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EXPLAINED VARIATION IN DYNAMIC NETWORK MODELS[1]

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RÉSUMÉ – Une mesure de la part de variation expliquée par les modèles dynamiques de réseau
On propose une mesure de la part de variation expliquée par un modèle stochastique de la dynamique des réseaux sociaux complets. Cette mesure est fondée sur l'entropie de la distribution des choix faits par les acteurs au cours du processus d'évolution du réseau. Elle a pour but d'aider à effectuer une meilleure interprétation et à sélectionner une spécification appropriée dans l'application des modèles statistiques s'appliquant aux données longitudinales concernant des relations.

MOTS CLÉS – Réseau complet, Dynamique, Analyse longitudinale, Variation expliquée, Coefficient de Détermination, Entropie.

SUMMARY – *A measure for explained variation is proposed for stochastic actor-driven models for data on social networks. The measure is based on the entropy of the distribution of the choices made by the actors during the network evolution process. This measure can be helpful in the specification and interpretation of statistical models for longitudinal network data.*

KEYWORDS – Complete network, Longitudinal study, Dynamics, Explained variation, Coefficient of Determination, Entropy.

1. INTRODUCTION

Social networks are representations of relations in groups of individuals or other social actors (e.g., [Degenne, Forsé, 1994; Wasserman and Faust, 1993]), and the longitudinal study of social networks can yield important insights in the context-dependent rules of individual behavior [Doreian, Stokman, 1997]. The present paper focuses on entire, or complete, networks, which represent the collection of all ties (according to some predefined criterion) between all members of a given group. The complex patterns of dependencies between the corresponding tie variables preclude the possibility of basing statistical modeling on straighforward independence assumptions, and therefore statististical models for entire networks have to be quite complex in order to be even slightly realistic.

Statistical models for longitudinal network data, collected at two or more discrete moments in time, were proposed by Snijders and van Duijn [1997] and Snijders

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[2001]. These models are based on the assumption that the network dynamics continue – unobserved – in between the observation moments, and that this dynamics is the result of choices made by the individual actors in the network. These choices, in their turn, are modeled as expressing a combination of purposeful action and random influences. Methods for parameter estimation in these models have been developed based on the Method of Moments, implemented by stochastic approximation [Snijders 2001; Snijders *et al.*, 2004].

For the interpretation of statistical models fitted in this way, it could be helpful to have available a measure of explained variation, analogous in some way to the familiar coefficient of determination, or explained variance, usually denoted by R^2, in linear regression analysis. In models more complicated than the linear regression model, such measures do not have the compelling nature of R^2, and for many models several different measures of explained variation of been proposed – e.g., see Cameron and Windmeijer [1997], Long [1997, Section 4.3], and Menard [2000]. This paper proposed such a measure for the stochastic actor-driven model for network evolution.

The paper is structured as follows. The actor-driven model is described in Section 2, and the proposed measure for explained variation is discussed in Section 3. An empirical example is presented in Section 4, and the paper is concluded with a brief discussion in Section 5.

2. STOCHASTIC ACTOR-DRIVEN MODELS FOR NETWORK DYNAMICS

We consider one binary relation on a set of n actors – who may be a group of individual humans, or animals, companies, countries, etc. Actors will be arbitrarily referred to by masculine pronouns. The relation can be represented equivalently by a digraph or its square adjacency matrix. The time-dependent adjacency matrix will be denoted $\mathbf{X}(t) = (X_{ij}(t))$, where i and j range from 1 to n, and t represents time. The variable $X_{ij}(t)$ indicates whether at time t there is a tie from i to j (value 1) or not (value 0), and the diagonal of the adjacency matrix is defined to be 0, $X_{ii}(t) = 0$ for all i. Although in practice there will be a strong dependence between the two reciprocal tie variables $X_{ij}(t)$ and $X_{ji}(t)$, these are not assumed to be necessarily equal to each other. In most instances a continuous time record is impossible to obtain (although there are exceptions, e.g., when ties represent links between companies of which the start and end moments are officially recorded). It is assumed here that observations at intermittent moments are available; then we have a network panel study. The observation moments are denoted t_m for $m = 1, \ldots, M$, where the number M of repeated observations is 2 or larger. In this section we sketch the model that is presented in more detail in Snijders [2001] and Snijders [2005], and applications of which can be found, e.g., in Van de Bunt et al. [1999] and van Duijn, Zeggelink, Huisman, Stokman, and Wasseur [2003].

