4. **Approximating Markov chains by differential equations**

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Brief Description: As you saw in Part A **Probability** and Part B *Applied Probability*, Markov chains (in both discrete and continuous time) are a very useful tool for modelling random phenomena which evolve in time in a great number of different application areas. To give a simple concrete example (taken from the Darling and Norris paper mentioned below), consider a gunfight between two gangs. On each side, each surviving gunman hits one of the opposing side randomly, with members of gang A hitting an opponent at rate alpha and at members of gang B hitting an opponent at rate beta. Write A_t and B_t for the numbers alive on each of the two sides at time t. Then $(A_t, B_t)_{t \ge 0}$ evolves as a (2-dimensional) continuous-time Markov chain (see if you can write down the rates!). Note that the numbers of gunmen decrease until one side has none left. Suppose now that the gangs are very large, so that $A_0 = N a_0$ and $B_0 = N b_0$, where N is big. Then it turns out that $(A_t/N,B_t/N)_{t \ge 0}$ is well approximated by the solution $(a(t), b(t))_{t \ge 0}$ to the pair of differential equations

da(t)/dt = - beta b(t), db(t) = - alpha a(t)

with $a(0) = a_0$ and $b(t) = b_0$. (Which side wins?!)

The purpose of this project is to give an account of the use of differential equations to approximate suitable Markov chains. The project should have two aspects: a theoretical part and an applications part. There are nice survey papers by Wormald (in the context of Markov chains arising in the analysis of random graphs and greedy algorithms) and by Darling and Norris (which is rather more technical); these would be a good starting point for the theory. The student should then aim to give an account of one or more applications of the method. These can be dictated by the student's own interests; two possibilities would be epidemic models and cores in random graphs.

Prerequisite courses/knowledge: Part A *Probability*, BS3a *Applied Probability*. Part B *Martingales Through Measure Theory* and Part C *Probabilistic Combinatorics* would tie in well with this project but are not essential; a willingness to learn about martingales, however, is essential!
Data available? (if relevant) N/A