# Manual for SIENA version 3

Tom A.B. Snijders Christian E.G. Steglich Michael Schweinberger Mark Huisman

University of Groningen: ICS, Department of Sociology Grote Rozenstraat 31, 9712 TG Groningen, The Netherlands

University of Oxford: Department of Statistics

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#### Abstract

SIENA (for Simulation Investigation for Empirical Network Analysis) is a computer program that carries out the statistical estimation of models for the evolution of social networks according to the dynamic actor-oriented model of Snijders (2001, 2005) and Snijders, Steglich, and Schweinberger (2007). It also carries out MCMC estimation for the exponential random graph model according to the procedures described in Snijders (2002) and Snijders, Pattison, Robins, and Handcock (2006).

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# 1 General information

SIENA<sup>1</sup>, shorthand for Simulation Investigation for Empirical Network Analysis, is a computer program that carries out the statistical estimation of models for repeated measures of social networks according to the dynamic actor-oriented model of Snijders and van Duijn (1997), Snijders (2001), and Snijders, Steglich, and Schweinberger (2007); also see Steglich, Snijders, and Pearson (2007). The model for network evolution is explained also in Snijders (2005). Some examples are presented, e.g., in van de Bunt (1999); van de Bunt, van Duijn, and Snijders (1999); and van Duijn, Zeggelink, Stokman,and Wasseur (2003); and Steglich, Snijders, and West (2006). A website for SIENA is maintained at http://stat.gamma.rug.nl/snijders/siena.html . Introductions in French and Spanish are given in de Federico de la Rúa (2004, 2005) and Jariego and de Federico de la Rúa (2006).

The program also carries out MCMC estimation for the exponential random graph model (abbreviated to *ERGM* or *ERG model*), also called  $p^*$  model, of Frank and Strauss (1986), Frank (1991), Wasserman and Pattison (1996), and Snijders, Pattison, Robins, and Handcock (2006). The algorithm is described in Snijders (2002). A good introduction is Robins, Snijders, Wang, Handcock, and Pattison (2007).

This manual is for SIENA version 3. Changes of this version compared to earlier versions are in Section 18. The program and this manual can be downloaded from the web site,

http://stat.gamma.rug.nl/stocnet/. One way to run SIENA is as part of the StOCNET program collection (Boer, Huisman, Snijders, Steglich, Wichers & Zeggelink, 2006), which can be downloaded from the same website. For the operation of StOCNET, the reader is referred to the corresponding manual. If desired, SIENA can be operated also independently of StOCNET, as is explained in Section 16.

This manual consists of two parts: the user's manual and the programmer's manual. It can be viewed and printed with the Adobe Acrobat reader. The manual is updated rather frequently, and it may be worthwhile to check now and then for updates.

The manual focuses on the use of SIENA for analysing the dynamics of directed networks. The case of non-directed networks is very similar, and at various points this case is described more in particular. Sections on data requirements, general operation, etc., apply as well to parameter estimation in the ERGM. Some sections are devoted specifically to this model.

For getting started, there are various options:

- 1. One excellent option is to read the User's Manual from start to finish (leaving aside the Programmer's Manual).
- 2. A second option is to read the Minimal Introduction contained in Sections 2 3, together with the table of contents to have an idea of what can be looked up later.
- 3. Another option is first to read the Minimal Introduction and further to focus on Sections 6 for the model specification, 7 to get a basic insight in what happens in the parameter estimation, 7.2 to understand the output file (which is meant to be as self-explanatory as possible), and 13 for the basis of getting started.

<sup>&</sup>lt;sup>1</sup>This program was first presented at the International Conference for Computer Simulation and the Social Sciences, Cortona (Italy), September 1997, which originally was scheduled to be held in Siena. See Snijders & van Duijn (1997).

We are grateful to Peter Boer, Bert Straatman, Minne Oostra, Rob de Negro, all (now or formerly) of SciencePlus, and Evelien Zeggelink, for their cooperation in the development of the StOCNET program and its alignment with SIENA. We also are grateful to NWO (Netherlands Organisation for Scientific Research) for their support to the integrated research program *The dynamics of networks and behavior* (project number 401-01-550), the project *Statistical methods for the joint development of individual behavior and peer networks* (project number 575-28-012), the project *An open software system for the statistical analysis of social networks* (project number 405-20-20), and to the foundation ProGAMMA, which all contributed to the work on SIENA and StOCNET.

# Part I Minimal Intro

The following is a minimal cookbook-style introduction for getting started with SIENA as operated from within StOCNET.

# 2 General remarks for StOCNET.

- 1. Ensure that the directories in Options Directories are existing, and that these are the directories where your data are stored, and where the output is to be stored.
- 2. Always keep in mind that, when the green Apply sign is visible, StOCNET expects you to press this button in order to confirm the most recent commands and to continue.

(You can choose to Cancel if you do not wish to confirm.)

3. The output file which you see in Results is the file, with extension .out, that is stored in the directory specified in Options - Directories as the Directory of session files.

#### 2.1 Operating StOCNET.

- 1. Start by choosing to enter a new session or open a previous session.
- You have to go sequentially through the various steps:
   Data Transformation (optional) Selection (optional) Model Results.
- 3. When starting a new session, you must select one or more network data sets as dependent variable(s), and optionally one or more network data sets as dyadic covariates (independent variables).

In addition, you can optionally select one or more files with actor-level covariates ('actor attributes') (as independent variables). If you do this, StOCNET will determined the number of variables in the data set and it is advisable to edit the names of the variables (which have the not very helpful default names of Attribute1, etc.).

- 4. After selecting the data files and clicking Apply, you are requested to save the session, and give it a name which serves later to identify this session.
- 5. If necessary, transform the data and indicate missing data values. This is self-explanatory (consult the StOCNETmanual if you need help). You have to note yourself how you transformed the variables. But it is recorded also in the session-tree on the right hand side of the StOCNETscreen.

It is also advisable to save the session (Session - Save session) after having transformed the data.

# 3 Using SIENA

#### 3.1 Steps for estimation: Choosing SIENA in StOCNET.

1. In the Model step, select SIENA.

Then select Data Specification, where the dependent network variable(s) must go to Digraphs in seq. order and the dyadic covariates (if any) to the box with that name. If you specify one file as dependent network variable, then the ERGM (p\*) model is applied. If you specify more than one file as dependent network variables, then the (longitudinal) actor-oriented model is applied, and the ordering of the files in the Digraphs in seq. order box

must be the correct order in time.

- 2. If you are analyzing only a network as the dependent variable, then the actor covariates (if any) must go to the box Constant covariates or Changing covariates; the 'changing' refers to change over time, and can be used only for the longitudinal option.
- 3. Next go to the Model specification and select the effects you wish to include in the model. When starting, choose a small number (e.g., 1 to 4) effects.
- 4. After clicking OK, you can then continue by estimating parameters: the Estimation option must be selected (which contrasts with Simulation), and the estimation algorithm then is started by clicking the Run button.
- 5. It will depend on the size of the data set and the number of parameters in the model, how long the estimation takes. The output file opens automatically in the **Results** step.
- 6. Below you see some points about how to evaluate the reliability of the results. If the convergence of the algorithm is not quite satisfactory but not extremely poor, then you can continue just by Running the estimation algorithm again.
- 7. If the parameter estimates obtained are very poor (not in a reasonable range), then it usually is best to start again, with a simpler model, and from a standardized starting value. The latter option must be selected in the Model specification Options screen.

SIENA estimates parameters by the following procedure:

- Certain statistics are chosen that should reflect the parameter values; the finally obtained parameters should be such, that the *expected values* of the statistics are equal to the *observed values*. Expected values are approximated as the averages over a lot of simulated networks. Observed values are calculated from the data set. These are also called the *target values*.
- 2. To find these parameter values, an *iterative stochastic simulation algorithm* is applied. This works as follows:
  - (a) In Phase 1, the sensitivity of the statistics to the parameters is roughly determined.
  - (b) In Phase 2, provisional parameter values are updated: this is done by simulating a network according to the provisional parameter values, calculating the statistics and the deviations between these simulated statistics and the *target values*, and making a little change (the 'update') in the parameter values that hopefully goes into the right direction.
    (Only a 'hopefully' good update is possible, because the simulated network is only a random draw from the distribution of networks, and not the expected value itself.)

(c) In Phase 3, the final result of Phase 2 is used, and it is checked if the average statistics of many simulated networks are indeed close to the target values. This is reflected in the so-called t statistics for deviations from targets.

#### 3.2 Steps for looking at results: Executing SIENA.

- 1. Look at the start of the output file for general data description (degrees, etc.), to check your data input.
- 2. When parameters have been estimated, first look at the t statistics for deviations from targets. These are good if they are all smaller than 0.1 in absolute value, and reasonably good if they are all smaller than 0.2.

We say that the algorithm has converged if they are all smaller than 0.1 in absolute value, and that it has nearly converged if they are all smaller than 0.2.

These bounds are indications only, and may be taken with a grain of salt.

Items 3–4 apply only to the ERGM (non-longitudinal) case and to estimation for longitudinal data using the ML (Maximum Likelihood) method.

- 3. In the ERGM (non-longitudinal) case and when using the ML (Maximum Likelihood) method for longitudinal data, it is often harder to obtain good convergence. This means that it may take several runs of the estimation algorithm, and that it may be necessary to fiddle with two parameters in the Model Specification Options: the Multiplication factor and the Initial value of gain parameter.
- 4. The Multiplication factor determines for these cases the number of Metropolis-Hastings steps taken for simulating each new network. When this is too low, the sequentially simulated networks are too similar, which will lead to high autocorrelation in the generated statistics. This leads to poor performance of the algorithm. These autocorrelations are given in the output file. When some autocorrelations are more than 0.1, it is good to increase the Multiplication factor.

When the Multiplication factor is unnecessarily high, computing time will be unnecessarily high.

- 5. (This item also is of interest mainly for the ERGM and ML cases).
- The Initial value of gain parameter determines the step sizes in the parameter updates in the iterative algorithm. A too low value implies that it takes very long to attain a reasonable parameter estimate when starting from an initial parameter value that is far from the 'true' parameter estimate. A too high value implies that the algorithm will be unstable, and may be thrown off course into a region of unreasonable (e.g., hopelessly large) parameter values. In the longitudinal case using the Method of Moments (the default estimation procedure), it usually is unnecessary to change this. In the ERGM case, when the autocorrelations are smaller than 0.1 but the t statistics for deviations from targets are relatively small (less than, say, 0.3) but do not all become less than 0.1 in absolute value in repeated runs of the estimation algorithm, then it will be good to decrease the Initial value of gain parameter. Do this by dividing it by, e.g., a factor 2 or a factor 5, and then try again a few estimation runs.
- 6. If all this is of no avail, then the conclusion may be that the model specification is incorrect for the given data set.
- 7. Further help in interpreting output is in Section 7.2 of this manual.

#### 3.3 Giving references

When using SIENA, it is appreciated that you refer to this manual and to one or more relevant references of the methods implemented in the program. The reference to this manual is the following.

Snijders, Tom A.B., Christian E.G. Steglich, Michael Schweinberger, and Mark Huisman. 2007. Manual for SIENA version 3. Groningen: University of Groningen, ICS. Oxford: University of Oxford, Department of Statistics. http://stat.gamma.rug.nl/stocnet

A basic reference for the network dynamics model is Snijders (2001) or Snijders (2005). Basic references for the model of network-behavior co-evolution are Snijders, Steglich, and Schweinberger (2007) and Steglich, Snijders, and Pearson (2007).

Basic references for the non-longitudinal (Exponential Random Graph) model are Frank and Strauss (1986); Wasserman and Pattison (1996); Snijders (2002); and Snijders, Pattison, Robins, and Handcock (2006). A more didactic reference here is Robins, Snijders, Wang, Handcock, and Pattison (2007).

More specific references are Schweinberger (2005) for the score-type goodness of fit tests; Schweinberger and Snijders (2007) for the calculation of standard errors of the Method of Moments estimators; and Snijders, Koskinen and Schweinberger (2007) for maximum likelihood estimation.

# Part II User's manual

The user's manual gives the information for using SIENA. It is advisable also to consult the user's manual of StOCNET because normally, the user will operate SIENA from within StOCNET.

## 4 Parts of the program

The operation of the SIENA program is comprised of four main parts:

- 1. input of basic data description,
- 2. model specification,
- 3. estimation of parameter values using stochastic simulation,
- 4. simulation of the model with given and fixed parameter values.

The normal operation is to start with data input, then specify a model and estimate its parameters, and then continue with new model specifications followed by estimation or simulation. For the comparison of (nested) models, statistical tests can be carried out.

The program is organized in the form of *projects*. A project consists of data and the current model specification. All files internally used in a given project have the same root name, which is called the project name, and indicated in this manual by *pname*. (In view of its use for special simulation purposes, it is advised not to use the project name *sisim*; also avoid to use the name *siena*, which is too general for this purpose.)

The main output is written to the text file *pname.out*, auxiliary output is contained in the text files *pname.log* and *pname.eff*.

## 5 Input data

The main statistical method implemented in SIENA is for the analysis of repeated measures of social networks, and requires network data collected at two or more time points. It is possible to include changing actor variables (representing behavior, attitudes, outcomes, etc.) which also develop in a dynamic process, together with the social networks. As repeated measures data on social networks, at the very least, two or more data files with digraphs are required: the observed networks, one for each time point. The number of time points is denoted M.

The other statistical method implemented in SIENA if the parameter estimation for the exponential random graph model ('ERGM'). For this method, one observed network data set is required.

In addition, various kinds of variables are allowed:

1. actor-bound or individual variables, also called actor attributes, which can be symbolized as  $v_i$  for each actor *i*; these can be constant over time or changing; the changing individual variables can be dependent variables (changing dynamically in mu-

tual dependence with the changing network) or independent variables (exogenously changing variables; then they are also called individual covariates).

2. dyadic covariates, which can be symbolized as  $w_{ij}$  for each ordered pair of actors (i, j); they are allowed only to have integer values ranging from 0 to 255. If one has real-valued dyadic covariates, then one option is to multiply them e.g. by 10 or 100 so that they still have a range between 0 and 255, and used the rounded values. These likewise can be constant over time or changing.

All variables must be available in ASCII ('raw text') data files, described in detail below. These files, the names of the corresponding variables, and the coding of missing data, must be made available to SIENA. In the StOCNET environment, files and variable names are entered in the Data dialog window, while missing data are identified in the Transformation dialog window. In the Model dialog window, network data and additional variables subsequently can be selected for SIENA analyses. This is done by first choosing SIENA from the list of available statistical methods, and then pushing the Data specification button.

Names of variables must be composed of at most 11 characters. This is because they are used as parts of the names of effects which can be included in the model, and the effect names should not be too long. The use of the default variable and file names proposed by **StOCNET** is not recommended.

#### 5.1 Digraph data files

Each digraph must be contained in a separate input file. Two data formats are allowed.

1. Adjacency matrices.

The first is an adjacency matrix, i.e., n lines each with n integer numbers, separated by blanks or tabs, each line ended by a hard return. The diagonal values are meaningless but must be present.

Although this section talks only about digraphs (directed graphs), it is also possible that all observed adjacency matrices are symmetric. This will be automatically detected by SIENA, and the program will then utilize methods for non-directed networks.

The data matrices for the digraphs must be coded in the sense that their values are converted by the program to the 0 and 1 entries in the adjacency matrix. A set of code numbers is required for each digraph data matrix; these codes are regarded as the numbers representing a present arc in the digraph, i.e., a 1 entry in the adjacency matrix; all other numbers will be regarded as 0 entries in the adjacency matrix. Of course, there must be at least one such code number. All code numbers must be in the range from 0 to 9, except for structurally determined values (see below).

This implies that if the data are already in 0-1 format, the single code number 1 must be given. As another example, if the data matrix contains values 1 to 5 and only the values 4 and 5 are to be interpreted as present arcs, then the code numbers 4 and 5 must be given.

2. Pajek format.

If the digraph data file has extension name .net, then the program assumes that the data file has Pajek format. This can not yet be done when using StOCNET, and therefore this option is available only when running SIENA outside of StOCNET, as described in Section 16. The keywords Arcs\*, Edges\*, Arcslist\*, and Edgeslist\* are allowed, followed by data lines according to the Pajek rules. All of these keywords may be used in one input file. The Edges\* and Edgeslist\* keywords announce that mutual ties are following. Codes 'for present arcs' as in the adjacency matrix format must be given in the .IN file, but this is only for consistency in the format for the .IN file, and these codes have no effect. Note that the Arcslist\* and Edgeslist\* formats produce binary data anyway. Tie values different from 1, which are used to indicate missings but can also be used for valued data, can only be input

in the Pajek format by using the keywords Arcs\* and Edges\*.

Code numbers for missing numbers also must be indicated – in the case of either input data format. These codes must, of course, be different from the code numbers representing present arcs.

Although this section talks only about digraphs (directed graphs), it is also possible that all observed ties (for all time points) are mutual. This will be automatically detected by SIENA, and the program will then utilize methods for non-directed networks.

#### 5.1.1 Structurally determined values

It is allowed that some of the values in the digraph are structurally determined, i.e., deterministic rather than random. This is analogous to the phenomenon of 'structural zeros' in contingency tables, but in SIENA not only structural zeros but also structural ones are allowed. A structural zero means that it is certain that there is no tie from actor i to actor j; a structural one means that it is certain that there is a tie. This can be, e.g., because the tie is impossible or formally imposed, respectively.