One of the basic assumptions in the model is that the network evolution continues between the observation moments, which is a natural assumption in the majority of applications. The network is assumed to change by only one tie at a time, at random moments, and the model is actor-driven in the sense that for each change in the network, the perspective is taken of the actor 'whose tie' is changing. Such changes are called *ministeps*. It is assumed that actor i controls the set of outgoing tie variables $(X_{i1}, \ldots, X_{in})$, collected in the i'th row of the adjacency matrix. The change made by the actor can depend on the network structure and on attributes represented by observed covariates. In the model of Snijders [2001], the moments when changes in the network are made, are stochastically determined by the *rate function*; the particular change made is determined by the *objective function* and the *gratification function*. In this paper, a simplified version of the model is explained where the rate function does not depend on the actor or the network, and there is no gratification function.

It is assumed that the moments where the actors may change one of their tie variables are randomly determined (i.e., the moments of the ministeps), and that for each actor the expected frequency of ministeps between the consecutive observation moments t_m and t_{m+1} is λ_m per unit of time.

Formally, this means that the ministeps occur as a Poisson process with parameter λ_m. We say that the rate function is equal to $\lambda(t) = \lambda_m$ for $t_m \leq t < t_{m+1}$. The moments where different actors make ministeps are independent, which implies that the probability is zero that two or more actors make a change at the same time.

The objective function can be regarded as a representation of the preference order, or utility, or objectives, of the actors, depending on their position in the network and on observed covariates. When actor i makes a ministep, this actor selects the change which gives the greatest increase in the objective function plus a random term. It is assumed that if there are differences between actors in their objective functions, these can be identified on the basis of covariates.

When actor i makes a ministep, he either does nothing or he changes how he is tied to exactly one of the $n-1$ other actors. Given that the present network is denoted by $\mathbf{x} = \mathbf{X}(t)$, the new network that would result by changing the single tie variable x_{ij} into its opposite $1 - x_{ij}$ is denoted $\mathbf{x}(i \rightsquigarrow j)$ (to be interpreted as "the digraph obtained from $\mathbf{x}$ when i changes the tie variable to j"). A convenient notation is obtained by formally defining $\mathbf{x}(i \rightsquigarrow i) = \mathbf{x}$, corresponding to no change. The choice is modeled as follows. Denote by $U(j)$ a random variable which indicates the unexplained, or residual, part of the attraction for i to j. These U_j are assumed to be random variables distributed symmetrically about 0 and independently generated for each ministep (this is left implicit in the notation). The actor chooses to change his tie variable with that other actor j for whom the value of

$$f_i(\mathbf{x}(i \rightsquigarrow j)) + U(j)$$

is highest; where $j = i$ means that no change is made. This can be regarded as a myopic stochastic optimization rule: myopic because only the situation obtained immediately after the ministep is considered, stochastic because the unexplained part is modeled by means of a random variable.

A convenient and traditional choice for the distribution of $U(j)$ is the type 1 extreme value distribution, or Gumbel distribution, with mean 0 and scale parameter 1 [Maddala, 1983]. Under this assumption, the probability that i chooses to change

x_{ij} for any particular j, given that i has a ministep, is given by

$$\begin{aligned} p_{ij}(\mathbf{x}) &= \frac{\exp\left(f_i(\mathbf{x}(i \leadsto j))\right)}{\sum_{h=1}^{n} \exp\left(f_i(\mathbf{x}(i \leadsto h))\right)} \\ &= \frac{\exp\left(f_i(\mathbf{x}(i \leadsto j)) - f_i(\mathbf{x})\right)}{\sum_{h=1}^{n} \exp\left(f_i(\mathbf{x}(i \leadsto h)) - f_i(\mathbf{x})\right)} \ . \end{aligned} \tag{1}$$

This probability is also used in multinomial logistic regression, cf. Maddala [1983, p. 60].

The model thus described defines a continuous-time Markov chain on the space of all digraphs, with intensity matrix defined by

$$\begin{aligned} q_{ij}(\mathbf{x}) &= \lim_{dt \downarrow 0} \frac{1}{dt} \, \mathrm{P}\{\mathbf{X}(t+dt) = \mathbf{x}(i \leadsto j) \mid \mathbf{X}(t) = \mathbf{x}\} \\ &= \lambda(t) \, p_{ij}(\mathbf{x}) \qquad\qquad (i \neq j) \end{aligned} \tag{2}$$

where $p_{ij}(\mathbf{x})$ is given by (1). Expression (2) is the rate at which actor i makes ministeps, multiplied by the probability that, *if* he makes a ministep, he changes the arc variable X_{ij}. This Markov process will in general not be stationary. The Markov chain is defined to be left-continuous, which means that the changes are defined to occur immediately *after* the ministep, i.e.,

$$\lim_{t' \uparrow t} \mathbf{X}(t') = \mathbf{X}(t) \ .$$

The option that, at an occasion to make a change, the actor prefers to leave his collection of outgoing ties as they are, is a difference with respect to Snijders [2001, 2005]. It is attractive to include this possibility because it expresses the plausible property that actors who are more satisfied with the current network will have a smaller probability of making changes.

