20. Title: **Branching processes and random trees**

Supervisor: Dr Matthias Winkel

Prerequisites: BS3a Applied Probability required
B10a Martingales Through Measure Theory desirable

Type: Theoretical project with simulation component if desired

Description: Galton-Watson processes are discrete-time Markov chains that model population sizes. One should think of time as generation. The rules are such that one starts with an ancestor; each individual gives birth to an independent number of offspring, drawn from an offspring distribution on N. Depending on the mean number of offspring, the population will die out in finite time or survive forever with positive probability. In the first case one can e.g. study the extinction time, in the second one can study the asymptotic behaviour of the population size and establish limit results.

There are variants of the model such as several types of individuals or lifetimes for the individuals. The latter leads to continuous-time Markov chains. There are even analogues with continuous population sizes (just as continuous random variables, in general, are often useful as approximations of large discrete random variables). Each of this could provide a possible direction of the project.

More explicitly, one can study the family tree of such a model (which contains more information than just the population size process), called a Galton-Watson tree. There are different codings of such random trees, and a possible direction of the project could be to study some of these.

References:
• T.E. Harris: *The theory of branching processes*. Springer 1963
• K.B. Athreya, P.E. Ney: *Branching processes*. Springer 1972
• S. Asmussen, H. Hering: B*ranching processes.* Birkhäuser 1983
• J.-F. Le Gall: Random trees and spatial branching processes. *MaPhySto Lecture Notes* no.9 (2000)