Structural zeros provide an easy way to deal with actors leaving or joining the network between the start and the end of the observations. Another way (more complicated but it gives possibilities to represent actors entering or leaving at specified moments between observations) is described in Section 5.7.

Structurally determined values are defined by reserved codes in the input data: the value 10 indicates a structural zero, the value 11 indicates a structural one. Structurally determined values can be different for the different time points. (The diagonal of the data matrix always is composed of structural zeros, but this does not have to be indicated in the data matrix by special codes.) The correct definition of the structurally determined values can be checked from the brief report of this in the output file, and by looking at the file *pname*.s01 (for the first time point), *pname*.s02 (second time point), etc. In these files, the structurally determined positions (structural zeros as well as structural ones) are indicated by the value 1, all others (i.e., the positions where ties are random) by the value 0.

Structural zeros offer the possibility of analyzing several networks simultaneously under the assumption that the parameters are identical. E.g., if there are three networks with 12, 20 and 15

actors, respectively, then these can be integrated into one network of 12 + 20 + 15 = 47 actors, by specifying that ties between actors in different networks are structurally impossible. This means that the three adjacency matrices are combined in one  $47 \times 47$  data file, with values 10 for all entries that refer to the tie from an actor in one network to an actor in a different network. In other words, the adjacency matrices will be composed of three diagonal blocks, and the off-diagonal blocks will have all entries equal to 10. In this example, the number of actors per network (12 to 20) is rather small to obtain good parameter estimates, but if the additional assumption of identical parameter values for the three networks is reasonable, then the combined analysis may give good estimates.

In such a case where K networks (in the preceding paragraph, the example had K = 3) are combined artificially into one bigger network, it will often be helpful to define K - 1 dummy variables at the actor level to distinguish between the K components. These dummy variables can be given effects in the rate function and in the evaluation function (for "ego"), which then will represent that the rate of change and the out-degree effect are different between the components, while all other parameters are the same.

#### 5.2 Dyadic covariates

As the digraph data, also each measurement of a dyadic covariate must be contained in a separate input file with a square data matrix, i.e., n lines each with n integer numbers, separated by blanks or tabs, each line ended by a hard return. The diagonal values are meaningless but must be present. Pajek input format is currently not possible for dyadic covariates.

A distinction is made between constant and changing dyadic covariates, where change refers to changes over time. Each constant covariate has one value for each pair of actors, which is valid for all observation moments, and has the role of an independent variable. Changing covariates, on the other hand, have one such value for each period between measurement points. If there are M waves of network data, this covers M-1 periods, and accordingly, for specifying a single changing dyadic covariate, M-1 data files with covariate matrices are needed.

The StOCNET interface requires the user to enter these in blocks of M - 1, and within each block in sequential order. This is done in the Data specification menu of the SIENA model page. For each such block, also a name must be provided to identify the changing dyadic covariate. For data sets with only two waves, the specification of changing dyadic covariates is meaningless, because there is only one period, hence there is no change over periods possible. Constant dyadic covariates can be selected in the respective section of the Data specification menu. They are identified by the name given to them in the initial Data step in StOCNET.

The reasons for restricting dyadic covariates to integer values from 0 to 255 are historical and have to do with how the constant dyadic covariate data are stored internally. If the user wishes to use a dyadic covariate with a different range, this variable first must be transformed to integer values from 0 to 255. E.g., for a continuous variable ranging from 0 to 1, the most convenient way probably is to multiply by 100 (so the range becomes 0-100) and round to integer values. In the current implementation, this type of recoding cannot easily be carried out within StOCNET, but the user must do it in some other program.

The mean is always subtracted from the covariates. See the section on *Centering*.

#### 5.3 Individual covariates

Individual (i.e., actor-bound) variables can be combined in one or more files. If there are k variables in one file, then this data file must contain n lines, with on each line k numbers which all are read as real numbers (i.e., a decimal point is allowed). The numbers in the file must be separated by blanks and each line must be ended by a hard return. There must not be blank lines after the last data line. Also here, a distinction is made between constant and changing actor variables. Each constant actor covariate has one value per actor valid for all observation moments, and has the role of an independent variable.

Changing variables can change between observation moments. They can have the role of dependent variables (changing dynamically in mutual dependence with the changing network) or of independent variables; in the latter case, they are also called 'changing individual covariates'. Dependent variables are treated in the section below, this section is about individual variables in the role of independent variables – then they are also called individual covariates.

When changing individual variables have the role of independent variables, they are assumed to have constant values from one observation moment to the next. If observation moments for the network are  $t_1, t_2, ..., t_M$ , then the changing covariates should refer to the M - 1 moments  $t_1$  through  $t_{M-1}$ , and the *m*-th value of the changing covariates is assumed to be valid for the period from moment  $t_m$  to moment  $t_{m+1}$ . The value at  $t_M$ , the last moment, does not play a role. Changing covariates, as independent variables, are meaningful only if there are 3 or more observation moments, because for 2 observation moments the distinction between constant and changing covariates is not meaningful.

Each changing individual covariate must be given in one file, containing k = M - 1 columns that correspond to the M - 1 periods between observations. It is not a problem if there is an M'th column in the file, but it will not be read.

The mean is always subtracted from the covariates. See the section on *Centering*.

#### 5.4 Interactions and dyadic transformations of covariates

For actor covariates, two kinds of transformations to dyadic covariates are made internally in SIENA. Denote the actor covariate by  $v_i$ , and the two actors in the dyad by i and j. Suppose that the range of  $v_i$  (i.e., the difference between the highest and the lowest values) is given by  $r_V$ . The two transformations are the following:

1. dyadic similarity, defined by  $1 - (|v_i - v_j|/r_V)$ , and centered so the mean of this similarity variable becomes 0;

note that before centering, the similarity variable is 1 if the two actors have the same value, and 0 if one has the highest and the other the lowest possible value;

2. dyadic identity, defined by 1 if  $v_i = v_j$ , and 0 otherwise (not centered).

In addition, SIENA offers the possibility of user-defined two- and three-variable interactions between covariates; see Section 6.6.

#### 5.5 Dependent action variables

SIENA also allows dependent action variables, also called dependent behavior variables. This can be used in studies of the co-evolution of networks and behavior, as described in Snijders, Steglich, and Schweinberger (2007) and Steglich, Snijders, and Pearson (2007). These action variables represent the actors' behavior, attitudes, beliefs, etc. The difference between dependent action variables and changing actor covariates is that the latter change exogenously, i.e., according to mechanisms not included in the model, while the dependent action variables change endogenously, i.e., depending on their own values and on the changing network. In the current implementation only one dependent network variable is allowed, but the number of dependent action variables are not assumed to be constant between observations.

Dependent action variables must have nonnegative integer values; e.g., 0 and 1, or a range of integers like 0,1,2 or 1,2,3,4,5. Each dependent action variable must be given in one file, containing k = M columns, corresponding to the M observation moments.

#### 5.6 Missing data

SIENA allows that there are some missing data on network variables, on covariates, and on dependent action variables. Missing data in changing dyadic covariates are not yet implemented. Missing data must be indicated by missing data codes (this can be specified in StOCNET, if SIENA is operated through StOCNET), *not* by blanks in the data set.

In the current implementation of SIENA, missing data are treated in a simple way, trying to minimize their influence on the estimation results. The simulations are carried out over all actors. Missing data are treated separately for each period between two consecutive observations of the network. In the initial observation for each period, missing entries in the adjacency matrix are set to 0, i.e., it is assumed that there is *no* tie. Missing covariate data as well as missing entries on dependent action variables are replaced by the variable's average score at this observation moment. In the course of the simulations, however, the adjusted values of the dependent action variables are allowed to change.

In order to ensure a minimal impact of missing data treatment on the results of parameter estimation (method of moments estimation) and/or simulation runs, the calculation of the target statistics used for these procedures is restricted to non-missing data. When for an actor in a given period, any variable is missing that is required for calculating a contribution to such a statistic, this actor in this period does not contribute to the statistic in question. For network and dependent action variables, an actor must provide valid data both at the beginning and at the end of a period for being counted in the respective target statistics.

#### 5.7 Composition change

SIENA can also be used to analyze networks of which the composition changes over time, because actors join or leave the network between the observations. This can be done in two ways: using the method of Huisman and Snijders (2003), or using structural zeros. (For the maximum likelihood estimation option, the Huisman-Snijders method is not implemented, and only the structural zeros method can be used.) Structural zeros can specified for all elements of the tie variables toward and from actors who are absent at a given observation moment. How to do this is described in subsection 5.1.1. This is straightforward and not further explained here.<sup>2</sup> This subsection explains the method of Huisman and Snijders (2003).

For this case, a data file is needed in which the *times of composition change* are given. For networks with constant composition (no entering or leaving actors), this file is omitted and the current subsection can be disregarded.

Network composition change, due to actors joining or leaving the network, is handled separately from the treatment of missing data. The digraph data files must contain all actors who are part of the network at any observation time (denoted by n) and each actor must be given a separate (and fixed) line in these files, even for observation times where the actor is not a part of the network (e.g., when the actor did not yet join or the actor already left the network). In other words, the adjacency matrix for each observation time has dimensions  $n \times n$ .

At these times, where the actor is not in the network, the entries of the adjacency matrix can be specified in two ways. First as missing values using missing value code(s). In the estimation procedure, these missing values of the joiners before they joined the network are regarded as 0

 $<sup>^{2}</sup>$ In the Siena01 program there is an option, which can be activated upon request by the programmers, to automatically convert

entries, and the missing entries of the leavers after they left the network are fixed at the last observed values. This is different from the regular missing data treatment. Note that in the initial data description the missing values of the joiners and leavers are treated as regular missing observations. This will increase the fractions of missing data and influence the initial values of the density parameter.

A second way is by giving the entries a regular observed code, representing the absence or presence of an arc in the digraph (as if the actor was a part of the network). In this case, additional information on relations between joiners and other actors in the network before joining, or leavers and other actors after leaving can be used if available. Note that this second option of specifying entries always supersedes the first specification: if a valid code number is specified this will always be used.

For joiners and leavers, crucial information is contained in the times they join or leave the network (i.e., the times of composition change), which must be presented in a separate input file. This data file must contain n lines, each line representing the corresponding actor in the digraph files, with on each line four numbers. The first two concern joiners, the last two concern leavers: 1) the last observation moment at which the actor is not yet observed, 2) the time of joining (expressed as a fraction of the length of the period), 3) the last observation moment at which the actor is observed, 4) the time of leaving (also expressed as a fraction). Also actors who are part of the network at all observation moments must be given values in this file. In the following example, the number of observation moments is considered to be M = 5, which means there are four periods; period m starts at observation moment m and ends at m + 1 for m = 1, 2, ..., 4 = M - 1.

Example of file with times of composition change				
Present at all five observation times	0	1.0	5	0.0
Joining in period 2 at fraction 0.6 of length of period	2	0.6	5	0.0
Leaving in period 3 at fraction 0.4 of length of period	0	1.0	3	0.4
Joining in per. 1 $(0.7)$ and leaving in per. 4 $(0.2)$	1	0.7	4	0.2
Leaving in per. 2 $(0.6)$ and joining in per. 3 $(0.8)$	3	0.8	2	0.6

Note that for joining, the numbers 0 1.0 have a different meaning than the numbers 1 0.0. The former numbers indicate that an actor is observed at time 1 (he/she joined the network right before the first time point), the latter indicate that an actor is not observed at observation time 1 (he/she joined just after the first time point). The same holds for leavers: 5 0.0 indicates that an actor is observed at time point 5, whereas 4 1.0 indicates that an actor left right before he/she was observed at time point 5.

From the example it follows that an actor is only allowed to join, leave, join and then leave, or leave and then join the network. The time that the actor is part of the network must be an uninterrupted period. It is not allowed that an actor joins twice or leaves twice. When there is no extra information about the time at which an actor joins or leaves (in some known period), there are three options: set the fraction equal to 0.0, 0.5, or 1.0. The second option is thought to be least restrictive.

The following special options are available for treatment of composition change by indicating this in the corresponding line in the .IN file (see Section 19.1):

- 2. The values of the joiners before joining are replaced by the value 0 (no ties), and the values of the leavers after leaving are treated as regular missing data.
- 3. The values of the joiners before joining and the values of the leavers after leaving are treated as regular missing data.

4. Before joining and after leaving, actors are treated as structural zeros.

Option 4 has the same effect as specifying the data for the absent actors as structural zeros; this option is useful for users who have a data set ready with joiners and leavers and wish to transform it automatically to a data set with structural zeros, e.g., because they wish to use the maximum likelihood estimation procedure.

#### 5.8 Centering

Individual as well as dyadic covariates are centered by the program in the following way.

For individual covariates, the mean value is subtracted immediately after reading the variables. For the changing covariates, this is the global mean (averaged over all periods). The values of these subtracted means are reported in the output.

For the dyadic covariates and the similarity variables derived from the individual covariates, the grand mean is calculated, stored, and subtracted during the program calculations. (Thus, dyadic covariates are treated by the program differently than individual covariates in the sense that the mean is subtracted at a different moment, but the effect is exactly the same.)

The formula for balance is a kind of dissimilarity between rows of the adjacency matrix. The mean dissimilarity is subtracted in this formula and also reported in the output. This mean dissimilarity is calculated by a formula given in Section 15.

The dependent network variable and the dependent action variables are not centered.

## 6 Model specification

After defining the data, the next step is to specify a model. In the StOCNET environment, this is done by clicking the Model specification button that is activated after a successful Data specification in StOCNET 's Model menu, provided that SIENA was selected from the list of available models.

The model specification consists of a selection of 'effects' for the evolution of each dependent variable (network or behavior). A list of all available effects for a given SIENA project is given in the secondary output file *pname.log*. A list of all effects in the objective function is given in the file *pname.eff*.

For the longitudinal case, three types of effects are distinguished (see Snijders, 2001; Steglich, Snijders and Pearson, 2007):

• rate function effects

The rate function models the speed by which the dependent variable changes; more precisely: the speed by which each network actor gets an opportunity for changing her score on the dependent variable.

Advice: in most cases, start modeling with a constant rate function without additional rate function effects. Constant rate functions are selected by exclusively checking the 'basic rate parameter' (for network evolution) and the main rate effects (for behavioral evolution) on the **model specification** screen. (When there are important size or activity differences between actors, it is possible that different advice must be given, and it may be necessary to let the rate function depend on the individual covariate that indicates this size; or on the out-degree.)

• evaluation function effects

The evaluation function<sup>3</sup> models the network actors' satisfaction with their local network neighborhood configuration. It is assumed that actors change their scores on the dependent variable such that they improve their total satisfaction – with a random element to represent the limited predictability of behavior. In contrast to the endowment function (described below), the evaluation function evaluates only the local network neighborhood configuration that results from the change under consideration. In most applications, the evaluation function will be the main focus of model selection.

The network evaluation function normally should always contain the 'density', or 'out-degree' effect, to account for the observed density. For directed networks, it mostly is also advisable to include the reciprocity effect, this being one of the most fundamental network effects. Likewise, behavior evaluation functions should normally always contain the tendency parameter, to account for the observed prevalence of the behavior.

• endowment function effects

The endowment function<sup>4</sup> is an extension of the evaluation function that allows to distinguish between new and old network ties (when evaluating possible network changes) and between increasing or decreasing behavioral scores (when evaluating possible behavioral changes). The function models the loss of satisfaction incurred when existing network ties are dissolved or when behavioral scores are decreased to a lower value (hence the label 'endowment'). Advice: start modeling without any endowment effects, and add them at a later stage.

The estimation and simulation procedures of SIENA operate on the basis of the model specification which comprises the set of effects included in the model as described above, together with the current parameter values and the Model Type (see Section 6.5). After data input, the

<sup>&</sup>lt;sup>3</sup>The evaluation function was called *objective function* in Snijders, 2001.

 $<sup>^4{\</sup>rm The}$  endowment function is similar to the gratification function in Snijders, 2001.

constant rate parameters and the density effect in the network evaluation function have default initial values, depending on the data. All other parameter values initially are 0. The estimation process changes the current value of the parameters to the estimated values. Values of effects not included in the model are not changed by the estimation process. It is possible for the user to change parameter values and to request that some of the parameters are fixed in the estimation process at their current value.

#### 6.1 Important structural effects for network dynamics

For the structural part of the model for network dynamics, the most important effects are as follows. The mathematical formulae for these and other effects are given in Section 15. Here we give a more qualitative description.

- 1. The *out-degree effect* which always must be included.
- 2. The *reciprocity effect* which practically always must be included.
- 3. There is a choice of four network closure effects. Usually it will be sufficient to express the tendency to network closure by including one or two of these. They can be selected by theoretical considerations and/or by their empirical statistical significance. Some researchers may find the last effect (distances two) less appealing because it expresses network closure inversely.
  - a. The transitive triplets effect, which is the classical representation of network closure by the number of transitive triplets. For this effect the contribution of the tie  $i \rightarrow j$ is proportional to the total number of transitive triplets that it forms – which can be transitive triplets of the type  $\{i \rightarrow j \rightarrow h; i \rightarrow h\}$  as well as  $\{i \rightarrow h \rightarrow j; i \rightarrow j\}$ ;



- b. The balance effect, which may also be called structural equivalence with respect to outgoing ties. This expresses a preference of actors to have ties to those other actors who have a similar set of outgoing ties as themselves. Whereas the transitive triplets effect focuses on how many same choices are made by ego (the focal actor) and alter (the other actor) — the number of h for which  $i \to h$  and  $j \to h$ , i.e.,  $x_{ih} = x_{jh} = 1$  where i is ego and j is alter — , the balance effect considers in addition how many the same non-choices are made —  $x_{ih} = x_{jh} = 0$ .
- c. The direct and indirect ties effect is similar to the transitive triplets effect, but instead of considering for each other actor j how many two-paths  $i \rightarrow h \rightarrow j$  there are, it is only considered whether there is at least one such indirect connection. Thus, one indirect tie suffices for the network embeddedness.
- d. The *distances two effect* expresses network closure inversely: stronger network closure (when the total number of ties is fixed) will lead to less geodesic distances equal to 2. When this effect has a negative parameter, actors will have a preference for having few others at a geodesic distance of 2 (given their out-degree, which is the number of others at distance 1); this is one of the ways for expressing network closure.