2.1. SPECIFICATION OF THE OBJECTIVE FUNCTION

In the model proposed in Snijders [2001], the objective function is represented as a weighted sum dependent on a parameter $\beta = (\beta_1, \ldots, \beta_L)$,

$$f_i(\beta, \mathbf{x}) = \sum_{k=1}^{L} \beta_k \, s_{ik}(\mathbf{x}) \ . \tag{3}$$

The functions $s_{ik}(\mathbf{x})$ represent meaningful aspects of the network, as seen from the viewpoint of actor i. Some possible functions $s_{ik}(\mathbf{x})$ are the following; further possibilities and explanations are given in the mentioned references.

1. *Out-degree effect*, defined by $s_{i1}(\mathbf{x}) = x_{i+} = \sum_j x_{ij}$;

2. *reciprocity effect*, defined by the number of reciprocated ties$s_{i2}(\mathbf{x}) = x_{i(r)} = \sum_j x_{ij} \, x_{ji}$;

3. *transitivity effect*, defined by the number of transitive triplets in i's ties. A transitive triplet for actor i is an ordered pairs of actors (j, h) to both of whom i is tied, while also j is tied to h. The transitivity effect is given by$s_{i3}(\mathbf{x}) = \sum_{j,h} x_{ij}\, x_{ih}\, x_{jh}$;

4. *number of geodesic distances two effect*, or indirect relations effect, defined by the number of actors to whom i is indirectly tied (through at least one intermediary, i.e., at geodesic distance 2),$s_{i4}(\mathbf{x}) = \sharp\{j \mid x_{ij} = 0,\ \max_h(x_{ih}\, x_{hj}) > 0\}$.

Positive transitivity and negative number-of-distances-two effects are two different mathematical representations of a tendency toward network closure.

When covariates are available, the functions $s_{ik}(\mathbf{x})$ can be dependent on them. For network data, a distinction should be made between actor-bound covariates v_i and dyadic covariates w_{ij} . The main effect for a dyadic covariate w_{ij} is defined as follows.

5. *Main effect of W (centered)*, defined by the sum of the values of w_{ij} for all others to whom i is tied,$s_{i5}(\mathbf{x}) = \sum_j x_{ij}\, (w_{ij} - \bar{w})$where $\bar{w}$ is the mean value of w_{ij} .

For each actor-dependent covariate V the following three effects can be considered:

6. *V-related popularity*, defined by the sum of the covariate over all actors to whom i is tied,$s_{i6}(\mathbf{x}) = \sum_j x_{ij}\, v_j$;

7. *V-related activity*, defined by i's out-degree weighted by his covariate value,$s_{i7}(\mathbf{x}) = v_i\, x_{i+}$;

8. *V-related similarity*, defined inversely by the sum of absolute covariate differences between i and the others to whom he is tied,

$$s_{i8}(\mathbf{x}) = \sum_j x_{ij} \left(1 - \frac{\mid v_i - v_j \mid}{m_v}\right) ,$$

   where m_v is the maximum of the absolute differences $\mid v_i - v_j \mid$.

The network evolution model is too complicated for explicit calculation of probabilities or expected values, but it can be simulated in a rather straighforward way. This is exploited in the method for parameter estimation, based on the method of moments and implemented by stochastic approximation, which was first proposed in Snijders [1996] and elaborated for the present model in Snijders [2001].

The null hypothesis that a single element of the parameter vector is zero,

$$H_0 :\ \beta_k\ = 0\ ,$$

can be tested by the t-statistic

$$t_k\ = \frac{\hat{\beta}_k}{\text{s.e. } (\hat{\beta}_k)} \tag{4}$$

in the standard normal distribution.

