Part B Statistical Lifetime-Models

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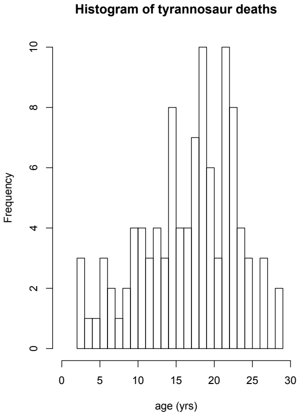
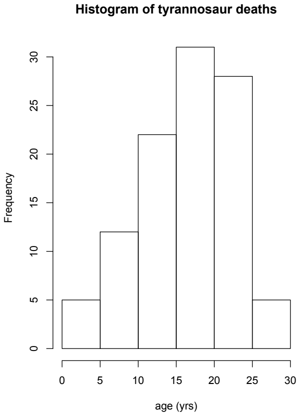
Estimated ages at death for 103 tyrannosaurs, from four different species, as reported in Erickson, Currie, Inouye and Winn (2006).

A. sarcophagus	2,4,6,8,9,11,12,13,14,14,15,15,16,17, 17,18,19,19,20,21,23,28
T. rex	2,6,8,9,11,14,15,16,17,18,18,18,18,18,19,21,21,21, 22,22,22,22,22,22,23,23,24,24,28
G. libratus	2,5,5,5,7,9,10,10,10,11,12,12,12,13,13, 14,14,14,14,14,15,16,16,17,17,17, 18,18,18,19,19,19,20,20,21,21,21,21,22
Daspletosaurus	3,9,10,17,18,21,21,22,23,24,26,26,26



(a) Narrow bins



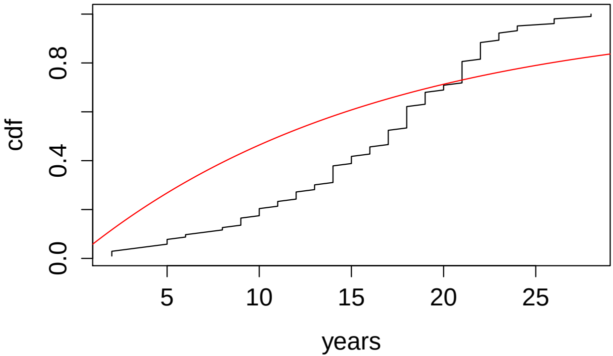
(b) Wide bins

Exponential model:

$$f(T;\mu) = \mu e^{-\mu T}.$$

Tyrannosaur data:

$$\begin{aligned} n &= 103, \\ \sum T_i &= 1652, \\ \bar{T} &= 16.03, \\ \hat{\mu} &= 0.062, \\ SE_{\hat{\mu}} &= 0.0061, \\ 95\% \text{ C.I. for } \hat{\mu} &: (0.050, 0.074). \end{aligned}$$



Comparison of empirical CDF for tyrannosaur data to exponential approximation.

Exponential model:

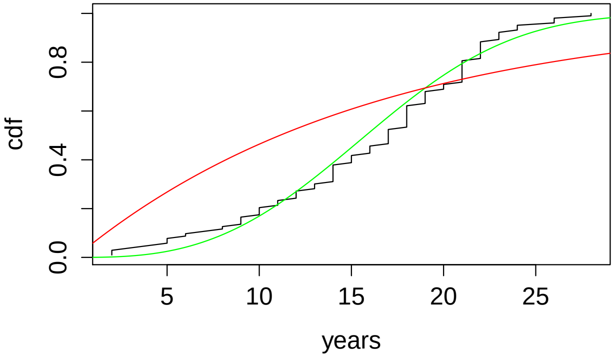
$$F(T;\mu) = 1 - e^{-\mu x}.$$

Weibull model:

$$F(T;\beta,\alpha) = 1 - e^{-(\beta x)^{\alpha}}.$$

Obtain MLE numerically:

$$(\hat{\alpha},\hat{\beta}) = (2.89, 0.056).$$



Comparison of empirical CDF for tyrannosaur data to exponential approximation (red) and Weibull approximation (green).

Age	Observed	Expected (exponential)	Expected (Weibull)
$[0, 5)$	5	22.7	2.6
$[5, 10)$	12	21.5	14.9
$[10, 15)$	22	15.7	28.9
$[15, 20)$	31	11.5	30.5
$[20, 25)$	28	8.4	18.6
$[25, \infty)$	5	23.1	7.5

$$X^2 = \sum_{j=1}^{m} \frac{(O_j - E_j)^2}{E_j}.$$

For the exponential case, we get $X^2 = 113.5$. Comparing to χ^2_4, we get a p-value around 10^{-20}.
For the Weibull case, we get $X^2 = 10.0$. Comparing to χ^2_3, we get a p-value of 0.018.