4. The three-cycles effect, which can be regarded as generalized reciprocity (in an exchange interpretation of the network) but also as the opposite of hierarchy (in a partial order interpretation of the network). A negative three-cycles effect sometimes may be interpreted as a tendency toward hierarchy. The three-cycles effect also contributes to network closure.

i

h

j

In a non-directed network, the three-cycles effect is identical to the transitive triplets effect.

- 5. Another triadic effect is the *betweenness effect*, which represents brokerage: the tendency for actors to position themselves between not directly connected others, i.e., a preference of i for ties  $i \to j$  to those j for which there are many h with  $h \neq j$ .
- 6. The distribution of degrees can be modeled more closely by using the effects sum of (1/(outdegree + 1 and/or the other effects defined by non-linear functions of out-degrees.

#### 6.2Effects for network dynamics associated with covariates

For each individual covariate, there are several effects which can be included in a model specification, both in the network evolution part and in the behavioral evolution part (should there be dependent behavior variables in the data).

- network rate function
  - 1. the covariate's effect on the rate of network change of the actor;
- network evaluation and endowment functions
  - 1. the covariate-similarity effect; a positive parameter implies that actors prefer ties to others with similar values on this variable - thus contributing to the network-autocorrelation of this variable not by changing the variable but by changing the network;
  - 2. the effect on the actor's activity (covariate-ego); a positive parameter will imply the tendency that actors with higher values on this covariate increase their out-degrees more rapidly;
  - 3. the effect on the actor's popularity to other actors (covariate-alter); a positive parameter will imply the tendency that the in-degrees of actors with higher values on this covariate increase more rapidly;
  - 4. the interaction between the value of the covariate of ego and of the other actor (covariate ego  $\times$  covariate alter); a positive effect here means, just like a positive similarity effect, that actors with a higher value on the covariate will prefer ties to others who likewise have a relatively high value; this effect is quite analogous to the similarity effect, and for dichotomous covariates, in models where the ego and alter effects are also included, it even is equivalent to the similarity effect (although expressed differently);
  - 5. the covariate identity effect, which expresses the tendency of the actors to be tied to others with exactly the same value on the covariate; whereas the preceding four effects are appropriate for interval scaled covariates (and mostly also for ordinal variables), the identity effect is suitable for categorical variables;
  - 6. the interaction effect of covariate-similarity with reciprocity.

The usual order of importance of these covariate effects on network evolution is: evaluation effects are most important, followed by endowment and rate effects. Inside the group of evaluation effects, it is the covariate-similarity effect that is most important, followed by the effects of covariate-ego and covariate-alter.

For each dyadic covariate, the following network evaluation effects can be included in the model for network evolution:

- network evaluation and endowment functions
  - 1. main effect of the dyadic covariate;
  - 2. the interaction effect of the dyadic covariate with reciprocity.

The main evaluation effect is usually the most important. In the current version of SIENA, there are no effects of dyadic covariates on behavioral evolution.

#### 6.3 Effects on behavior evolution

For models with a dependent behavior variable in models for the co-evolution of networks and behavior, the most important effects for the behavior dynamics are the following. In these descriptions, with the 'alters' of an actor we refer to the other actors to whom the focal actor has an outgoing tie.

- 1. The tendency effect, expressing the basic drive toward high values. A zero value for the tendency will imply a drift toward the midpoint of the range of the behavior variable.
- 2. The effect of the behavior on itself, which is relevant only if the number of behavioral categories is 3 or more. This can be interpreted as giving a quadratic preference function for the behavior. With a negative coefficient, this represents that the most desired behavior can lie somewhere between the minimum and maximum values of the behavioral variable.
- 3. The average similarity effect, expressing the preference of actors to being similar to their alters, where the total influence of the alters is the same regardless of the number of alters.
- 4. The total similarity effect, expressing the preference of actors to being similar to their alters, where the total influence of the alters is proportional to the number of alters.
- 5. The average alter effect, expressing that actors whose alters have a higher average value of the behavior, also have themselves a stronger tendency toward high values on the behavior.
- 6. The indegree effect, expressing that actors with a higher indegree (more 'popular' actors) have a stronger tendency toward high values on the behavior.
- 7. The outdegree effect, expressing that actors with a higher outdegree (more 'active' actors) have a stronger tendency toward high values on the behavior.

Effects 1 and 2 will practically always have to be included as control variables. (For dependent behavior variables with 2 categories, this applies only to effect 1.)

The average similarity, total similarity, and average alter effects are different specifications of social influence. The choice between them will be made on theoretical grounds and/or on the basis of statistical significance.

#### 6.4 Exponential Random Graph Models

For the non-longitudinal ('ERGM') case, default advice is given in Snijders et al. (2006) and in Robins et al. (2007). The basic structural part of the model is comprised, for directed networks, by the following effects.

- 1. The reciprocity effect.
- 2. The alternating k-out-stars effect to represent the distribution of the out-degrees.
- 3. The alternating k-in-stars effect to represent the distribution of the in-degrees.
- 4. For alternating transitive k-triangles effect to represent the tendency to transitivity.
- 5. The alternating independent two-paths effect to represent the preconditions for transitivity, or alternatively, the association between in-degrees and out-degrees.
- 6. The number of three-cycles to represent cyclicity, or generalized reciprocity, or the converse of hierarchy.

For nondirected networks. the basic structural part is comprised by the following, smaller, set.

- 1. The alternating k-stars effect to represent the distribution of the degrees.
- 2. For alternating transitive k-triangles effect to represent the tendency to transitivity.
- 3. The alternating independent two-paths effect to represent the preconditions for transitivity, or alternatively, the association between in-degrees and out-degrees.

Other effects can be added to improve the fit.

To obtain good convergence results for the ERGM case, it will usually be necessary to increase the default value of the multiplication factor; see Sections 3.2 and 11.

#### 6.5 Model Type

When the data is perfectly symmetric, this will be detected by SIENA. Then the analysis options for nondirected networks will be followed.

#### 6.5.1 Model Type: directed networks

For directed networks, the Model Type distinguishes between the model of Snijders (2001) (Model Type 1), that of Snijders (2003) (Model Type 2), and the tie-based model described in Snijders (2006) (Model Type 3). Model Type 1 is the default model and is described in the basic publications on Stochastic Actor-Oriented Models for network dynamics.

Model type 2 is at this moment not implemented in SIENA version 3.

In Model Type 2, the 'decisions' by the actors consist of two steps: first a step to increase or decrease their out-degree; when this step has been taken, the selection of the other actor towards whom a new tie is extended (if the out-degree rises) or from a an existing tie is withdrawn (if the out-degree drops). The decision by an actor to increase or decrease the number of outgoing ties is determined on the basis of only the current degree; the probabilities of increasing or decreasing the out-degree are expressed by the distributional tendency function  $\xi$  (indicated in the output as xi) and the volatility function  $\nu$  (indicated as nu). Which new tie to create, or which existing tie to withdraw, depends in the usual way on the evaluation and endowment functions. Thus, the outdegree distribution is governed by parameters that are not connected to the parameters for the

structural dynamics. The use of such an approach in statistical modeling minimizes the influence of the observed degrees on the conclusions about the structural aspects of the network dynamics. This is further explained in Snijders (2003).

For Model Type 2, in the rate function, effects connected to these functions  $\xi$  and  $\nu$  are included. On the other hand, effects in the evaluation function that depend only on the outdegrees are canceled from the model specification, because they are not meaningful in Model Type 2. To evaluate whether Model Type 1 or Model Type 2 gives a better fit to the observed degree distribution, the output gives a comparison between the observed out-degrees and the fitted distribution of the out-degrees (as exhibited by the simulated out-degrees). For Model Type 2 this comparison is always given. For Model Type 1, this comparison is given by adding 10 to the Model Code in the advanced options. (For IATEX users: the log file contains code that can be used to make a graph of the type given in Snijders, 2003).

For using Model Type 2, it is advised to first estimate some model according to Model Type 1 (this may be a simple model containing a reciprocity effect only, but it could also include more effects), and then – using the parameters estimated under Model Type 1 – change the specification to Model Type 2, and use the unconditional estimation method (see Section 7.3.4) (instead of the conditional method which is the default). It is likely that the very first model estimated under Model Type 2 will have results with poor convergence properties, but in such cases it is advised just to estimate the same model another time, now using the parameter values obtained under the previous Model Type 2 run as the initial values for the estimation.

To obtain a good model specification with respect to the rates of change in dependence of the out-degrees, three effects can be included:

- 1. the out-degrees effect
- 2. the factorial out-degree effect
- 3. the logarithmic out-degree effect.

These are the effects defined in formula (18) of Snijders (2003b) and indicated with the parameters  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$ , respectively. The user has to see from the estimation results which, or which two, out of these effects should be included to yield a good fit for the out-degrees.

In addition these types, there is Model Type 6 which implements the reciprocity model of Wasserman (1979) and Leenders (1995) (also see Snijders, 1999, 2005) — provided that no other effects are chosen than the outdegree effect, the reciprocity effect and perhaps the reciprocity endowment effect, and possible also effects of actor covariates or dyadic covariates. This model is meaningful only as a "straw man" model to provide a test of the null hypothesis that the dynamics of the dyads are mutually independent, against the alternative hypothesis that there do exist network effects (which make the dyad processes mutually dependent). For this purpose, Model Type 6 can be chosen, while for one or more network effects such as the effects representing transitivity, the null hypothesis is tested that their coefficients are zero (see Section 9).

The Model Type is specified in the model options as (part of) the Model Code.

#### 6.5.2 Model Type: non-directed networks

Non-directed networks are an undocumented option (there currently only is the presentation Snijders 2007), and therefore mentioned here reluctantly for those users who want to use this option anyway.

SIENA detects automatically when the networks all are non-directed, and then employs a model for this special case. For non-directed networks, the Model Type has seven possible values, as described in Snijders (2007).

1. Forcing model:

one actor takes the initiative and unilaterally imposes that a tie is created or dissolved.

- 2. Unilateral initiative and reciprocal confirmation: one actor takes the initiative and proposes a new tie or dissolves an existing tie; if the actor proposes a new tie, the other has to confirm, otherwise the tie is not created.
- 3. Tie-based model: a random pair of actors is chosen (actor-specific rate functions are not used here), and the average change in objective function (1) for toggling (i, j) and (j, i) is the log-odds of the probability of changing the tie variable.
- 4. Pairwise conjunctive model: a pair of actors is chosen and reconsider whether a tie will exist between them; a new tie is formed if both agree.
- 5. Pairwise disjunctive (forcing) model: a pair of actors is chosen and reconsider whether a tie will exist between them; a new tie is formed if at least one wishes this.
- 6. Pairwise compensatory (additive) model: a pair of actors is chosen and reconsider whether a tie will exist between them; this is based on the sum of their utilities for the existence of this tie.

In Models 1-2, where the initiative is one-sided, the rate function is comparable to the rate function in directed models. In Models 4-6, however, the pair of actors is chosen at a rate which is the *product* of the rate functions  $\lambda_i$  and  $\lambda_j$  for the two actors. This means that opportunities for change of the single tie variable  $x_{ij}$  occur at the rate  $\lambda_i \times \lambda_j$ . The numerical interpretation is different from that in Models 1-2.

#### 6.6 Additional interaction effects

It is possible for the user to define additional interaction effects for the network. This applies both to longitudinal and non-longitudinal (ERG) modeling. The basis is provided by the initial definition, by SIENA, of "unspecified interaction effects". Modifying the internal effect parameters of these effects allows the definition of two-way or three-way interactions. Not all user-defined interactions are possible:

• For *longitudinal models*:

Ego effects of actor variables can interact with all effects. Further, interaction effects are permitted which are combinations of actor variables, dyadic variables, and reciprocity.

• For *non-longitudinal* (ERG) models: Actor covariates and dyadic covariates can interact with each other and with reciprocity.

The specification is made by changing the *internal effect parameter* for the interaction effects. The values of these internal parameters can be changed in the *pname.mo* file described in Section 19.2.1. In StOCNET, they are accessible as the "par." column in the Advanced Options screen of the Model Specification.

For interaction effects, this parameter represents a code for the two or three interacting effects. Each effect is represented by its index number in three digits (including leading zeros) as reported in the file called *pname*.eff (recall that *pname* stands for your project name). Thus, two-way interactions are represented by twice three digits: e.g., the code 020003 refers to the interaction

between the effects numbered 20 and 3, where the numbers are the rank numbers in this list of all effects. Leading zeros of the total parameter can be skipped, so that the code 020003 can also be represented by 20003 (but not by 203!). The order does not matter, so that the codes 020003, 003020, 20003, and 3020 all are equivalent. Three-way interactions similarly are represented by thrice three digits. For example, the code 020028003 represents the interaction effects between the effects numbered 20, 28, and 3. E.g., to implement the interaction effect between the effects numbered 20 and 3, in the *pname*.mo file the lines that initially are

unspecified interaction effect 0 0 0 0 0 0.000000 020003 0 0 0 0 0 0.000000 0 0 0 0 0 0 0.000000 0

After this is done, SIENA will automatically replace the name by the suitable interaction effect name.

The calculation of user-defined effects is slightly more time-consuming than the calculation of internally defined effects. Therefore, when there is the choice between two equivalent effects – e.g., in longitudinal modeling, interactions of actor covariates with reciprocity – then it is advisable to use the predefined interaction effects.

# 7 Estimation

The model parameters are estimated under the specification given during the model specification part, using a stochastic approximation algorithm. Three estimation procedures are implemented: the Method of Moments (MoM) (Snijders, 2001; Snijders, Steglich, and Schweinberger, 2007); the Method of Maximum Likelihood (ML) (Snijders, Koskinen and Schweinberger, 2007); and a Bayesian method (Koskinen, 2005; Koskinen and Snijders, 2007; Schweinberger and Snijders, 2007). For non-constant rate functions, currently only MoM estimation is available. The Method of Moments is the default; the other two methods require much more computing time. Given the greater efficiency but longer required computing time for the ML and Bayesian methods, these can be useful especially for smaller data sets and relatively complicated models (networks and behavior; endowment effects).

In the following, the number of parameters is denoted by p. The algorithms are based on repeated (and repeated, and repeated...) simulation of the evolution process of the network. These repetitions are called 'runs' in the following. The MoM estimation algorithm is based on comparing the observed network (obtained from the data files) to the hypothetical networks generated in the simulations.

Note that the estimation algorithm is of a stochastic nature, so the results can vary! This is of course not what you would like. For well-fitting combinations of data set and model, the estimation results obtained in different trials will be very similar. It is good to repeat the estimation process at least once for the models that are to be reported in papers or presentations, to confirm that what you report is a stable result of the algorithm.

The initial value of the parameters normally is the current value (that is, the value that the parameters have immediately before you start the estimation process); as an alternative, it is possible to start instead with a standard initial value. Usually, a sequence of models can be fitted without problems, each using the previously obtained estimate as the starting point for the new estimation procedure. Sometimes, however, problems may occur during the estimation process, which will be indicated by some kind of warning in the output file or by parameter estimates being outside a reasonably expected range. In such cases the current parameter estimates may be unsatisfactory, and using them as initial values for the new estimation process might again lead to difficulties in estimation. Therefore, when the current parameter values are unlikely and also when they were obtained after a divergent estimation algorithm, it is advisable to start the estimation algorithm with a *standard initial value*. The use of standard initial values is one of the model options.

#### 7.1 Algorithm

During the estimation process, StOCNET transfers control to the SIENA program. The estimation algorithm has for both the MoM and ML method three phases:

- 1. In phase 1, the parameter vector is held constant at its initial value. This phase is for having a first rough estimate of the matrix of derivatives.
- 2. Phase 2 consists of several subphases. More subphases means a greater precision. The default number of subphases is 4. The parameter values change from run to run, reflecting the deviations between generated and observed values of the statistics. The changes in the parameter values are smaller in the later subphases.

The program searches for parameter values where these deviations average out to 0. This is reflected by what is called the 'quasi-autocorrelations' in the output screen. These are averages of products of successively generated deviations between generated and observed statistics. It is a good sign for the convergence of the process when the quasi-autocorrelations

are negative (or positive but close to 0), because this means the generated values are jumping around the observed values.

3. In phase 3, the parameter vector is held constant again, now at its final value. This phase is for estimating the covariance matrix and the matrix of derivatives used for the computation of standard errors.

The default number of runs in phase 3 is 1000. This requires a lot of computing time, but when the number of phase 3 runs is too low, the standard errors computed are rather unreliable.

The number of subphases in phase 2, and the number of runs in phase 3, can be changed in the model options.