3. AN ENTROPY-BASED MEASURE FOR EXPLAINED VARIATION

For models as complex as the present one, it is possible in principle to propose various different measures of explained variation. In many models, an attractive principle for defining such measures is the principle of reduction of predictive uncertainty. However, the endogenous feedback processes inherent in this network evolution model imply that the network as such is highly unpredictable, and mainly the structure of the network is predictable to some extent. E.g., in a situation of an digraph with originally a very low average degree, it could be predicted that the digraph produced after some time will have a high degree of reciprocity, but it will be very hard to predict accurately *which* dyads will be mutually tied.

For this reason the measure of explained variation will not be based on the accuracy with which $\mathbf{x}(t_{m+1})$ can be predicted from $\mathbf{x}(t_m)$. Instead the measure will be based on the local or instantaneous predictability, more specifically, the degree of uncertainty in the decisions taken by the actors in their ministeps. This corresponds well with the actor-driven nature of the model. Each ministep amounts to a choice among n different possibilities. For discrete probability distributions with probabilities p_j $(j = 1, \ldots, n)$, the uncertainty in the outcome is defined by the well-known entropy measure [Shannon, 1948]

$$H(p) \;=\; -\sum_{j=1}^{n} p_j \, {}^2\!\log p_j \; . \tag{5}$$

The entropy is minimal and equal to 0 in the case of certainty, where one outcome j has probability 1 and all others probability 0. It is maximal and equal to ${}^2\!\log(n)$ if all outcomes are equiprobable. For a given ministep made by actor i in a current network $\mathbf{x}$, the degree of certainty in the outcome can be measured by

$$R_H(i, \mathbf{x}) \;=\; 1 \;-\; \frac{H(p_i(\mathbf{x}))}{{}^2\!\log(n)} \tag{6}$$

where $p_i(\mathbf{x}) = (p_{i1}, \ldots, p_{in})$ is given by (1). This measure is bounded between 0 and 1, and 0 is obtained under the greatest uncertainty – the discrete uniform distribution –, while 1 is obtained under complete certainty concerning the choice made by i. The constancy of the rate function implies that at each moment all actors have the same probability of making ministep. Hence the proposed measure of explained variation, or reduction of uncertainty, is defined by

$$R_H(t) \;=\; \frac{1}{n} \sum_{i=1}^{n} \mathrm{E} R_H(i, \mathbf{X}(t)) \; . \tag{7}$$

This is a time-dependent measure since the non-stationarity of the Markov process $\mathbf{X}(t)$ implies that the degree of uncertainty is not necessarily constant over time. An aggregate measure R_H can be obtained by averaging this over t.

The statistical analysis of data according to this model is based on simulations, as described in Snijders [2001, 2005]. Denote the random finite set of time moments of ministeps in the N^{th} simulated network evolution by $\mathcal{T}_N$ and for $t \in \mathcal{T}_N$ denote by $I_N(t)$ the actor making the ministep. The measure (7) can be estimated from simulations by the average of the simulated values (6) for all ministeps at time points between $t-\epsilon$ and $t+\epsilon$,

$$\hat{R}_H(t) = \frac{\sum_N \sum_{t' \in \mathcal{T}_N, t-\epsilon \le t' < t+\epsilon} R_H(I_N(t'), \mathbf{X}_N(t'))}{\sum_N \sharp\{t' \in \mathcal{T}_N,\, t-\epsilon \le t' < t+\epsilon\}}\ , \tag{8}$$

where $\sharp A$ denotes the number of elements in the set A and ϵ is a small positive number. Note that, since the Markov process is defined to be left-continuous, the digraph $\mathbf{X}_N(t')$ is the simulated digraph immediately before the ministep at moment t'. Averaging these values computed over disjoint time intervals of the same length yields an overall measure of explained variation, denoted $\hat{R}_H$.

In the estimation algorithm described in Snijders [2001], after the estimate has been obtained, a number of simulations are done for this estimated parameter value in the so-called Phase 3. From the simulations, the estimate (8) can be computed. In the SIENA program [Snijders, Huisman, Steglich, Schweinberger, 2004], the default number of simulations in Phase 3 is 500. To have a reasonable precision for estimate (8), the number of simulations and the value of ϵ have to be such that the denominator of (8) is at least a few hundred.

4. EXAMPLE APPLICATION TO THE EVOLUTION OF A FRIENDSHIP NETWORK

As an example, the network of 32 freshmen students is used that was studied and described more extensively by van de Bunt [1999] and also by van de Bunt, van Duijn, and Snijders [1999]. The network consists of 32 freshmen students in the same discipline at a university in The Netherlands, who answered a questionnaire with sociometric (and other) questions at seven times points during the academic year, coded t_0 to t_6, spaced three to six weeks apart. This data set is distributed with the SIENA program [Snijders *et al.*, 2004]. The relation studied here is defined as a 'friendly relation', as defined in van de Bunt [1999].

