For the case k=1 of no two disjoint cycles a much fuller story is known, characterising the graphs with no blocker consisting of a single vertex. Also, in very recent work (Kurauskas and McDiarmid, as yet unpublished) it has been shown that most graphs with which do not have k+1 disjoint cycles have a blocker of size just k. Indeed, if we consider all the graphs on n vertices which do not have k+1 disjoint cycles, the proportion of these graphs for which the minimum size of a blocker is not k is exponentially small as n tends to infinity.

The project is to survey what is known and to investigate this area further, in one or both of the following ways:
(a) experimentally, for fixed small k and for modest values of n, how does the least size of a blocker usually behave?
(b) theoretically, what can we say about the case k=2?
There are also related questions which could be addressed experimentally or theoretically.

Prerequisite courses: Part C Graph Theory strongly advised
Type of project: theoretical/computational/simulation?
Computing required? Yes for (a), at a modest level of programming skill