The user can break in and modify the estimation process in three ways:

- 1. it is possible to terminate the estimation;
- 2. in phase 2, it is possible to terminate phase 2 and continue with phase 3;
- 3. in addition, it is possible to change the current parameter values and restart the whole estimation process.

For the ML estimation option and for the non-longitudinal case, tuning the 'multiplicaton factor' and the 'initial gain parameter' can be important for getting good results; for Bayesian estimation the 'multiplicaton factor' can likewise be important; this is briefly described in Section 3.2.

#### 7.2 Output

There are three output files. All are ASCII ('text') files which can be read by any text editor. The main output is given in the *pname.out* file (recall that *pname* is the project name defined by the user). A brief history of what the program does is written to the file *pname.log*. The latter file also contains some supplementary output that usually is disregarded but sometimes is helpful. Some diagnostic output containing a history of the estimation algorithm which may be informative when there are convergence problems is written to the file *pname.cck* (for 'check'). This file is overwritten for each new estimation. Normally, you only need to look at *pname.out*.

The output is divided into sections indicated by a line @1, subsections indicated by a line @2, subsubsections indicated by @3, etc. For getting the main structure of the output, it is convenient to have a look at the @1 marks first.

The primary information in the output of the estimation process consists of the following three parts. Results are presented here which correspond to Table 2, column " $t_1$ ,  $t_3$ " of Snijders (2001). The results were obtained in an independent repetition of the estimation for this data set and this model specification; since the repetition was independent, the results are slightly different, illustrating the stochastic nature of the estimation algorithm.

#### 1. Convergence check

In the first place, a convergence check is given, based on Phase 3 of the algorithm. This check considers the deviations between simulated values of the statistics and their observed values (the latter are called the 'targets'). Ideally, these deviations should be 0. Because of the stochastic nature of the algorithm, when the process has properly converged the deviations are small but not exactly equal to 0. The program calculates the averages and standard deviations of the deviations and combines these in a *t*-statistic (in this case, average divided by standard deviation). For longitudinal modeling, convergence is excellent when these *t*-values are less than 0.1 in absolute

value, good when they are less than 0.2, and moderate when they are less than 0.3. (These bounds are indications only, and are not meant as severe limitations.) The corresponding part of the output is the following.

```
Total of 1954 iterations.
Parameter estimates based on 954 iterations,
basic rate parameter as well as
convergence diagnostics, covariance and derivative matrices based on 1000 iterations.
Information for convergence diagnosis.
Averages, standard deviations, and t-ratios for deviations from targets:
  1.
         -0.236
                   7.006
                            -0.034
                   7.059
  2.
          0.204
                             0.029
  З.
         -1.592
                  22.242
                            -0.072
```

Good convergence is indicated by the t-ratios being close to zero.

In this case, the t-ratios are -0.034, -0.029, and -0.072, which is less than 0.1 in absolute value, so the convergence is excellent. In data exploration, if one or more of these t-ratios are larger in absolute value than 0.3, it is advisable to restart the estimation process. For results that are to be reported, it is advisable to carry out a new estimation when one or more of the t-ratios are larger in absolute value than 0.1. Large values of the averages and standard deviations are in themselves not at all a reason for concern.

For the exponential random graph (or  $p^*$ ) model, the convergence of the algorithm is more problematic than for longitudinal modeling. A sharper value of the *t*-ratios must be found before the user may be convinced of good convergence. It is advisable to try and obtain *t*-values which are less than 0.15. If, even with repeated trials, the algorithm does not succeed in producing *t*-values less than 0.15, then the estimation results are of doubtful value.

2. Parameter values and standard errors

The next crucial part of the output is the list of estimates and standard errors. For this data set and model specification, the following result was obtained.

5.4292 (

0.6920)

# @3Estimates and standard errors0. Rate parameter

Other para	meters:			
1. eval:	outdegree (density)	-0.7648	(	0.2957)
2. eval:	reciprocity	2.3071	(	0.5319)
<pre>3. eval:</pre>	number of actors at distance 2	-0.5923	(	0.1407)

The rate parameter is the parameter called  $\rho$  in section 15.1.3 below. The value 5.4292 indicates that the estimated number of changes per actor (i.e., changes in the choices made by this actor, as reflected in the row for this actor in the adjacency matrix) between the two observations is 5.43 (rounded in view of the standard error 0.69). Note that this refers to unobserved changes, and that some of these changes may cancel (make a new choice and then withdraw it again), so the average observed number of differences per actor will be somewhat smaller than this estimated number of unobserved changes.

The other three parameters are the weights in the *evaluation function*. The terms in the evaluation function in this model specification are the out-degree effect defined as  $s_{i1}$  in Section

15.1.1, the reciprocity effect  $s_{i2}$ , and the number of distances 2 (indirect relations) effect, defined as  $s_{i5}$ . Therefore the estimated evaluation function here is

$$-0.76 s_{i1}(x) + 2.31 s_{i2}(x) - 0.59 s_{i5}(x)$$
.

The standard errors can be used to test the parameters. For the rate parameter, testing the hypothesis that it is 0 is meaningless because the fact that there are differences between the two observed networks implies that the rate of change must be positive. The weights in the evaluation function can be tested by t-statistics, defined as estimate divided by its standard error. (Do not confuse this t-test with the t-ratio for checking convergence; these are completely different although both are t ratios!) Here the t-values are, respectively, -0.7648/0.2957 = -2.59, 2.3071/0.5319 = 4.34, and -0.5923/0.1407 = -4.21. Since these are larger than 2 in absolute value, all are significant at the 0.05 significance level. It follows that there is evidence that the actors have a preference for reciprocal relations and for networks with a small number of other actors at a distance 2. The value of the density parameter is not very important; it is important that this parameter is included to control for the density in the network, but as all other statistics are correlated with the density, the density is difficult to interpret by itself.

When for some effects the parameter estimate as well as the standard error are quite large, say, when both are more than 2, and certainly when both are more than 5, then it is possible that this indicates poor convergence of the algorithm: in particular, it is possible that the effect in question does have to be included in the model to have a good fit, but the precise parameter value is poorly defined (hence the large standard error) and the significance of the effect cannot be tested with the *t*-ratio. This can be explored by estimating the model without this parameter, and also with this parameter fixed at some large value (see section 13.1) – whether the value is large positive or large negative depends on the direction of the effect. For the results of both model fits, it is advisable to check the fit by simulating the resulting model and considering the statistic corresponding to this particular parameter. (The indicative sizes of 2 and 5 result from experience with network effects and with effects of covariates on usual scales with standard deviations ranging between, say, 0.4 and 2. These numbers have to be modified for covariates with different standard errors.)

#### 3. Collinearity check

After the parameter estimates, the covariance matrix of the estimates is presented. In this case it is

```
Covariance matrix of estimates (correlations below diagonal):

0.087 -0.036 0.003

-0.230 0.283 -0.033

0.078 -0.440 0.020
```

The diagonal values are the variances, i.e., the squares of the standard errors (e.g., 0.087 is the square of 0.2957). Below the diagonal are the correlations. E.g., the correlation between the estimated density effect and the estimated reciprocity effect is -0.230. These correlations can be used to see whether there is an important degree of collinearity between the effects. Collinearity means that several different combinations of parameter values could represent the same data pattern, in this case, the same values of the network statistics. When one or more of the correlations are very close to -1.0 or +1.0, this is a sign of collinearity. This will also lead to large standard errors of those parameters. It is then advisable to omit one of the corresponding effects from the model, because it may be redundant given the other (strongly correlated) effect. It is possible that the standard error of the retained effect becomes much smaller by omitting the other effect, which can also mean a change of the *t*-test from non-significance to significance.

#### 7.3 Other remarks about the estimation algorithm

#### 7.3.1 Changing initial parameter values for estimation

When you wish to change initial parameter values for running a new estimation procedure, this can be done in StOCNET as one of the model options. It can also be done by 'breaking in' into the SIENA program.

#### 7.3.2 Fixing parameters

Sometimes an effect must be present in the model, but its precise numerical value is not welldetermined. E.g., if the network at time  $t_2$  would contain only reciprocated choices, then the model should contain a large positive reciprocity effect but whether it has the value 3 or 5 or 10 does not make a difference. This will be reflected in the estimation process by a large estimated value and a large standard error, a derivative which is close to 0, and sometimes also by lack of convergence of the algorithm. (This type of problem also occurs in maximum likelihood estimation for logistic regression and certain other generalized linear models; see Geyer and Thompson (1992, Section 1.6) and Albert and Anderson (1984).) In such cases this effect should be fixed to some large value and not left free to be estimated. This can be specified in the model specification under the Advanced button. As another example, when the network observations are such that ties are formed but not dissolved (some entries of the adjacency matrix change from 0 to 1, but none or hardly any change from 1 to 0), then it is possible that the density parameter must be fixed at some high positive value.

#### 7.3.3 Automatic fixing of parameters

If the algorithm encounters computational problems, sometimes it tries to solve them automatically by fixing one (or more) of the parameters. This will be noticeable because a parameter is reported in the output as being fixed without your having requested this. This automatic fixing procedure is used, when in phase 1 one of the generated statistics seems to be insensitive to changes in the corresponding parameter.

This is a sign that there is little information in the data about the precise value of this parameter, when considering the neighborhood of the initial parameter values. However, it is possible that the problem is not in the parameter that is being fixed, but is caused by an incorrect starting value of this parameter or one of the other parameters.

When the warning is given that the program automatically fixed one of the parameter, try to find out what is wrong.

In the first place, check that your data were entered correctly and the coding was given correctly, and then re-specify the model or restart the estimation with other (e.g., 0) parameter values. Sometimes starting from different parameter values (e.g., the default values implied by the model option of "standard initial values") will lead to a good result. Sometimes, however, it works better to delete this effect altogether from the model.

It is also possible that the parameter does need to be included in the model but its precise value is not well-determined. Then it is best to give the parameter a large (or strongly negative) value and indeed require it to be fixed (see Section 13.1).

#### 7.3.4 Conditional and unconditional estimation

SIENA has two methods for MoM estimation and simulation: conditional and unconditional. They differ in the *stopping rule* for the simulations of the network evolution. In unconditional estimation, the simulations of the network evolution in each time period (and the co-evolution of the behavioral

dimensions, if any are included) carry on until the predetermined time length (chosen as 1.0 for each time period between consecutive observation moments) has elapsed.

In conditional estimation, in each period the simulations run on until a stopping criterion is reached that is calculated from the observed data. Conditioning is possible for each of the dependent variables (network, or behavior), where 'conditional' means 'conditional on the observed number of changes on this dependent variable'.

Conditioning on the network variable means running simulations until the number of different entries between the initially observed network of this period and the simulated network is equal to the number of entries in the adjacency matrix that differ between the initially and the finally observed networks of this period.

Conditioning on a behavioral variable means running simulations until the sum of absolute score differences on the behavioral variable between the initially observed behavior of this period and the simulated behavior is equal to the sum of absolute score differences between the initially and the finally observed behavior of this period.

Conditional estimation is slightly more stable and efficient, because the corresponding rate parameters are not estimated by the Robbins Monro algorithm, so this method decreases the number of parameters estimated by this algorithm. Therefore, it is the default for models that do not include any dependent behavior variables. For models including dependent behavior variables, the default estimation type is unconditional (because in most applications, there will be no straightforward choice for the conditioning variable). The possibility to choose between unconditional and the different types of conditional estimation is one of the model options.

If there are changes in network composition (see Section 5.7), only the unconditional estimation procedure is available.

#### 7.3.5 Automatic changes from conditional to unconditional estimation

Even though conditional estimation is slightly more efficient than unconditional estimation, there is one kind of problem that sometimes occurs with conditional estimation and which is not encountered by unconditional estimation.

It is possible (but luckily rare) that the initial parameter values were chosen in an unfortunate way such that the conditional simulation does not succeed in ever attaining the condition required by its stopping rule (see Section 7.3.4). This is detected by SIENA, which then switches automatically to unconditional estimation; after some time it switches back again to conditional estimation.

## 8 Standard errors

The estimation of standard errors for the MoM estimates requires the estimation of derivatives, which indicate how sensitive the expected values of the statistics (see Section 7.1) are with respect to the parameters. The derivatives can be estimated by four methods:

- 0: finite differences method with common random numbers,
- 1: score function method 1,
- 2: score function method 2.
- 4: score function method 4.

Schweinberger and Snijders (2007) point out that the finite differences method is associated with a bias-variance dilemma, and proposed the unbiased score function methods. These are also much more efficient than Method 0 in terms of computation time. Method 1 estimates the derivatives per observation period separately by the simulated sample covariance of the complete data score function and the generated statistics; this is then added over the observation periods. Especially for more than 2 observations, method 1 has a much smaller standard error of the estimated standard errors than the other methods. This method is the default. Methods 2 and 4 are included only for methodological research purposes. It is advisable to use at least 1000 iterations (default) in phase 3.

What is the best recommendation for the number of phase 3 iterations still is under study currently.

### 9 Tests

Three ways of testing are available in SIENA.

- 1. For a given estimated model, the parameters can be tested by *t*-ratios, calculated as estimate divided by standard error. Under the null hypothesis that the parameter is 0, these have approximately a standard normal distribution.
- 2. In the maximum likelihood estimation methods (both the ERGM case and the longitudinal case provided, for the latter, that the maximum likelihood option has been chosen) it is possible to request likelihood ratio tests. The log likelihood ratio is computed by bridge sampling (Gelman and Meng, 1998; Handcock and Hunter, 2006). This can be requested (a bit deviously) by the number of runs in phase 3 (defined in the specification options):
  - (a) If the number of phase 3 runs is a multiple of 100 plus 1 (e.g., 101, 501, etc.), then the log likelihood ratio is calculated comparing the estimates obtained with the standard initial values.
  - (b) If the number of phase 3 runs is a multiple of 100 plus 2 (e.g., 102, 502, etc.), then the log likelihood ratio is calculated comparing the estimates obtained with the initial values used in the current estimation procedure.

The first option will be the most frequently useful, because it yields log likelihood ratios which, for different models fitted to a given data set, all are comparable.

3. Score-type tests of single and multiple parameters are described in the next section.

#### 9.1 Goodness of fit testing

To compare competing models for network and behavior evolution, goodness-of-fit tests are indispensable. A generalized Neyman-Rao score test is implemented for the MoM estimation method in SIENA which can be used for goodness-of-fit tests (see Schweinberger, 2005); for the ML method (including the ERGM case), following the same steps produces the Rao efficient score test. In section 9.2, it is described how goodness-of-fit tests can be carried out, and in section 9.3 an example is given including interpretation. Section 9.4 considers an alternative application of the test, which may be of interest when facing convergence problems.

#### 9.2 How-to-do

Most goodness-of-fit tests will have the following form: some model is specified and one or more parameters are restricted to some constant, in most cases 0. Such restrictions on parameters can be imposed in the StOCNET program collection by pressing the Model specifications button on the main SIENA interface, selecting the parameter of interest, pressing the Advanced button, checking the box in the column with label t corresponding to the parameter of interest, and specifying the value to which the parameter is restricted. Outside the StOCNET program collection, parameters can be restricted by opening the *pname*.MO file, going to the parameters of interest and setting the values in the fourth column equal to 1. The goodness-of-fit test proceeds by simply estimating the restricted model (not the unrestricted model, with unrestricted parameters) by the standard SIENA estimation algorithm. No more information needs to be communicated. When the model is restricted, SIENA by default assumes that the restricted model is to be tested against the unrestricted model, and by default SIENA evaluates the generalized Neyman-Rao score test statistic.

#### 9.3 Example: one-sided tests, two-sided tests, and one-step estimates

Suppose that it is desired to test the goodness-of-fit of the model restricted by the null hypothesis that the transitivity parameter is zero. The following output may be obtained:

02 Generalised score test  $\langle c \rangle$ \_\_\_\_\_ Testing the goodness-of-fit of the model restricted by = 0.0000 (1) eval: reciprocity ----c = 3.9982 d.f. = 1 p-value = 0.0455 one-sided (normal variate): 1.9996 -----One-step estimates: 1: constant network rate (period 1) 6.3840 1: constant network rate (period 2) 6.4112 0.9404 eval: outdegree (density) eval: reciprocity 1.2567

To understand what test statistic  $\langle c \rangle$  is about, consider the case where the network is observed at two time points, and let R be the number of reciprocated ties at the second time point. Then it can be shown that the test statistic is some function of

Expected R under the restricted model - observed R.

Thus, the test statistic has some appealing interpretation in terms of goodness-of-fit: when reciprocated ties do have added value for the firms—which means that the reciprocity parameter is not 0, other than the model assumes—then the deviation of the observed R from the R that is expected under the model will be large (large misfit), and so will be the value of the test statistic. Large values of the test statistic imply low p-values, which, in turn, suggests to abandon the model in favor of models incorporating reciprocity.

Under some conditions, the distribution of the test statistic tends, as the number of observations increases, to the chi-square distribution, where the number of degrees of freedom is equal to the number of restricted parameters. The corresponding p-value is given in the output file.

In the present case, one parameter is restricted (reciprocity), hence there is one degree of freedom d.f. = 1. The value of the test statistic c = 3.9982 at one degree of freedom gives p

= 0.0455. That is, it seems that reciprocity should be included into the model and estimated as the other parameters.

The one-sided test statistic, which can be regarded as normal variate, equals 1.9996 indicating that the value of the transitivity parameter is positive.

The one-step estimates are approximations of the unrestricted estimates (that is, the estimates that would be obtained if the model were estimated once again, but without restricting the transitivity parameter). The one-step estimate of reciprocity, 1.2567, hints that the transitivity parameter is positive, which agrees with the one-sided test.