Descriptive statistics are presented in Table 1. The mutuality index is defined as the fraction of reciprocated ties among all ties.

Time	t_0	t_1	t_2	t_3	t_4	t_5	t_6
Average degree	0.19	3.78	4.63	5.60	6.95	7.73	6.96
Mutuality index	0.67	0.66	0.67	0.64	0.66	0.74	0.71

Table 1. Basic descriptives.

The average degree, starting at virtually nil, rises rapidly to a value about 7. The mutuality index is quite high at approximately 0.7.

This example was analysed using SIENA version 2.0 [Snijders et al., 2004]. In addition to the structural effects, effects of three covariates were considered: gender, programme, and smoking. Gender and smoking are dummy variables coded 1 for female and 2 for male and, respectively, 1 for smoking and 2 for non-smoking. Programme is a numerical variable coded 2, 3, 4 for the length in years of the programme followed by the students. Greater similarity on this variable indicates a greater opportunity for interaction. All covariates are centered by SIENA (i.e., the mean is subtracted), including the similarity variables internally defined as $(1 - (| v_i - v_j |)/m_v)$, where m_v is the average of all $| v_i - v_j |$ values.

Models were fitted sequentially, starting with a very limited specification and then adding effects to obtain eventually a specification that is close to the specification reported in Snijders [2005]. Compared to the latter specification, the difference is that the rate function is constant and in order to obtain a good fit also a non-linear function of the out-degrees is included in the objective function,

$$s_{ik}(\mathbf{x}) = \frac{1}{x_{i+} + 1} \; .$$

For the definition of the rate parameters, the numerical values of the total time length is arbitrarily set equal to 1.0, equally divided between the 6 periods. The parameter estimates are presented in Table 2.

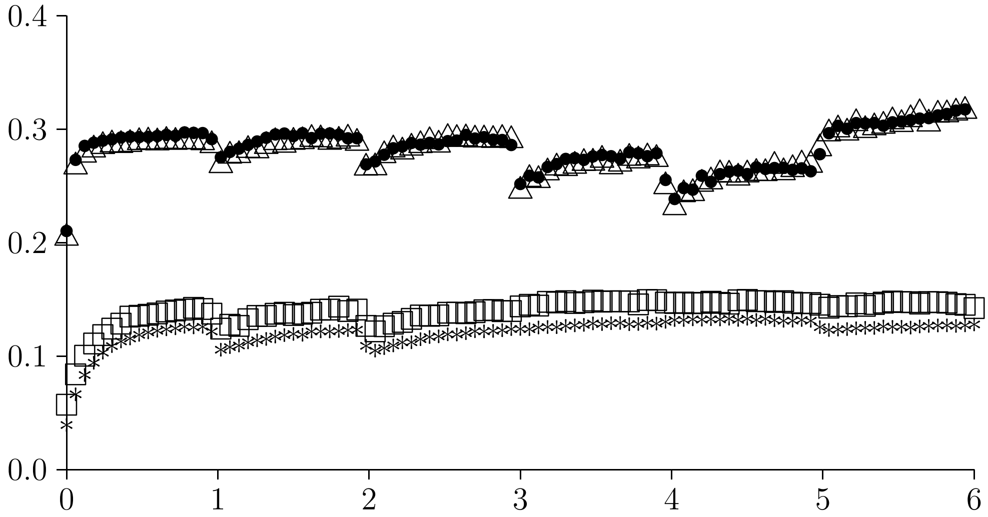
	Effect	Estimate	Standard error
Rate function			
λ_0	Rate parameter t_0–t_1	29.79	8.94
λ_1	Rate parameter t_1–t_2	4.55	0.72
λ_2	Rate parameter t_2–t_3	5.55	0.99
λ_3	Rate parameter t_3–t_4	3.71	0.61
λ_4	Rate parameter t_4–t_5	5.05	0.70
λ_5	Rate parameter t_5–t_6	4.08	0.58
Objective function			
β_1	Out-degree x_{i+}	−2.22	0.22
β_2	Reciprocity	1.90	0.16
β_3	Transitive triplets	0.13	0.07
β_4	Number of distances 2	−0.49	0.05
β_5	Transformed out-degree $1/(x_{i+} + 1)$	1.29	0.50
β_6	Gender similarity	0.38	0.15
β_7	Gender popularity	0.41	0.11
β_8	Gender activity	−0.02	0.13
β_9	Program similarity	0.64	0.15
β_{10}	Smoking similarity	0.30	0.12

Table 2. Parameter estimates.

