A.1 cash-flows, discounting, interest rate models

1. Discuss the cash-flows of a typical final-year undergraduate over 5 years. Aim for a concise presentation.

2. If the effective annual rate of interest is 9%, calculate the total accumulated value at 1 January 2010 of payments of £100 made on 1 January 2009, 1 April 2009, 1 July 2009 and 1 October 2009.

3. A man stipulates in his will that £50,000 from his estate is to be placed in a fund from which his three children are each to receive an equal amount when they reach age 21. When the man dies, the children are ages 19, 15 and 13. If this fund earns 6% interest per half-year (i.e. nominal 12% p.a. compounded each 6 months), what is the amount that they each receive? Is the distribution fair?

4. (i) Establish a table of relationships between (constant) δ, i, v and d.

   (ii) Show that $d = vi$ and interpret this.

   (iii) Show that $\delta \approx i - i^2/2$ and $d \approx i - i^2$ for small i, and that $d \approx \delta - \delta^2/2$ for small δ.

5. Calculate the equivalent effective annual rate of interest of

   (a) a force of interest of 7.5% p.a.

   (b) a discount rate of 9% p.a.

   (c) a nominal rate of interest of 8% p.a. convertible half-yearly

   (d) a nominal rate of interest of 9% p.a. convertible monthly.

6. A borrower is under an obligation to repay a bank £6,280 in four years' time, £8,460 in seven years' time and £7,350 in thirteen years' time.

   As part of a review of his future commitments the borrower now offers either

   (a) to discharge his liability for these three debts by making an appropriate single payment five years from now; or

   (b) to repay the total amount owed (i.e. £22,090) in a single payment at an appropriate future time.

   On the basis of a constant interest rate of 8% per annum, find the appropriate single payment if offer (a) is accepted by the bank, and the appropriate time to repay the entire indebtedness if offer (b) is accepted. (You should work on the basis that the present value of the single payment under the revised arrangement should equal the present value of the three payments due under the current obligation).

7. Calculate the accumulated value of £1,000 after 2 years if $\delta(t) = 0.06(t + 1)$ for $0 \leq t \leq 2$.

8. *Stoodley's formula.* Suppose the force of interest is given by

$$\delta(t) = p + \frac{s}{1 + re^{st}}, \qquad t \in \mathbb{R}_+$$

where $p, r \in \mathbb{R}_+$ and $s \geq -p$.

Calculate the discount factor $v(t)$, $t \in \mathbb{R}_+$, and show that the model can be reparametrized such that $v(t) = \lambda v_1^t + (1-\lambda)v_2^t$. Interpret this!

Optional question for further practice:

9. In valuing future payments an investor uses the formula

$$v(t) = \frac{\alpha(\alpha+1)}{(\alpha+t)(\alpha+t+1)}, \qquad t \in \mathbb{R}_+$$

where α is a given positive constant, for the value at time 0 of 1 due at time t (measured in years).

Show that the above formula implies that

(i) the force of interest per annum at time t will be

$$\delta(t) = \frac{2t + 2\alpha + 1}{(\alpha+t)(\alpha+t+1)}$$

(ii) the effective rate of interest for the period r to $r+1$ will be

$$i(r) = \frac{2}{r+\alpha}$$

(iii) the present value of a series of n payments, each of amount 1 (the rth payment being due at time r) is

$$a(n) = \frac{n\alpha}{n+\alpha+1}.$$

(iv) Suppose now that $\alpha = 15$. Find the level annual premium, payable in advance for twelve years, which will provide an annuity of £1,800 per annum, payable annually for ten years, the first annuity payment being made one year after payment of the final premium. What is the value at time 12 of the series of annuity payments, what at time 0?

Course webpage, where all other assignments and solutions will be posted:
`http://www.stats.ox.ac.uk/~winkel/bs4a.html`

Class allocations and hand-in times:
`https://minerva.stats.ox.ac.uk/perl/classlists.pl`