#### 9.3.1 Multi-parameter tests

In the case where K > 1 model parameters are restricted, SIENA evaluates the test statistic with K degrees of freedom. A low p-value of the joint test would indicate that the goodness-of-fit of the model is intolerable. However, the joint test with K degrees of freedom gives no clue as to what parameters should be included into the model: the poor goodness-of-fit could be due to only one of the K restricted parameters, it could be due to two of the K restricted parameters, or due to all of them. Hence SIENA carries out, in addition to the joint test with K degrees of freedom, additional tests with one degree of freedom that test the single parameters one-by-one. The goodness-of-fit table looks as follows:

C2 Generalised score test  $\langle c \rangle$ 

#### Testing the goodness-of-fit of the model restricted by

```
(1)
     eval: covariate_ij (centered)
                                                = 0.0000
(2)
     eval: covariate_i alter
                                                = 0.0000
(3)
     eval: covariate_i similarity
                                                = 0.0000
       _____
Joint test:
_____
  c = 92.5111 d.f. = 3 p-value [ 0.0001
(1) tested separately:
_____
- two-sided:
  c = 62.5964 d.f. = 1 p-value [ 0.0001
- one-sided (normal variate): 7.9118
(2) tested separately:
- two-sided:
  c = 16.3001 d.f. = 1 p-value [ 0.0001
- one-sided (normal variate): 4.0373
(3) tested separately:
_____
- two-sided:
  c = 23.4879 d.f. = 1 p-value [ 0.0001
- one-sided (normal variate): 4.8464
```

One-step estimates:

l: con	stant network rate (period 1)	7.4022
l: con	stant network rate (period 2)	6.4681
eval:	outdegree (density)	-0.4439
eval:	reciprocity	1.1826
eval:	transitive triplets	0.1183
eval:	covariate_ij (centered)	0.4529
eval:	covariate_i alter	0.1632
eval:	covariate_i similarity	0.4147

In the example output, three parameters are restricted. The *p*-value corresponding to the joint test indicates that the restricted model is not tenable. Looking at the separate tests, it seems that the misfit is due to all three parameters. Thus, it is sensible to improve the goodness-of-fit of the baseline model by including all of these parameters, and estimate them.

#### 9.3.2 Testing homogeneity assumptions

SIENA by default assumes that the parameter values are constant across actors and periods. Such assumptions are sometimes hardly credible in the light of substantive insight and empirical data, and it may be desired to test them by including suitable dummy variables. See Schweinberger (2005) for examples.

#### 9.4 Alternative application: convergence problems

An alternative use of the score test statistic is as follows. When convergence of the estimation algorithm is doubtful, it is sensible to restrict the model to be estimated. Either "problematic" or "non-problematic" parameters can be kept constant at preliminary estimates (estimated parameters values). Though such strategies may be doubtful in at least some cases, it may be, in other cases, the only viable option besides simply abandoning "problematic" models. The test statistic can be exploited as a guide in the process of restricting and estimating models, as small values of the test statistic indicate that the imposed restriction on the parameters is not problematic.
# 10 Simulation

The simulation option simulates the network evolution for fixed parameter values. This is meaningful mainly at the point that you have already estimated parameters, and then either want to check again whether the statistics used for estimation have expected values very close to their observed values, or want to compute expected values of other statistics. The statistics to be simulated can be specified in a special screen in StOCNET.

The number of runs is set at a default value of 1,000, and can be changed in the simulation options. The user can break in and terminate the simulations early. When only 1 run is requested, an entire data set is generated and written to file in SIENA format and also in Pajek format.

The output file contains means, variances, covariances, and correlations of the selected statistics. The output file also contains *t*-statistics for the various statistics; these can be regarded as tests for the simple null hypothesis that the model specification with the current parameter values is correct.

The simulation feature can be used in the following way. Specify a model and estimate the parameters. After this estimation (supposing that it converged properly), add a number of potential effects. This number might be too large for the estimation algorithm. Therefore, do not Estimate but choose Simulate instead. The results will indicate which are the statistics for which the largest deviations (as measured by the *t*-statistics) occurred between simulated and observed values. Now go back to the model specification, and return to the specification for which the parameters were estimated earlier. The effects corresponding to the statistics with large *t*-values are candidates for now being added to the model. One should be aware, however, that such a data-driven approach leads to capitalization on chance. Since the selected effects were chosen on the basis of the large deviation between observed and expected values, the *t*-tests, based on the same data set, will tend to give significant results too easily.

The generated statistics for each run are also written to the file *pname.sdt* ('sdt' for 'simulation data'), so you can inspect them also more precisely. This file is overwritten each time you are simulating again. A brief history of what the program does is again written to the file *pname.log*.

## 10.1 Conditional and unconditional simulation

The distinction between conditional and unconditional simulation is the same for the simulation as for the estimation option of SIENA, described in Section 7.3.4.

If the conditional simulation option was chosen (which is the default) and the simulations do not succeed in achieving the condition required by its stopping rule (see Section 7.3.4), then the simulation is terminated with an error message, saying *This distance is not achieved for this parameter vector*. In this case, you are advised to change to unconditional simulation.

# 11 One observation: exponential random graph models

By choosing only one observation moment, the user specifies that not a model for network evolution is studied, but an exponential random graph model ('ERGM'), also called a  $p^*$  model (Frank & Strauss, 1986; Frank, 1991; Wasserman & Pattison, 1996; Snijders, Pattison, Robins, and Handcock, 2006; Snijders, 2002; Robins, Snijders, Wang, Handcock, and Pattison, 2007). A good introduction to the current knowledge about this model is Robins et al. (2007). SIENA carries out Markov chain Monte Carlo (MCMC) estimation for this model, as described in Snijders (2002). This algorithm computes a Monte Carlo approximation of the maximum likelihood estimate. However, if the model specification is not well in accordance with the data set, then the algorithm will not converge properly. This is discussed in Snijders (2002) and Handcock (2002). How to specify the model is discussed in Snijders, Pattison, Robins and Handcock (2006), focusing on how to specify transitivity. To model this concept more complicated effects are required than the traditional transitive triplet count. Also when the model specification is good, however, it may require repeated SIENA runs, each using the previously obtained estimate as the new starting value, to obtain satisfactory convergence of the algorithm. This means in practice that (1) great care is required for the model specification, (2) the user may have to tune two constants for the algorithm, called the *multiplication factor* and the *initial gain parameter*, which are discussed below. In any case, it is advisable always to choose the conditional estimation/simulation option, which means here that the total number of ties is kept fixed. For unconditional estimation, the total number of ties is a random variable. The choice between these two is made in the advanced options.

If there are structural zeros (see Section 5.1.1) and the elements of the adjacency matrix that are *not* structurally determined split the network into two or more components (which will happen in the case where several smaller networks are artificially combined into one network with structural zeros between the original smaller networks), then the conditional estimation option keeps the total number of ties constant within each component.

The program recognizes automatically if the data set is symmetric (a non-directed graph, with  $x_{ij} = x_{ji}$  for all i, j) or anti-symmetric (a tournament, with  $x_{ij} \neq x_{ji}$  for all  $i \neq j$ ). In such cases, this is respected by the Metropolis-Hastings algorithm (numbers 6 or 7 in the list below) chosen by SIENA for the MCMC estimation, and the exponential random graph model is considered only on the set of all symmetric or all antisymmetric graphs, respectively.

The program has the following possibilities for the definition of the steps in the MCMC procedure (cf. Snijders, 2002):

- 1. Gibbs steps for single tie variables  $x_{ij}$ ;
- 2. Gibbs steps for dyads  $(x_{ij}, x_{ji})$ ;
- 3. Gibbs steps for triplets  $(x_{ij}, x_{jh}, x_{ih})$  and  $(x_{ij}, x_{jh}, x_{hi})$ ;
- 4. Metropolis Hastings steps for single tie variables  $x_{ij}$ , version A;
- 5. Metropolis Hastings steps for single tie variables  $x_{ij}$ , version B;
- 6. Metropolis Hastings steps for single tie variables  $x_{ij}$ , version A, for non-directed graphs;
- 7. Metropolis Hastings steps for single tie variables  $x_{ij}$ , for antisymmetric graphs ('tournaments');
- Metropolis Hastings steps keeping the in-degrees and out-degrees fixed; see Snijders and van Duijn (2002).

The choice between these types of steps is made in the model options. The default for directed networks is step type 4, which is represented by code 14 because of the necessity to use a continuous chain (see below). Some other options are available by modifying the *pname*.MO file as indicated in Section 19.2.1 below. When there are structurally determined positions, options 6 and 7 should be used only if these positions also are placed symmetrically.

In the conditional option (where the number of arcs is fixed), options 1 and 4–6 exchange values of arcs  $x_{ij}$  and  $x_{hk}$  with  $(i, j) \neq (h, k)$  with probabilities defined by the Gibbs and Metropolis-Hastings rules, respectively; option 2 changes values of dyads  $(x_{ij}, x_{ji})$  and  $(x_{hk}, x_{kh})$  with  $(i, j) \neq$ (h, k), keeping  $x_{ij} + x_{ji} + x_{hk} + x_{kh}$  constant; and option 3 changes the value of one triplet  $(x_{ij}, x_{jh}, x_{ih})$  or  $(x_{ij}, x_{jh}, x_{hi})$ , keeping the sum  $x_{ij} + x_{jh} + x_{ih}$  or  $x_{ij} + x_{jh} + x_{hi}$  constant.

The number of steps, or run length, for generating one exponential random graph is  $r n^2/2d(1-d)$ , where r is a constant which can be changed in the model options, and called *multiplication* factor in the model and estimation options screen; n is the number of actors; and d is the density of the graph, truncated to lie between 0.05 and 0.95. The default value of r can be increased when it is doubted that the run length is sufficient to achieve convergence of the MCMC algorithm. It can be helpful to start the estimation on a given data set by specifying a very simple model, with only the out-degree and the reciprocity effects – for non-directed networks, only the degree effect. Then by some trial and error determine the multiplication factor so that the autocorrelations reported in the SIENA output are less than .4. This will presumably be a suitable value of the multiplication factor also for other, more complicated models. If later on the largest reported autocorrelations become much smaller or larger, then tune the multiplication factor such that the largest reported autocorrelation is smaller than 0.4.

The algorithm uses by default a continuous chain to make successive draws from the ERGM, even for different parameter values. This is in order to improve convergence speed. For this purpose, the possibilities 1–8 mentioned above should be communicated in the model options (see p. 68) by the values 11–18.

When convergence (as evidenced by all *t*-ratios for convergence being less than 0.1 in absolute value) is not easy to obtain, then one can try to improve convergence in repeated runs of SIENA, with the following options, When some autocorrelations are markedly higher than 0.1, then it can help to increase the multiplication factor. When the provisional parameter estimate (used as initial value for the estimation algorithm) seems to be reasonably close to a satisfactory value, then decrease the initial gain parameter (see Section 12), e.g., to the value 0.001 or 0.0001. Some guidance for how to do this is also given in Section 3.2.

# 12 Options for model type, estimation and simulation

There are several options available in SIENA. The main options concern the model type and the estimation procedure used.

Options concerning model type and estimation procedure can be accessed in the StOCNET environment via the Model specification screen's 'option' page. More detailed information is given starting at page 67.

1. There is a choice between conditional (1) and unconditional (0) estimation. If there are dependent action variables, the default for conditional estimation is to condition on the observed distance for the network variable; but it then is possible also to condition on the distances observed for the dependent action variables.

In addition, there are options for maximum likelihood (2) and Bayesian (3) estimation; these are beginning to be documented.

2. The Model Code.

This defines the Model Type and an associated output option.

In the longitudinal case, the meaning of this code is as follows.

Model Codes 10 or more give extra output for evaluating the fit of the out-degree distribution and for the explained variation (Snijders, 2004);

the integer Model Code in the unit position (i.e., Model Code itself if it is less than 10, and Model Code - 10 if the code is more than 10) defines the Model Type defined in Section 6.5. In the ERGM (non-longitudinal) case, the Model Code defines the kind of steps made in the MCMC algorithm. It is advised to use one of the values 11-16, because these generate a continuous chain which yields much better convergence.

- The number of subphases in phase 2 of the estimation algorithm. This determines the precision of the estimate. Advice: 3 for quick preliminary investigations, 4 or 5 for serious estimations.
- 4. The number of runs in phase 3 of the estimation algorithm.

This determines the precision of the estimated standard errors (and covariance matrix of the estimates), and of the *t*-values reported as diagnostics of the convergence. Advice: 200 for preliminary investigations when precise standard errors and *t*-values are not important, 1000 for serious investigations, 2000 for estimations of which results are to be reported.

(These numbers can be twice as low if, instead of the new (from Version 2.3) default option of estimation by the Score Function method, the older method of Finite Differences is used. The latter method has runs that take more time, but needs less runs.)

- 5. A constant used in other estimation procedures. In the ERGM (non-longitudinal) case, this is the multiplication factor r for the run length used in the MCMC algorithm.
- 6. The initial gain value, which is the step size in the starting steps of the Robbins-Monro procedure, indicated in Snijders (2001) by  $a_1$ .
- 7. The choice between standard initial values (suitable estimates for the density and reciprocity parameters and zero values for all other parameters) or the current parameter values as initial values for estimating new parameter values.
- 8. The selection of the period for which a goodness-of-fit on period homogeneity is to be carried out.

- 9. The selection of the effect for which a goodness-of-fit on actor homogeneity is to be carried out (1 for the out-degree effect, 2 for the reciprocity effect); if this is selected, a list of actors also has to be supplied.
- 10. A random number seed. If the value 0 is chosen, the program will randomly select a seed. This is advised to obtain truly random results. If results from an earlier run are to be exactly replicated, the random number seed from this earlier run can be used.
- 11. The method to estimate derivatives; 0 is the older finite differences method (this is the method used in SIENA versions 1 and 2, which has a bias); 1 and 2 are the more efficient and unbiased methods proposed by Schweinberger and Snijders (2007); the preferred method is number 1. See Section 8.

Options about the simulation runs can be accessed in the StOCNET environment via the simulation specification button on the SIENA model's main screen. This button only is activated when 'simulation' is chosen as the 'Run model'. There is one option for simulations that can be chosen here.

1. The number of runs in the straight simulations. Advice: the default of 1000 will usually be adequate.

Depending on the choice for conditional or unconditional estimation in the estimation options, also the simulations are run conditionally or unconditionally.

# 13 Getting started

For getting a first acquaintance with the model, one may use the data set collected by Gerhard van de Bunt, discussed extensively in van de Bunt (1999), van de Bunt, van Duijn, and Snijders (1999), and used as example also in Snijders (2001) and Snijders (2005). The data files are provided with the program. The digraph data files used are the two networks vrnd32t2.dat, vrnd32t4.dat. The networks are coded as 0 = unknown, 1 = best friend, 2 = friend, 3 = friendly relation, 4 = neutral, 5 = troubled relation, 6 = item non-response, 9 = actor non-response. In the Transformations screen of StOCNET, choose the values '1 2 3' as the values to be coded as 1 for the first as well as the second network. Choose '6 9' as missing data codes.

The actor attributes are in the file vars.dat. Variables are, respectively, gender (1 = F, 2 = M), program, and smoking (1 = yes, 2 = no). See the references mentioned above for further information about this network and the actor attributes.

Specify the data in StOCNET by using subsequently the Data and Transformation menus (do not forget to click Apply when finishing each of these parts), then select SIENA in the Model menu and click the Data specification button. Select the network data, in temporal sequence, on the left side of the Data specification screen, then click on OK. Back on the SIENA main screen, now click on the Model specification button.

You will be requested to make some choices for the specification, the meaning of which should be clear given what is explained above. On the left hand side of the **Model specification** screen, you can specify evaluation and endowment effects, indicated by two columns of checkboxes marked 'u' and 'e', respectively. In the specification of the evaluation function, choose the out-degree effect, the reciprocity effect, and one other effect. In the specification of the endowment function, choose no effects at all. On the right hand side of the screen, you can specify rate function effects. At first, leave the specification of the rate function as it is (see Section 6, in which it was advised to start modeling with a constant rate function).

Then let the program estimate the parameters. You will see a screen with intermediate results: current parameter values, the differences ('deviation values') between simulated and observed statistics (these should average out to 0 if the current parameters are close to the correct estimated value), and the quasi-autocorrelations discussed in Section 7.

It is possible to intervene in the algorithm by clicking on the appropriate buttons: the current parameter values may be altered or the algorithm may be restarted or terminated. In most cases this is not necessary.

Some patience is needed to let the machine complete its three phases. How this depends on the data set and the number of parameters in the model is indicated in Section 17. After having obtained the outcomes of the estimation process, the model can be respecified: non-significant effects may be excluded (but it is advised always to retain the out-degree and the reciprocity effects) and other effects may be included.

## 13.1 Model choice

For the selection of an appropriate model for a given data set it is best to start with a simple model (including, e.g., 2 or 3 effects), delete non-significant effects, and add further effects in groups of 1 to 3 effects. Like in regression analysis, it is possible that an effect that is non-significant in a given model may become significant when other effects are added or deleted!

When you start working with a new data set, it is often helpful first to investigate the main endogenous network effects (reciprocity, transitivity, etc.) to get an impression of what the network dynamics looks like, and later add effects of covariates. The most important effects are discussed in Section 6; the effects are defined mathematically in Section 15.