The table shows, judging by the t-ratios of parameter estimate divided by standard error, that there is strong evidence for the reciprocity effect and the network closure effect expressed by a relatively low number of distances two, and weak evidence that in addition there is a network closure effect expressed by a relatively high number of transitive triplets. The covariate effects show that male students tend to attract more choices than females, and that similarity on gender, program, and smoking behavior leads to a higher likelihood of a tie; male and female students do not differ in the propensity to make choices.

Figure 1 exhibits the estimated explained variation (8) as a function of time, estimated for four models, differing in the sets of effects included in the objective function: the almost trivial model with only the out-degree and reciprocity effects, represented by $*$; the model with these two effects together with the five covariate effects, represented by $\square$; the model with only the five structural effects (out-degree, reciprocity, transformed out-degrees, transitive triplets, and geodesic distances equal to 2), represented by $\triangle$; and the full model including all effects of Table 2, represented as $\bullet$. Time is arbitrarily scaled in such a way that $t_m = m$ for all m.

Figure 1. Explained variation for four models (see text).



We see that the covariates have a very small extra effect on the explained variation, especially when taken in addition to the five structural effects. The effect of the three structural effects taken together – transitive triplets, number of geodesic distances equal to 2, and transformed out-degree – has a much larger contribution to the explained variation than the covariates. The averages of the four curves are $\hat{R}_H = 0.119$, 0.137, 0.280, and 0.284. At $t = 0$, where the average degree is small, there is much uncertainty in the choices made by the actors, especially in the models without the network closure effects; but immediately after $t = 0$ the certainty increases fast as time progresses. There are noticeable downward jumps in the two higher curves at the observation moments, which perhaps points to a lack of fit in the model in the sense that there is a trend in some of the parameter values which the model does not represent.

Finally, a comparison is made between the statistical significance and the effect size of variables, indicated respectively by the t-ratio for an added effect, and its contribution to $\hat{R}_H$. Models were estimated consecutively adding effects one by one. The average values of the explained variation measures, the increase $\Delta\hat{R}_H$ in $\hat{R}_H$, and the t-values for the added effects, are presented in Table 3.

Added effect	$\hat{R}_H$	$\Delta\hat{R}_H$	t (added)
Reciprocity	0.1190		13.5
Transitive triplets	0.2057	0.0867	2.9
Distances 2	0.2676	0.0619	11.8
Transformed out-degrees	0.2801	0.0125	1.9
Sex similarity	0.2816	0.0015	2.2
Sex popularity	0.2807	–0.0009	3.1
Sex activity	0.2806	–0.0001	0.4
Program similarity	0.2823	0.0017	3.9
Smoking similarity	0.2840	0.0017	2.4

Table 3. Explained variation $\hat{R}_H$ and t-values for added effects, for consecutively fitted models. $\Delta\hat{R}_H$ is the difference between $\hat{R}_H$ in the current and the preceding line.

It can be concluded from this table that the t-ratios for added effects are not at all nearly proportional to $\Delta\hat{R}_H$. E.g., adding the transitive triplets has a t ratio of only 2.9, but increases $\hat{R}_H$ from 0.1190 to 0.2057, whereas the t ratio for the number of distances 2 is 11.8 but this effect only increases $\hat{R}_H$ further to 0.2676, a smaller difference than the preceding one. Adding the covariate effects leaves the explained variation hovering about 0.28, while some of them are clearly significant. This underscores that statistical significance and effect size should not be confused. We also see that sometimes, adding an effect to the model does not lead to a higher value for $\hat{R}_H$. This may be attributable to sampling error. In addition, it should be noted that we have no proof that R_H will increase when effects are added.

5. DISCUSSION

This paper has proposed a measure for explained variation of actor-driven models for network dynamics. By utilizing an inverse measure of entropy, it reflects the degree of certainty of the actors when making changes in their pattern of outgoing ties. Intuitively this is an appealing measure, although a disadvantage is that it is strongly model-based and a long story needs to be told to explain how it is obtained from the data.

Further experience with this measure will have to be collected to obtain better insights into what may be considered low and high values. The measure is included in version 2.0 of the SIENA program which is included in the StOCNET system and can be downloaded from http://stat.gamma.rug.nl/stocnet/, [Snijders *et al.*, 2004].

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