P.4 Insurance and saving

Remember, only section B questions need to be handed in for marking.

A.1. Consider an economy where each agent faces a statistically independent risk of losing 100 with probability p. A *pool* of N agents decide to create a mutual agreement where the aggregate loss in the pool will be equally split among its members.

(a) Describe the possible losses and associated probabilities for each member of the pool when $N = 2$ and $N = 3$.

(b) Show that an increase in the pool size from 2 to 3 would be preferred by all members as long as they are risk averse
(*Hint: Denoting $\tilde{x}$ as the loss borne by a member of a pool with $N = 2$ and $\tilde{y}$ as the loss borne by a member of a pool with $N = 3$, show that $\tilde{x} \sim \tilde{y} + \tilde{\varepsilon}$ for some white noise $\tilde{\varepsilon}$ where $\mathbb{E}[\tilde{\varepsilon}|\tilde{y}] = 0$.*)

B.1. An agent with current wealth X has the opportunity to bet any amount on the occurrence of an event that she believes will occur with probability $p \in (0, 1)$. If she wagers w, she will receive (the gross amount) $2w$ if the event occurs and 0 if it does not. Her utility function is given by $u(x) = -e^{-rx}$ with $r > 0$. How much should she wager?

B.2. Arabella has a logarithmic utility function. She owns an asset with value £12 million, but which is subject to a potential loss of £8 million with probability $\frac{1}{4}$. Suppose that Arabella can purchase coinsurance with level $\beta \in [0, 1]$ and that insurance is priced with a loading of 0.2.

(a) What is the insurance premium if Arabella chooses $\beta = 1$?

(b) Show that expected utility is concave in β and calculate the optimal value of β, denoted β^*.

(c) Compute Arabella's expected utility for $\beta = 0$, $\beta = 1$ and $\beta = \beta^*$.

(d) What would happen to β^* if the loading fell to zero?

B.3. Consider the standard portfolio problem with a zero return on the risk-free investment and a random return $\tilde{x}$ on the risky investment.

(a) How do you react to the announcement that the return of the risky investment is not $\tilde{x}$, but rather $\tilde{x}$ with probability q and 0 with probability $1 - q$?

(b) How do you react to the announcement that you cannot directly invest in the risky project, but rather in a risky asset which is a portfolio containing a proportion q of the risky investment project, and a proportion $1 - q$ of the risk free investment project?

B.4. Consider a logarithmic investor ($u(z) = \ln z$) who can invest in a risk-free asset with return r and in a risky asset whose distribution of return is $(p, a; 1-p, b)$ with $a < r < b$.

(a) Derive an analytical formula for the optimal demand for the risky asset

(b) Is it always finite?

(c) Examine the effect of change in wealth on the demand for the risky asset

(d) Examine the effect of an increase of the risk free rate on the demand for the risky asset

(e) Examine the effect of a change in a or b on the optimal demand.

(f) Show that an increase in b combined with a reduction of a that leaves the expected excess return unchanged reduces the demand.

B.5 Sally retires at $t = 0$ with total net worth of 1 and lives for a maximum of 2 periods. The probability of her still being alive at times 1 and 2 are $p_1 = 0.75$ and $p_2 = 0.4$ respectively. Sally has a logarithmic felicity function and subjectively discounts future utility with discount factor $1/(1 + \delta)$, where $\delta = 10\%$. At $t = 0$ she chooses c_1 and c_2 to maximise utility

$$U(c_1, c_2) = \frac{p_1}{1+\delta} \ln(c_1) + \frac{p_2}{(1+\delta)^2} \ln(c_2).$$

(a) Suppose that Sally can only invest her earnings in the risk free asset, which gives a rate of return of $i = 10\%$. Calculate her optimal consumption schedule (c_1^*, c_2^*) and utility $U(c_1^*, c_2^*)$. Is c_t^* increasing, decreasing or constant with t? Why?

(b) Now suppose that she can purchase annuities whereby an investment of $c_t \frac{p_t}{(1+i)^t}$ at time 0 returns c_t at time t if Sally is still alive and zero otherwise. Calculate her new optimal consumption schedule (c_1^{**}, c_2^{**}) and utility $U(c_1^{**}, c_2^{**})$.

(c) Calculate c^* such that $U(c^*, c^*) = U(c_1^*, c_2^*)$ and compare with (c_1^{**}, c_2^{**}).

C.1. Nya has preferences exhibiting CARA, and acts to maximise expected utility at the end of one year. She can choose to invest her initial wealth w_0 in a risk-free asset with return r and a risky-asset with excess return $\tilde{y}$, with $\mathbb{E}\tilde{y} > 0$. She chooses to invest half her wealth in the risky asset.

Lloyd has the same preferences as Nya, the same starting wealth, and has the same available assets, but faces a background risk. Assume the background risk $\tilde{\varepsilon}$ is added to his wealth at the end of the year, has zero mean and is independent of $\tilde{y}$. Is Lloyd's optimal investment in the risky asset less than, the same as, or greater than Nya's?

C.2. An individual owns assets of value $W_0 = 10$, which may suffer a random loss $\tilde{x}$ described by $\left(0, \frac{7}{10}; 4, \frac{1}{10}; 8, \frac{1}{10}; 10, \frac{1}{10}\right)$.

(a) Compute the actuarially fair premium for full insurance.

(b) What is the actuarially fair premium when a deductible $D = 3$ is selected? What happens to the premium when $D = 6$? Why does the premium not fall by 50%?

(c) For each deductible compute the coinsurance rate β that yields the same actuarially fair premium.

(d) Draw the cumulative distribution function of final wealth first if $D = 6$ and then if the policy is characterized by the coinsurance rate β that yields the same premium as $D = 6$. By reference to the Rothschild-Stiglitz integral condition, show that the policy with a coinsurance rate induces a riskier distribution of final wealth.