#### 13.1.1 Exploring which effects to include

The present section describes an exploratory approach to model specification. A more advanced approach to testing model specifications is described in Section 9.

For an exploration of further effects to be included, the following steps may be followed:

- 1. Estimate a model which includes a number of basic effects;
- 2. Simulate the model for these parameter values but also include some other relevant statistics among the simulated statistics;
- 3. Look at the *t*-values for these other statistics; effects with large *t*-values are candidates for inclusion in a next model.

It should be kept in mind, however, that this exploratory approach may lead to capitalization on chance, and also that the *t*-value obtained as a result of the straight simulations is conditional on the fixed parameter values used, without taking into account the fact that these parameter values are estimated themselves.

It is possible that for some model specifications the data set will lead to divergence, e.g., because the data contains too little information about this effect, or because some effects are 'collinear' with each other. In such cases one must find out which are the effects causing problems, and leave these out of the model. Simulation can be helpful to distinguish between the effects which should be fixed at a high positive or negative value and the effects which should be left out because they are superfluous.

When the distribution of the out-degrees is fitted poorly (which can be investigated by the extra output requested by selecting Model Code larger than 10 in the model options), an improvement usually is possible either by including non-linear effects of the out-degrees in the evaluation function, or by changing to Model Type 2 (see Section 6.5).

## 13.2 Convergence problems

If there are convergence problems, this may have several reasons.

- The data specification was incorrect (e.g., because the coding was not given properly).
- The starting values were poor. Try restarting from the standard initial values (a certain non-zero value for the density parameter, and zero values for the other parameters); or from values obtained as the estimates for a simpler model that gave no problems. The initial default parameter values can be obtained by choosing the model option "standard initial values".

When starting estimations with Model Type 2 (see Section 6.5), there may be some problems to find suitable starting values. For Model Type 2, it is advised to start with unconditional estimation (see the model options) and a simple model, and to turn back to conditional estimation, using the current parameter values as initial estimates for new estimation runs, only when satisfactory estimates for a simple model have been found.

• The model does not fit well in the sense that even with well-chosen parameters it will not give a good representation of the data.

This can be the case, e.g., when there is a large heterogeneity between the actors which is not well represented by effects of covariates. The out-degrees and in-degrees are given in the begin of the SIENA output to be able to check whether there are outlying actors having very high in- or out-degrees, or a deviating dynamics in their degrees. Strong heterogeneity between the actors will have to be represented by suitable covariates; if these are not available, one may define one or a few dummy variables each representing an outlying actor, and give this dummy variable an ego effect in the case of deviant out-degrees, and an alter effect in the case of deviant in-degrees.

Another possibility is that there is time heterogeneity. Indications about this can be gathered also from the descriptives given in the start of the output file: the number of changes upward and downward, in the network and also – if any – in the dependent behavioral variable. If these do not show a smooth or similar pattern across the observations, then it may be useful to include actor variables representing time trends. These could be smooth – e.g., linear – but they also could be dummy variables representing one or more observational periods; these must be included as an ego effect to represent time trends in the tendency to make ties (or to display higher values of the behavior in question).

- Too many weak effects are included. Use a smaller number of effects, delete non-significant ones, and increase complexity step by step. Retain parameter estimates from the last (simpler) model as the initial values for the new estimation procedure, provided for this model the algorithm converged without difficulties.
- Two or more effects are included that are almost collinear in the sense that they can both explain the same observed structures. This will be seen in high absolute values of correlations between parameter estimates. In this case it may be better to exclude one of these effects from the model.
- An effect is included that is large but of which the precise value is not well-determined (see above: section on fixing parameters). This will be seen in estimates and standard errors both being large and often in divergence of the algorithm. Fix this parameter to some large value. (Note: large here means, e.g., more than 5 or less than -5; depending on the effect, of course.)

If the algorithm is unstable, with parameter values (the left hand list in the SIENA window) changing too wildly, or with the algorithm suddenly seeming stuck and not moving forward, the a solution may be to simplify the model (perhaps later on making it more complex again in forward parameter estimation steps); another solution may be to decrease the initial gain parameter (see Section 12).

If there are problems you don't understand, but you do know something about the operation of SIENA, you could take a look at the file *pname*.log; and, if the problems occur in the estimation algorithm, at the file *pname*.cck. These files give information about what the program did, which may be helpful in diagnosing the problem. E.g., you may look in the *pname*.cck file to see if some of the parameters are associated with positive values for the so-called quasi-autocorrelations. If this happens from subphase 2.2 onward for some parameters, these may be the parameters that led to problems in the estimation algorithm (e.g., because the corresponding effect is collinear with other effects; or because they started from unfortunate starting values; or because the data set contains too little information about their value).

## 13.3 Composition change

Example data files for a network of changing composition are also provided with the program. These files are called vtest2.dat, vtest3.dat, and vtest4.dat. They contain the same network data as the friendship data files of van de Bunt (for these three observation times and with the same coding), except that in these data some joiners and leavers were artificially created. These actors were given the code '9' for the observation moment at which they were not part of the network. The attribute file vtestexo.dat contains the times at which the network composition changes (see

also the example in Section 5.7). This file is necessary for the program to correctly include the times at which actors join or leave the network. For example, the first line of the file contains the values

1 0.7 3 0.0

which indicates that the first actor joins the network at fraction 0.7 of period 1 (the period between the first and second observation moments) and leaves the network right after the beginning of the third period, i.e., he/she does not leave the network before the last observation at the third time point. Thus, the first actor joins the network and then stays in during the whole period being analyzed.

# 14 Multilevel network analysis

The program Siena08.exe is a relatively simple multilevel extension to SIENA. This program must be run independently, i.e., not through StOCNET, after having obtained estimates for a common model estimated for several data sets. Siena08 combines the estimates in a meta-analysis or multilevel analysis according to the methods of Snijders and Baerveldt (2003), and according to a Fisher-type combination of one-sided *p*-values.

All SIENA output files to be used must already exist, and the last estimation runs in these output files will be used. It is required that all these last estimation runs have the same set of estimated parameters.

An easy way to operate Siena08 is to make a shortcut in Windows, right-click on the shortcut and open the "properties" tab, and in the "Target" – which already contains the path and filename of the Siena08.exe file – add the projectname after the filename (separated by a space). E.g., suppose the projectname is ABC. Then there must be a project file with the name ABC.mli (the root name "ABC" can be chosen by the user, the extension name "mli" is prescribed.) If the number of network evolution projects combined in this Siena08 run is given by K, e.g. the K = 3 projects with names A, B and C, then the file ABC.mli must give the project names on separate lines and in addition the options, as indicated in the following example file:

```
[This file contains specifications for the meta-analysis of Siena projects.]
[It serves as input for the Siena08 program.]
@1 [general information about the Siena project list ]
10 [number of projects, names follow:]
A
B
C
@2 [options for estimation of projects]
5 [upper bound for standard error in meta-analysis]
1 [code 0=estimate, 1=aggregate from .out-files, 2=generate .dsc-file]
1 [code 1=extra output]
```

To get started, try this out with a small data set.

# 15 Mathematical definition of effects

Here, the mathematical formulae for the definition of the effects are given. In Snijders (2001, 2005) and Steglich, Snijders and Pearson, (2007), further background to these formulae can be found. The effects are grouped into effects for modelling network evolution and effects for modelling behavioral evolution (i.e., the dynamics of dependent actor variables). Within each group of effects, the effects are listed in the order in which they appear in SIENA.

Some of the effects contain a number which is denoted in this section by c, and called in this manual an *internal effect parameter*. (These are totally different from the statistical parameters which are the weights of the effects in the objective function.) These numbers can be determined by the user as the "par." column in the advanced model specification options of StOCNET, or by changing the *pname.mo* file described in Section 19.2.1.

#### 15.1 Network evolution

The model of network evolution consists of the model of actors' decisions to establish new ties or dissolve existing ties (according to *evaluation* and *endowment functions*) and the model of the timing of these decisions (according to the *rate function*). The objective function of the actor is the sum of the network evaluation function and the network endowment function

$$u^{\text{net}}(x) = f^{\text{net}}(x) + g^{\text{net}}(x) , \qquad (1)$$

and a random term; where the evaluation function  $f^{\text{net}}(x)$  and the endowment function  $g^{\text{net}}(x)$  are as defined in the following subsections.

(It may be noted that the network evaluation function was called objective function, and the endowment function was called gratification function, in Snijders, 2001.)

#### 15.1.1 Network evaluation function

The network evaluation function for actor i is defined as

$$f^{\rm net}(x) = \sum_{k} \beta_k s_{ik}^{\rm net}(x) \tag{2}$$

where  $\beta_k$  are parameters and  $s_{ik}^{\text{net}}(x)$  are effects as defined below.

The potential effects in the network evaluation function are the following. Note that in all effects where a constants c occurs, this constant can be chosen and changed by the user. Also note that the evaluation effects which are a function only of the out-degree of actor i are excluded for Model Type 2. For non-directed networks, the same formulae are used, unless a different formula is given explicitly.

- 1. out-degree effect or density effect, defined by the out-degree  $s_{i1}^{\text{net}}(x) = x_{i+} = \sum_j x_{ij}$ , where  $x_{ij} = 1$  indicates presence of a tie from *i* to *j* while  $x_{ij} = 0$  indicates absence of this tie;
- 2. reciprocity effect, defined by the number of reciprocated ties  $s_{i2}^{\text{net}}(x) = \sum_{j} x_{ij} x_{ji};$
- 3. transitivity effect, defined by the number of transitive patterns in *i*'s relations (ordered pairs of actors (j, h) to both of whom *i* is tied, while also *j* is tied to *h*), for directed networks,  $s_{i3}^{\text{net}}(x) = \sum_{j,h} x_{ij} x_{ih} x_{jh}$ ; and for non-directed networks,  $s_{i3}^{\text{net}}(x) = \sum_{j < h} x_{ij} x_{ih} x_{jh}$ ;

4. (direct and indirect) ties effect, defined by the number of actors to whom i is directly as well as indirectly tied,  $\sum_{i=1}^{net} (i) = \sum_{i=1}^{net} (i) = \sum_{$ 

 $s_{i4}^{\text{net}}(x) = \sum_{j} x_{ij} \, \max_h(x_{ih} \, x_{hj});$ 

5. *balance*, defined by the similarity between the outgoing ties of actor i and the outgoing ties of the other actors j to whom i is tied,

$$s_{i5}^{\text{net}}(x) = \sum_{j=1}^{n} x_{ij} \frac{1}{n-2} \sum_{\substack{h=1\\h\neq i,j}}^{n} (b_0 - |x_{ih} - x_{jh}|) ,$$

where  $b_0$  is a constant included to reduce the correlation between this effect and the density effect, defined by

$$b_0 = \frac{1}{(M-1)n(n-1)(n-2)} \sum_{m=1}^{M-1} \sum_{\substack{i,j=1\\h\neq i,j}}^n \sum_{\substack{h=1\\h\neq i,j}}^n |x_{ih}(t_m) - x_{jh}(t_m)| .$$

- 6. number of distances two effect, defined by the number of actors to whom *i* is indirectly tied (through one intermediary, i.e., at sociometric distance 2),  $s_{i6}^{\text{net}}(x) = \#\{j \mid x_{ij} = 0, \max_h(x_{ih} x_{hj}) > 0\};$
- 7. popularity effect, defined by 1/n times the sum of the in-degrees of the others to whom *i* is tied,  $s_{i7}^{\text{net}}(x) = \frac{1}{n} \sum_{j} x_{ij} x_{+j} = \frac{1}{n} \sum_{j} x_{ij} \sum_{h} x_{hj};$
- 8. popularity of alter (sqrt measure) effect, defined by 1/n times the sum of the square roots of the in-degrees of the others to whom i is tied,

 $s_{i8}^{\text{net}}(x) = \frac{1}{n} \sum_{j} x_{ij} \sqrt{x_{+j}} = \frac{1}{n} \sum_{j} x_{ij} \sqrt{\sum_{h} x_{hj}};$ this often works better in precisive than the popularity effect itself; also it is often reasonable

to assume that differences between high in-degrees are relatively less important than the same differences between low in-degrees;

9. activity effect, defined by 1/n times the sum of the out-degrees of the others to whom i is tied,

$$s_{i9}^{\text{net}}(x) = \frac{1}{n} \sum_{j} x_{ij} x_{j+} = \frac{1}{n} \sum_{j} x_{ij} \sum_{h} x_{jh}$$

- ⊙ for non-directed networks, the popularity and activity effects are taken together as "degree effects", since in-degrees and out-degrees are the same in this case;
- 10. *out-degree up to c*, where *c* is some constant, defined by  $s_{i10}^{\text{net}}(x) = \max(x_{i+}, c);$
- 11. square root out-degree  $-c \times o.d.$ , where c is some constant, defined by  $s_{i11}^{\text{net}}(x) = \sqrt{x_{i+}} cx_{i+},$  where c is chosen to diminish the collinearity between this and the density effect;
- 12. squared (out-degree c), where c is some constant, defined by  $s_{i12}^{\text{net}}(x) = (x_{i+} c)^2$ , where c is chosen to diminish the collinearity between this and the density effect.
- 13. sum of (1/(out-degree + c)), where c is some constant, defined by  $s_{i13}^{\text{net}}(x) = 1/(x_{i+} + c);$

- 14. sum of (1/(out-degree + c)(out-degree + c + 1)), where c is some constant, defined by  $s_{i_{14}}^{\text{net}}(x) = 1/(x_{i_{1}} + c)(x_{i_{1}} + c + 1);$
- 15. number of 3-cycles,  $s_{i15}^{\text{net}}(x) = \sum_{j,h} x_{ij} x_{jh} x_{hi};$
- 16. betweenness count,  $s_{i16}^{\text{net}}(x) = \sum_{j,h} x_{hi} x_{ij} (1 - x_{hj});$
- 17. number of dense triads, defined as triads containing at least c ties,  $s_{i17}^{\text{net}}(x) = \sum_{j,h} x_{ij} I\{x_{ij} + x_{ji} + x_{ih} + x_{hi} + x_{jh} + x_{hj}\} \ge c\}$ , where the 'indicator function'  $I\{A\}$  is 1 if the condition A is fulfilled and 0 otherwise, and where c is either 5 or 6; (this effect is superfluous and undefined for symmetric networks);
- 18. number of (unilateral) peripheral relations to dense triads,  $s_{i18}^{\text{net}}(x) = \sum_{j,h,k} x_{ij}(1-x_{ji})(1-x_{hi})(1-x_{ki})I\{(x_{jh}+x_{hj}+x_{jk}+x_{kj}+x_{hk}+x_{kh}) \ge c\},$ where c is the same constant as in the dense triads effect; for symmetric networks, the 'unilateral' condition is dropped, and the definition is  $s_{i18}^{\text{net}}(x) = \sum_{j,h,k} x_{ij}(1-x_{hi})(1-x_{ki})I\{(x_{jh}+x_{hj}+x_{jk}+x_{kj}+x_{hk}+x_{kh}) \ge c\}.$

The effects for a dyadic covariate  $w_{ij}$  are

- 19. covariate (centered) main effect,  $s_{i19}^{\text{net}}(x) = \sum_{j} x_{ij} (w_{ij} - \bar{w})$ where  $\bar{w}$  is the mean value of  $w_{ij}$ ;
- 20. covariate (centered) × reciprocity,  $s_{i20}^{\text{net}}(x) = \sum_{j} x_{ij} x_{ji} (w_{ij} - \bar{w}).$

For actor-dependent covariates  $v_j$  (recall that these are centered internally by SIENA ) as well as for dependent behavior variables (for notational simplicity here also denoted  $v_j$ ), these are the possible effects:

- 21. covariate-alter or covariate-related popularity, defined by the sum of the covariate over all actors to whom *i* has a tie,  $s_{i21}^{\text{net}}(x) = \sum_{j} x_{ij} v_j;$
- 22. covariate-ego or covariate-related activity, defined by i's out-degree weighted by his covariate value,  $s_{i22}^{net}(x) = v_i x_{i+};$
- 23. covariate-related similarity, defined by the sum of centered similarity scores  $\sin_{ij}^{v}$  between i and the other actors j to whom he is tied,

 $s_{i23}^{\text{net}}(x) = \sum_{j} x_{ij} (\sin_{ij}^{v} - \sin^{v}),$ where  $\widehat{\sin^{v}}$  is the mean of all similarity scores, which are defined as  $\sin_{ij}^{v} = \frac{\Delta - |v_i - v_j|}{\Delta}$  with  $\Delta = \max_{ij} |v_i - v_j|$  being the observed range of the covariate v;

24. covariate-related similarity × reciprocity, defined by the sum of centered similarity scores for all reciprocal dyads in which *i* is situated,  $s_{i24}^{\text{net}}(x) = \sum_{i} x_{ij} x_{ji} (\sin_{ij}^{v} - \widehat{\sin^{v}});$  25. covariate-related identity, defined by the number of ties of *i* to all other actors *j* who have exactly the same value on the covariate,  $s_{i25}^{\text{net}}(x) = \sum_{j} x_{ij} I\{v_i = v_j\},$ 

where the indicator function  $I\{v_i = v_j\}$  is 1 if the condition  $\{v_i = v_j\}$  is satisfied, and 0 if it is not;

- 26. covariate-related identity × reciprocity, defined by the number of reciprocated ties between i and all other actors j who have exactly the same value on the covariate,  $s_{i26}^{net}(x) = \sum_{i} x_{ij} x_{ji} I\{v_i = v_j\};$
- 27. covariate-ego  $\times$  alter, defined by the product of *i*'s covariate and the sum of those of his alters,

 $s_{i27}^{\text{net}}(x) = v_i \sum_j x_{ij} v_j;$ 

- 28. covariate-ego × alter × reciprocity, defined by the product of *i*'s covariate and the sum of those of his reciprocated alters,  $s_{i28}^{net}(x) = v_i \sum_j x_{ij} x_{ji} v_j;$
- 29. covariate-related similarity × popularity alter, defined by the sum of centered similarity scores between *i* and the other actors *j* to whom he is tied, weighted by the indegree of these other actors,  $s_{i29}^{\text{net}}(x) = \sum_{i} x_{ij} x_{+j} (\sin_{ij}^{v} - \widehat{\sin^{v}}).$
- 30. user-defined interaction effects as described in Section 6.6. The internal effect parameter is decomposed by SIENA into its two or three constituents, as described in the mentioned section. The interaction is defined on a tie basis: if two interacting effects are defined by  $s_{ia}^{\text{net}}(x) = \sum_{j} s_{ij}^{a}(x)$  and  $s_{ib}^{\text{net}}(x) = \sum_{j} s_{ij}^{b}(x)$  (where *a* and *b* care calculated from the internal effect parameter *c*), then the interaction is defined by  $s_{i30}^{\text{net}}(x) = \sum_{j} s_{ij}^{a}(x) s_{ij}^{b}(x)$ .

