

## A.4 Inflation, taxation and project appraisal

If needed, you may wish to treat Question 1 as an optional question for further practice.

1. On 15 May 1997 the government of a country issued an index-linked bond of term 15 years. Coupons are payable half-yearly in arrears, and the annual nominal coupon rate is  $D = 4\%$ .

Interest and capital repayments are indexed by reference to the value of a retail price index with a time lag of 8 months. The retail price index value in September 1996 was  $Q(-8/12) = 200$  and in March 1997 was  $Q(-2/12) = 206$ .

The issue price of the bond was such that, if the retail price index  $Q$  were to increase continuously at a rate of  $e_1 = 7\%$  p.a. from March 1997, a tax exempt purchaser of the bond at the issue date would obtain a real yield of  $y = 3\%$  p.a. convertible half-yearly.

- (i) Determine  $Q(t)$  for all  $t \geq 0$  and hence the coupon and redemption payments of the bond per 100 nominal, assuming inflation at constant rate  $e_1$ .
  - (ii) Derive the formula for the price of the bond at issue to a tax-exempt investor and show that the issue price of the bond is £111.53 per 100 nominal.
  - (iii) An investor purchases a bond at the price calculated in (ii) and holds it to redemption. The actual rate of increase of the retail price index is  $e_2 = 5\%$  p.a. from March 1997. A new tax is introduced such that the investor pays tax at 40% on any real capital gain, where the real capital gain is the difference between the redemption money and the purchase price revalued according to the retail price index to the redemption date. Tax is only due if the real capital gain is positive. Calculate the real annual yield convertible half-yearly actually obtained by the investor.
2. A zero-coupon bond was purchased  $m$  years ago by investor  $A$  who is liable to capital gains tax at rate  $t$ . At the time of purchase the outstanding term of the bond was  $n$  years ( $n > m$ ). The price paid by  $A$  will provide him with a net effective annual yield of  $i > 0$  if he holds the bond until it is redeemed.

Investor  $A$  now wishes to sell the bond. He will be liable to capital gains tax on the excess of his selling price over his purchase price.

- (a) Derive an expression in terms of  $t$ ,  $n$ , and  $i$  for the purchase price (per unit redemption money) paid by  $A$ .
- (b) Derive also an expression in terms of  $t$ ,  $n$ ,  $m$ , and  $i$  for the price (per unit redemption money) at which  $A$  should now sell the bond in order to obtain a net annual yield of  $i$  on the completed transaction.
- (c) Assume that in fact the bond is sold by  $A$  to a second investor, who is also liable to capital gains tax at rate  $t$ , at a price which will provide the *new* purchaser with a net annual yield of  $i$ , if he holds the bond until it is redeemed.

Derive an equation from which can be found the value of  $j$ , the net annual yield obtained by  $A$  on the completed transaction.

Find the value of  $j$  when  $n = 10$ ,  $m = 5$ ,  $t = 0.4$ , and  $i = 0.1$ .

3. A bond pays coupons twice yearly in arrears at nominal annual rate  $j^{(2)} = 5\%$ , and will be redeemed at par after 5 years.

An investor will be liable to capital gains tax at 40% on the difference between redemption price and purchase price, adjusted for inflation over the 5-year period.

Under the assumption of a constant inflation rate of 2% p.a., find the purchase price which provides the investor with a yield (after capital gains tax) of (i) 6% (ii) 8%. What is the corresponding *real* yield in each case?

4. An investor purchases a bond 3 months after issue. The bond will be redeemed at par ten years after issue and pays coupons of 6% p.a. annually in arrears. The investor pays tax of 25% on both income and capital gains (no relief for indexation).
- Calculate the purchase price of the bond per £100 nominal to provide the investor with a rate of return of 8% per annum effective.
  - The real rate of return expected by the investor from the bond is 3% per annum effective. Calculate the annual rate of inflation expected by the investor.

5. Two business projects, each of which takes two years to complete, produce the following cash-flows:

Project A:	Project B
<ul style="list-style-type: none"> <li>• initial income of £2000;</li> <li>• after one year, expenditure of £3900;</li> <li>• after two years, income of £2000;</li> </ul>	<ul style="list-style-type: none"> <li>• initial income of £360;</li> <li>• after one year, expenditure of £4000;</li> <li>• after two years, income of £4000.</li> </ul>

An investor considering the two projects has no spare cash, but can borrow or invest money at rate  $i > 0$  for any desired term. For what range of  $i$  is each of the projects profitable? What can you say about the yields of the two projects?

If both projects are profitable and the investor must choose between them, which is the more profitable (for various possible values of  $i$ )?

6. An investor has decided to purchase a leasehold property for £80,000, with a further payment of £5000 for repairs in one year's time. The income associated with letting the property will be £10,000 per annum, payable continuously for 20 years commencing in two years' time.
- Given that the venture will be financed by bank loans on the basis of an effective annual interest rate of 7% and that the loans may be repaid continuously, find the discounted payback period for the project.
    - Given, further, that after the loans have been repaid the investor will deposit all the available income in an account which will earn interest at 6% per annum effective, find the accumulated amount of the account in 22 years' time.
  - Suppose that the bank loans may be repaid partially, but only at the end of each complete year, and that the investor may still deposit money at any time for any term at an annual rate of interest of 6% effective. Find
    - the discounted payback period for the project, and
    - the accumulated amount in the investor's account in 22 years' time.