Additional possible effects are documented in Steglich, Snijders and Pearson (2007) and for Model Type 2 in Snijders (2003).

#### 15.1.2 Network endowment function

The network endowment function is the way of modeling effects which operate in different strengths for the creation and the dissolution of relations. The network endowment function is zero for creation of ties, and is given by

$$g^{\text{net}}(x) = \sum_{k} \gamma_k s_{ik}^{\text{net}}(x) \tag{3}$$

for dissolution of ties. In this formula, the  $\gamma_k$  are the parameters for the endowment function. The potential effects  $s_{ik}^{\text{net}}(x)$  in this function, and their formulae, are the same as in the evaluation function. For further explication, consult Snijders (2001, 2005), Snijders, Steglich, and Schweinberger (2007), and Steglich, Snijders and Pearson (2007).

#### 15.1.3 Network rate function

The network rate function  $\lambda^{\text{net}}$  (lambda) is defined for Model Type 1 (which is the default Model Type) as a product

$$\lambda_i^{\rm net}(\rho, \alpha, x, m) = \lambda_{i1}^{\rm net} \lambda_{i2}^{\rm net} \lambda_{i3}^{\rm net}$$

of factors depending, respectively, on period m, actor covariates, and actor position (see Snijders, 2001, p. 383). The corresponding factors in the rate function are the following:

1. The dependence on the period can be represented by a simple factor

$$\lambda_{i1}^{\rm net} = \rho_m^{\rm net}$$

for m = 1, ..., M - 1. If there are only M = 2 observations, the basic rate parameter is called  $\rho^{\text{net}}$ .

2. The effect of actor covariates with values  $v_{hi}$  can be represented by the factor

$$\lambda_{i2}^{\text{net}} = \exp(\sum_h \alpha_h \, v_{hi}) \,.$$

3. The dependence on the position of the actor can be modeled as a function of the actor's outdegree, in-degree, and number of reciprocated relations, the 'reciprocated degrees'. Define these by

$$x_{i+} = \sum_{j} x_{ij}, \ x_{+i} = \sum_{j} x_{ji}, \ x_{i(r)} = \sum_{j} x_{ij} x_{ji}$$

(recalling that  $x_{ii} = 0$  for all i).

The contribution of the out-degrees to  $\lambda_{i3}^{\text{net}}$  is a factor

 $\exp(\alpha_h x_{i+}),$ 

if the associated parameter is denoted  $\alpha_h$  for some h, and similarly for the contributions of the in-degrees and the reciprocated degrees.

Also an exponential dependence on reciprocals of out-degrees can be specified; this can be meaningful because the rate effect of the out-degree becoming a value 1 higher might become smaller and smaller as the out-degree increases. Denoting again the corresponding parameter by  $\alpha_h$  (but always for different index numbers h), this effect multiplies the factor  $\lambda_{i3}^{\text{net}}$  by

 $\exp(\alpha_h/x_{i+})$ .

## 15.1.4 Network rate function for Model Type 2

For Model Type 2 (see Section 6.5), the network rate function is defined according to Snijders (2003) by

$$\rho_m \,\lambda_{i+}(s) = \rho_m \,\frac{\nu(s)\,\xi(s)}{1\,+\,\xi(s)},$$
  
$$\rho_m \,\lambda_{i-}(s) = \rho_m \,\frac{\nu(s-1)}{1\,+\,\xi(s-1)}$$

where  $\rho_m \lambda_{i+}(s)$  and  $\rho_m \lambda_{i-}(s)$  represent, respectively, the rate at which an actor of current outdegree s increases, or decreases, his out-degree by 1. The parameter  $\rho_m$  is a multiplicative effect of the observation period.

Function  $\xi$  (xi) is called the distributional tendency function and is represented according to Snijders (2003, formula (17)) by

$$\xi(s) = \exp\left(\alpha_1 - \alpha_2 \log(s+1) - \frac{\alpha_3}{s+1}\right) \ .$$

where the names given in  $\mathsf{SIENA}$  are

- $\alpha_1$  : out-degrees effect;
- $\alpha_2$  : logarithmic out-degree effect;
- $\alpha_3$  : factorial out-degree effect.

The reasons for these names and interpretation of the effects can be found in Snijders (2003). To the exponent also effects of actor covariates can be added.

The so-called volatility function  $\nu$  (nu) is defined as

$$\nu(s) = \left(1 + \alpha_4 \frac{1}{s+1}\right) \ .$$

Also to this exponent effects of actor covariates can be added.

## 15.2 Behavioral evolution

The model of the dynamics of a dependent actor variable consists of a model of actors' decisions (according to *evaluation* and *endowment functions*) and a model of the timing of these decisions (according to a *rate function*), just like the model for the network dynamics. The decisions now do not concern the creation or dissolution of network ties, but whether an actor increases or decreases his score on the dependent actor variable by one, or keeps it as it is.

## 15.2.1 Behavioral evaluation function

Effects for the behavioral evaluation function  $u^{\text{beh}}$  can be selected from the following. Here the dependent variable is transformed to have a minimum value of 0; in other words, z denotes the original input variable minus the minimum of its range.

- 1. behavioral tendency effect,  $s_{i1}^{\text{beh}}(x) = z_i$ , where  $z_i$  denotes the value of the dependent behavior variable of actor i;
- 2. average similarity effect, defined by the average of centered similarity scores  $\sin_{ij}^z$  between i and the other actors j to whom he is tied,  $s_{i2}^{\text{beh}}(x) = x_{i+}^{-1} \sum_j x_{ij} (\sin_{ij}^z - \widehat{\sin}^z);$ (and 0 if  $x_{i+} = 0$ );
- 3. total similarity effect, defined by the sum of centered similarity scores  $\sin_{ij}^{z}$  between *i* and the other actors *j* to whom he is tied,  $s_{i3}^{\text{beh}}(x) = \sum_{j} x_{ij} (\sin_{ij}^{z} - \widehat{\sin}^{z});$
- 4. indegree effect,  $s_{i4}^{\text{beh}}(x) = z_i \sum_j x_{ji};$
- 5. outdegree effect,  $s_{i5}^{\text{beh}}(x) = z_i \sum_j x_{ij};$
- 6. isolate effect, the differential attractiveness of the behavior for isolates,  $s_{i6}^{\text{beh}}(x) = z_i I\{x_{+i} = 0\},$ where again  $I\{A\}$  denotes the indicator function of the condition A;

- 7. average similarity × reciprocity effect, defined by the sum of centered similarity scores  $\sin_{ij}^{z}$ between *i* and the other actors *j* to whom he is reciprocally tied,  $s_{i7}^{\text{beh}}(x) = x_{i(r)}^{-1} \sum_{j} x_{ij} x_{ji} (\sin_{ij}^{z} - \widehat{\sin}^{z});$ (and 0 if  $x_{i(r)} = 0$ );
- 8. total similarity × reciprocity effect, defined by the sum of centered similarity scores  $\sin_{ij}^{z}$  between *i* and the other actors *j* to whom he is reciprocally tied,  $s_{i8}^{\text{beh}}(x) = \sum_{j} x_{ij} x_{ji} (\sin_{ij}^{z} - \sin^{z});$
- 9. average similarity × popularity alter effect, defined by the sum of centered similarity scores  $\sin_{ij}^{z}$  between *i* and the other actors *j* to whom he is tied, multiplied by their indegrees,  $s_{i9}^{beh}(x) = x_{i+}^{-1} \sum_{j} x_{ij} x_{+j} (\sin_{ij}^{z} - \widehat{\sin}^{z});$ (and 0 if  $x_{i+} = 0$ );
- 10. total similarity  $\times$  popularity alter effect, defined by the sum of centered similarity scores  $\sin_{ij}^z$  between *i* and the other actors *j* to whom he is reciprocally tied, multiplied by their indegrees,  $s_{i10}^{\text{beh}}(x) = \sum_j x_{ij} x_{+j} (\sin_{ij}^z - \widehat{\sin}^z);$
- 11. average similarity  $\times$  reciprocity  $\times$  popularity alter effect, defined by the sum of centered similarity scores  $\sin_{ij}^{z}$  between *i* and the other actors *j* to whom he is reciprocally tied, multiplied by their indegrees,

$$s_{i11}^{\text{beh}}(x) = x_{i(r)}^{-1} \sum_{j} x_{ij} x_{ji} x_{+j} (\sin_{ij}^{z} - \widehat{\sin^{z}});$$
  
(and 0 if  $x_{i(r)} = 0$ );

12. total similarity  $\times$  reciprocity  $\times$  popularity alter effect, defined by the sum of centered similarity scores  $\sin_{ij}^{z}$  between *i* and the other actors *j* to whom he is reciprocally tied, multiplied by their indegrees,

 $s_{i12}^{\text{beh}}(x) = \sum_{j} x_{ij} x_{ji} x_{+j} (\sin_{ij}^z - \widehat{\sin}^z);$ 

- 13. average alter effect, defined by the product of *i*'s behavior multiplied by the average behavior of his alters (a kind of ego-alter behavior covariance),  $s_{i13}^{\text{beh}}(x) = z_i \left(\sum_j x_{ij} z_j\right) / \left(\sum_j x_{ij}\right)$ (and  $\bar{z}$  if the ratio is 0/0);
- 14. average reciprocated alter effect, defined by the product of *i*'s behavior multiplied by the average behavior of his reciprocated alters,  $s_{i14}^{\text{beh}}(x) = z_i \left(\sum_j x_{ij} x_{ji} z_j\right) / \left(\sum_j x_{ij} x_{ji}\right)$ (and  $\bar{z}$  if the ratio is 0/0);
- 15. effect of the behavior upon itself, which is like a non-linear tendency effect, where the attractiveness of further steps up the behavior 'ladder' depends on where the actor is on the ladder:  $s_{i15}^{\text{beh}}(x) = (z_i - \bar{z})^2$
- 16. dense triads effect, defined by the number of dense triads in which actor *i* is located,  $s_{i16}^{\text{beh}}(x) = z_i \sum_{j,h} I\{x_{ij} + x_{ji} + x_{ih} + x_{hi} + x_{jh} + x_{hj}\} \ge c\},$ where *c* is either 5 or 6;
  this is currently not correctly implemented in SIENA 3;
- 17. *peripheral effect*, defined by the number of dense triads to which actor *i* stands in a unilateral-peripheral relation,

$$s_{i17}^{\text{beh}}(x) = z_i \sum_{j,h,k} x_{ij} (1 - x_{ji}) (1 - x_{hi}) (1 - x_{hi}) I\{x_{ij} + x_{ji} + x_{ih} + x_{hi} + x_{jh} + x_{hj} \} \ge c\},$$

where c is the same constant as in the *dense triads* effect; for directed networks, the unilateral condition is dropped, and the effect is  $s_{i17}^{\text{beh}}(x) = z_i \sum_{j,h,k} x_{ij}(1-x_{hi})(1-x_{ki})I\{x_{ij} + x_{ji} + x_{hi} + x_{hi} + x_{jh} + x_{hj}) \ge c\};$ this is currently not correctly implemented in SIENA 3;

For each actor-dependent covariate  $v_j$  (recall that these are centered internally by SIENA ) as well as for each of the other dependent behavior variables (for notational simplicity here also denoted  $v_i$ ), there is one main effect:

18. covariate effect,  $s_{i18}^{\text{beh}}(x) = z_i v_i$ .

Additional possible effects are documented in Steglich, Snijders and Pearson (2007).

#### 15.2.2 Behavioral endowment function

Also the behavioral model knows the distinction between evaluation and endowment effects. The formulae of the effects that can be included in the behavioral endowment function  $e^{\text{beh}}$  are the same as those given for the behavioral evaluation function. However, they enter calculation of the endowment function only when the actor considers decreasing his behavioral score by one unit (downward steps), not when upward steps (or no change) are considered. For more details, consult Snijders, Steglich, and Schweinberger (2007) and Steglich, Snijders and Pearson (2007).

#### 15.2.3 Behavioral rate function

The behavioral rate function  $\lambda^{\text{beh}}$  consists of a constant term per period,

$$\lambda_i^{\rm beh} = \rho_m^{\rm beh}$$

for m = 1, ..., M - 1.

## 15.3 Exponential random graph model

The exponential random graph model, which is used if there is only one observation (i.e., for the non-longitudinal case), is defined by the probability function

$$P_{\theta}\{X=x\} = \exp\left(\theta' u(x) - \psi(\theta)\right) ,$$

where u(x) is a vector of statistics. The following statistics are available. The selection of statistics is discussed extensively in Snijders, Pattison, Robins, and Handcock (2006), with attention especially to statistics 16–19.

Note that SIENA will note whether the observed graph  $(x_{ij})$  is symmetric or not, and choose accordingly between the statistics for undirected and directed graphs.

1. For undirected graphs, the number of edges  $\sum_{i < j} x_{ij}$ ;

for directed graphs, the number of arcs  $\sum_{i \neq j} x_{ij}$ .

- 2. The number of reciprocated relations  $\sum_{i < j} x_{ij} x_{ji}$ .
- 3. The number of out-two stars  $\sum_{i} \sum_{k < k} x_{ik} x_{ik}$ .
- 4. The number of in-two stars  $\sum_{i} \sum_{h < k} x_{hi} x_{ki}$ .

- 5. The number of two-paths (mixed twostars) given for undirected graphs by  $\frac{1}{2} \sum_{i} \sum_{h \neq k} x_{hi} x_{ik}$  and for directed graphs by  $\sum_{i} \sum_{h \neq k} x_{hi} x_{ik}$ .
- 6. For undirected graphs, the number of transitive triads  $\frac{1}{6} \sum_{i,j,h} x_{ij} x_{ih} x_{jh}$ ; for directed graphs, the number of transitive triplets  $\sum_{i,j,h} x_{ij} x_{ih} x_{jh}$ .
- 7. The number of three-cycles given for undirected graphs by  $\frac{1}{6} \sum_{i,j,h} x_{ij} x_{jh} x_{hi}$ and for directed graphs by  $\frac{1}{3} \sum_{i,j,h} x_{ij} x_{jh} x_{hi}$ .
- 8. The number of out-three stars  $\sum_{i} {x_{i+} \choose 3}$ .
- 9. The number of in-three stars  $\sum_{i} {x_{+i} \choose 3}$ .
- 10. The number of out-four stars  $\sum_{i} \binom{x_{i+}}{4}$  .
- 11. The number of in-fourstars  $\sum_{i} {x_{+i} \choose 4}$ .
- 12. The sum of reciprocal out-degrees  $\sum_{i} 1/(x_{i+} + c)$  for some constant c.
- 13. The sum of transformed out-degrees  $\sum_{i} 1/[(x_{i+}+c)(x_{i+}+c+1)]$  for some constant c.
- 14. The number of pairs directly and indirectly connected,
  i.e., tied pairs (i, j) for which there exists at least one h such that x<sub>ih</sub> = x<sub>hj</sub> = 1,
  i.e., tied pairs for which there is at least one twopath from i to j;
  for undirected graphs this is ∑<sub>i<j</sub> x<sub>ij</sub> max<sub>h≠i,j</sub> {x<sub>ih</sub> x<sub>hj</sub>}, for directed graphs it is ∑<sub>i≠j</sub> x<sub>ij</sub> max<sub>h≠i,j</sub> {x<sub>ih</sub> x<sub>hj</sub>}.
- 15. The number of indirectly connected pairs, i.e., pairs (i, j) for which there is at least one two path from i to j;

for undirected graphs this is  $\sum_{i < j} \max_{h \neq i, j} \{x_{ih} x_{hj}\}$ , for directed graphs it is  $\sum_{i \neq j} \max_{h \neq i, j} \{x_{ih} x_{hj}\}$ .

16. The alternating k-out-stars combination

$$c^{2} \sum_{i=1}^{n} \left\{ \left(1 - \frac{1}{c}\right)^{x_{i+}} + \frac{x_{i+}}{c} - 1 \right\} ,$$

for some value c.

17. The alternating k-in-stars combination

$$c^{2} \sum_{i=1}^{n} \left\{ \left(1 - \frac{1}{c}\right)^{x_{+i}} + \frac{x_{+i}}{c} - 1 \right\}$$

for some value c.

18. For undirected graphs, the related k-triangles statistic

$$c\sum_{i< j} x_{ij} \left\{ 1 - \left(1 - \frac{1}{c}\right)^{L_{2ij}} \right\}$$

for some value c, where  $L_{2ij}$  is the number of two-paths from i to j,  $L_{2ij} = \sum_h x_{ih} x_{hj}$ ; for directed graphs, the formula is

$$c\sum_{i,j} x_{ij} \left\{ 1 - \left(1 - \frac{1}{c}\right)^{L_{2ij}} \right\}$$

with the same definition of  $L_{2ij}$ .

19. For undirected graphs, the k-parallel two-paths statistic

$$c \sum_{i < j} \left\{ 1 - \left( 1 - \frac{1}{\lambda} \right)^{L_{2ij}} \right\}$$

which formula is replaced for directed graphs by

$$c \sum_{i,j} \left\{ 1 - \left(1 - \frac{1}{\lambda}\right)^{L_{2ij}} \right\} .$$

- 20. For each dyadic covariate  $w_{ij}$ , the sum  $\sum_{i,j} x_{ij} w_{ij}$ .
- 21. For each dyadic covariate  $w_{ij}$ , the associated reciprocity effect defined by  $\sum_{i,j} x_{ij} x_{ji} w_{ij}$ .
- 22. For each individual covariate  $v_i$  (changing or constant; recall that all covariates are centered), four effects are included. The first is the  $v_i$ -related popularity effect  $\sum_i x_{+i} v_i$ ;
- 23. next is the  $v_i$ -related activity effect  $\sum_i x_{i+} v_i$ ;
- for non-directed networks, these together are replaced by the single covariate effect  $\sum_{i} x_{+i} v_{i} = \sum_{i} x_{i+} v_{i}$ ;
- 24. the  $v_i$ -related similarity effect, defined by the sum of centered similarity scores  $\sum_{i,j} x_{ij} (\sin_{ij}^v - \widehat{\sin^v}),$ where  $\widehat{\sin^v}$  is the mean of all similarity scores, which are defined as  $\sin_{ij}^v = \frac{\Delta - |v_i - v_j|}{\Delta}$  with  $\Delta = \max_{ij} |v_i - v_j|$  being the observed range of the covariate v;
- 25. and fourth the  $v_i$ -related identity is defined by  $\sum_{i,j} x_{ij} I\{v_i = v_j\}$ , where the indicator function  $I\{v_i = v_j\}$  is 1 if the condition  $\{v_i = v_j\}$  is satisfied, and 0 if it is not;
- 26. user-defined interaction effects are possible as described in Section 6.6. The internal effect parameter is decomposed by SIENA into its two or three constituents, as described in the mentioned section. The change statistic for the interaction is the product of the change statistics of the two or three components.

# 16 Running Siena outside of StOCNET

The SIENA program consists of a basic computation part, which is associated with the StOCNET windows shell. The computational part can be used both directly and from StOCNET. The StOC-NET windows shell is easy for data specification and model definition. However, especially for frequent users it can be more convenient to run SIENA outside of the StOCNET environment. How to do this, is explained in this section.

The computational part of SIENA is composed of six executable programs. These programs are:

- 1. Siena01.exe for the basic data input, using an existing basic information file (see below);
- 2. Siena02.exe for data description;
- 3. Siena04.exe for changing the preset (non-estimated) internal effect parameters in the definitions of the effects;
  - and for copying the model definition from one project to another;
- 4. Siena05.exe for simulations with fixed parameter values;
- 5. Siena07.exe for parameter estimation;
- 6. Siena08.exe for multilevel analysis, which is a stand-alone program anyway, and not discussed here (see Section 14).

The mainly used programs are Siena01.exe for initiating the project, which only needs to be done once; and Siena07.exe for parameter estimation.

These programs can be used also independently of StOCNET. To run them, the project name *must* be given in a command line, e.g.

#### Siena01 bunt

if **bunt** is the name of the project, and there exists a **bunt**.in file. This **bunt** is called a *command line parameter*. There are the following four ways to specify a command with a command line parameter in Windows:

- 1. The command line can be given at the DOS prompt (in a Windows environment);
- 2. it can be given in the Windows "Run" command (for Windows 98 and higher);
- 3. a batch file with extension BAT can be created, e.g., with filename SIE1.BAT, containing the single line
  - Siena01 bunt

so that the program Siena01 will be executed when the batch file is called (it may be necessary to close the DOS window after the program is ready);

4. or a shortcut to the executable file can be made, where the project name is indicated in the "target" in the "properties" of the shortcut.

The third and fourth ways are the most straightforward in Windows. I (T.S.) prefer the use of the batch file. Note that a shortcut is made by opening Windows Explorer, giving a right mouse-click on the executable file, and then giving a left mouse-click on "Create Shortcut". When the shortcut has been made, the project name of the SIENA project must be added to the shortcut as follows: give a right mouse-click on the shortcut, then give a left mouse-click on "properties", and in the "Target" field add a space and the project name after the path-plus-filename of the executable file.

To run SIENA outside of StOCNET, the steps taken are the following.

- Write the basic information file *pname*.IN which describes the data files and variable names, according to Section 19.1. This file must be in ASCII (text) format. It is also possible to run SIENA once through StOCNET, which will produce the basic information file (recognizable from the extension name .in), instead of writing the basic information file oneself.
- 2. Make shortcuts or batch files as indicated above for each of the programs Siena01 and Siena07; and, if desired, also for Siena02, Siena04, and Siena05.
- 3. Give the session name (indicated here as *pname*) as the command line parameter in the shortcuts or batch files.
- 4. Click on the shortcut or batch file for Siena01. (This should be done only once to create the project, because calling Siena01 for an existing project will overwrite the output file!)
- 5. Open the file *pname*.MO in a text editor, edit it to obtain the desired model specification (see section 19.2.1), and save it as a text (ASCII, TXT) file. (Here it is more convenient to use a light-weight text editor such as Notepad, Textpad or Wordpad, which do not have the inbuilt preferences for formatting text that MS-Word has.)
- 6. Click on the shortcut or batch file for Siena07.

The last two steps – modifying the *pname*.MO file and running Siena07 – can be repeated as much as one likes.

Other possibilities are:

- 7. Get some basic descriptive statistics by running Siena02.
- 8. Simulate the model for fixed parameter values by running Siena05. The statistics to be simulated are indicated in the file *pname*.si, which can be modified to add to or delete from the list of simulated statistics.

The program Siena04 is of minor importance. Some of the model-defining statistics in the SIENA model contain numbers, or preset (non-changing)<sup>5</sup> internal effect parameters, which can be set at other values by the user. Examples are the value 3 in "out-degree up to 3" for the longitudinal version of SIENA, and the value 2 in the linear combination of k-out-stars in the non-longitudinal ("exponential random graph" or "p\*" model) version of SIENA. (These numbers should not be confused with the statistical parameters that are estimated by Siena07.) These values are represented in the pname.MO file by the last of the 6 values in the line for this effect. When these values are modified, the change must be put into effect by calling Siena04. This will update the names of the corresponding effects in the pname.MO file.

Another use of Siena04 is to call it with two parameters, e.g.,

#### Siena04 bunt bunthelp

if bunt is the name of the project, and there also exists a bunthelp.mo file. Then the model definition for the *pname* project, i.e., the effects included in the model as well their current parameter values and the preset parameters, and all the options, are taken from the bunthelp.mo file. This is useful, e.g., if one wishes to change a large number of projects to get an identical definition.

 $<sup>{}^{5}</sup>$ They are non-changing in the sense that they are not modified by the estimation algorithm, although they can be modified by the user.

# 17 Limitations and time use

The estimation algorithm, being based on iterative simulation, is time consuming. The time needed for the default way of running the program is approximately proportional to  $pn^aC$  where p is the number of parameters, n is the number of actors, the power a is some number between 1 and 2, and C is the number of tie (and behavior) variables that changed between time m and time m + 1, summed over m = 1 to M - 1. For data sets with 30 to 40 actors and something like 5 parameters, the estimation process takes a minute or so on a fast PC. The number of actors n should not give a problem up to, a few hundreds. For large data sets and models, the estimation process may take more minutes up to several hours.

Section 24 indicates the constants in the program that define limitations for the data sets used.

# 18 Changes compared to earlier versions

There are a few as yet undocumented options that will be disclosed when papers on these methods will be available; e.g., Bayesian estimation procedures and models for signed digraphs.

Version 3 is a major overhaul of the program. It contains

- 1. an entirely different representation of graphs and digraphs, employing edge lists instead of adjacency matrices to store them, which gives considerable speed increases for large networks; (input still is by adjacency matrices; the possibility of input through edge lists is one of the lans for the future);
- 2. a faster algorithm for ERGM estimation;
- 3. a different representation of components of the objective function ('effects'), which should make it easier to add new effects to the program;
- 4. ML estimation procedures for network evolution models as described in Snijders, Koskinen and Schweinberger (in preparation);
- 5. the possibility to use the program for a number of actors that is limited only by computing time and available memory, not by constraints in the software;
- 6. a more efficient procedure for calculating derivatives (and, hence, standard errors);
- 7. a corrected and expanded way of modeling longitudinal data of symmetric (i.e., non-directed) networks;
- 8. the possibility to use changing dyadic covariates;
- 9. the option of user-defined interactions;
- 10. the option to use Pajek format for reading network data files;
- 11. various other innovations.

The main innovations in version 2.2 are

- more efficient procedure for estimating standard errors, based on unbiased derivatives estimators (Schweinberger and Snijders, 2007);
- 2. extension of model specification possibilities, especially for dependent behavior variables.

The main changes in version 2.1 compared to version 1.98 are

- 1. extension by allowing dependent actor variables, inclusion of effects related to group position and an update of the similarity effects, implementing methods in Steglich, Snijders and Pearson (2007), see Section 5.5;
- the addition of Neyman-Rao goodness-of-fit tests according to Schweinberger (2005), see Section 9;
- 3. the possibility to analyse dynamics of non-directed networks, according to Snijders (2007);
- 4. statistical Monte Carlo studies, see Section 23;
- 5. extension of the specifications of the exponential random graph (" $p^*$ ") model in line with Snijders, Pattison, Robins, and Handcock (2006), and slight modifications of the algorithm for this case to increase efficiency;
- 6. missing data handling is extended to covariates and dependent actor variables;
- 7. addition of the program Siena08 for the multilevel analysis of multiple network evolution processes, implementing methods in Snijders and Baerveldt (2003);
- 8. analysing the dynamics of non-directed networks (not yet documented);
- 9. possibility to specify structural (i.e., non-random) zeros and structural ones in the adjacency matrices, see Section 5.1.1;
- a new format for the project definition file *pname*.IN and the replacement of the internal project files *pname*.mo1 through *pname*.mo4 by files *pname*.MO for model definition and *pname*.SI for simulation directives (old project files still can be read);
- 11. correction of various errors.

The main changes in version 1.98 compared to version 1.95 are

- 1. the advanced option modeltype is added, implementing methods in Snijders (2003);
- 2. maximum number of actors increased to 500.

The main changes in version 1.95 compared to version 1.90 are

- 1. for the exponential random graph model some extra simulation options were added, and inversion steps were added to the algorithm;
- 2. some effects (3-star and 4-star counts) added to the exponential random graph model;
- 3. for changing covariates, the global rather than the periodwise mean is subtracted;
- 4. the program Siena02 for data description was added.

The main changes in version 1.90 compared to version 1.70 are

- 1. possibility to use more than two observation moments;
- 2. inclusion of the exponential random graph (" $p^*$ ") model, corresponding to one observation moment;
- 3. possibility to have changes of composition of the network (actors leaving and/or entering);

- 4. changing actor covariates;
- 5. arbitrary codes allowed for missing data (instead of the automatic use of 6 and 9 as codes for missing data, the user now has to supply these codes explicitly);
- 6. small improvements in the user interface.

# Part III Programmer's manual

The programmer's manual will not be important for most users. It is intended for those who want to know what all the *pname*.\* files are all about, and for those who wish to have a look inside the source code.

The SIENA program consists of a basic computation part programmed by the authors of this manual in Turbo Pascal and Delphi; associated with the StOCNET windows shell, programmed by Peter Boer and Rob de Negro in Delphi, with first Evelien Zeggelink, then Mark Huisman, and later Christian Steglich as the project leader.

# **19** SIENA files

Internally the following files are used. Recall that *pname* is the name of the project, which the user can choose at will. The extension file names cannot be changed.

## **19.1** Basic information file

The basic information file is called *pname*.IN, and contains the definition of the numbers of cases and variables, the names of the files in which data are initially stored and their codes (including missing data identification), and the names of the variables. The requirements for the input data files are given in section 5. The basic information file is written by StOCNET when the data are defined, and can also be written by any text editor that can produce ASCII (TXT) files; note that it *must* have extension name .IN. It is read by Siena01.exe. This file contains up to eight sections, each starting with a line containing the section number (@1 through @8). These sections must have the following contents:

- 1. Section @1 contains basic information about type and amount of data. This section is required for all SIENA projects. It must contain nine rows, each starting with an integer number:
  - number of observations of the network (1 for exponential random graph modeling, 2 or more for modeling network evolution over time; denoted by M);
  - number of actors (denoted further by n);
  - number of dependent network variables (must be equal to one in the current version of SIENA ). The network data are further specified in Section @2;
  - number of dependent actor variables. Possible dependent actor variables are further specified in Section @3;
  - number of files with constant actor covariates (further specified in Section @4);
  - number of files with changing actor variables (further specified in Section @5);
  - number of constant dyadic covariates (further specified in Section @6);
  - number of exogenous changing dyadic covariates (from version 2.4 of SIENA onward). Section @7 contains the specification of this type of covariate data;
  - indicator of file with times of composition change (0 means no change of network composition; 1-4 mean composition change; 1 is the default treatment of composition change, 2-3 are alternatives using missing data, 4 effectively transforms the composition change information to structural zeros). See Section 5.7. If there is composition change, the file must be further specified in Section **@8**.

- 2. Section @2 contains information about the network data, as follows:
  - for each of the M network observations, the following three lines:
    - a line with the name of the data file;
    - a line with the codes that are regarded as a present arc in the digraph;
    - a line with the codes that are regarded as missing data;
  - a line with the name of the network variable.

All codes should be in the range from 0 to 9.

- 3. Section @3 contains information about the dependent actor variables, in this format:
  - for each dependent actor variable, the following three lines:
    - a line with the name of the data file;
    - a line with the code that is regarded as missing data;
    - a line with the name of the variable.
- 4. Section **@4** contains information about constant actor covariates, as follows:
  - For each file containing such covariates, there must be the following lines:
    - a line with the name of the data file;
    - a line with the number of variables in this file;
    - for each variable:
      - \* a line with the code for missing data;
      - $\ast\,$  a line with the name of the variable.

Note that this format differs from the one used in Sections **@3** and **@5** through **@7** because here, data files can potentially contain more than one covariate, while in these other sections, only one variable is given per file.

- 5. Section **@5** contains information about changing actor covariates, in the same format as the dependent actor variables are given:
  - for each changing actor covariate, the following three lines:
    - a line with the name of the data file;
    - a line with the code that is regarded as missing data.
    - a line with the name of the covariate.
- 6. Section @6 contains information about constant dyadic covariates, in the same format:
  - for each constant dyadic covariate, the following three lines:
    - a line with the name of the data file;
    - a line with the code that is regarded as missing data.
    - a line with the name of the covariate.
- 7. Section **@7** refers to changing dyadic covariates, with for each changing dyadic covariate the following lines:
  - for each observation moment 1 to M 1 (therefore not for the last!), the following two lines:

- a line with the name of the data file;

- a line with the code that is regarded as missing data.

followed by

- a line with the name of the covariate.

E.g., if there are 2 changing dyadic covariates and M = 4 observation moments, this requires a total of  $2 \times (M - 1) = 6$  data files.

8. Section @8 contains information about network composition change, namely:

• a line with the name of the file containing times of network composition change.

Whenever a certain type of data is not present, leave out the entire section in the file *pname*.IN corresponding to this type. For example, if you do not have any files containing changing actor covariates, leave out Section **@5**. If there are problems in reading the basic input file, try deleting superfluous blanks and/or empty rows. See to it that the basic input file is an ASCII text file, with numbers separated by blanks, lines separated by hard returns.

The variable names given in the input file will be used in the output files. If no names are provided and SIENA is run in the StOCNET environment, SIENA uses the default variable names generated by StOCNET in the StOCNET Data menu. This is not recommendable because it can lead to identical names for different variables. If no names are provided and SIENA is run outside of the StOCNET environment, SIENA uses its own default variable names, and this problem does not occur.

An example for the basic input file is the following file bunt.IN. This refers to data files that are included with the program, collected by Gerhard van de Bunt. This example, which contains a file with three covariates, is used in van de Bunt (1999) and in van de Bunt, van Duijn, and Snijders (1999).

```
@1[general information about SIENA project \langle bunt \rangle:]
2 [number of waves]
32 [number of actors]
1 [number of dependent network variables]
0 [number of dependent actor variables]
1 [number of files with constant actor covariates]
0 [number of exogenous changing actor covariates]
0 [number of constant dyadic covariates]
0 [number of exogenous changing dyadic covariates]
0 [indicator for file with composition change directives]
@2[network files in temporal order; names follow:]
vrnd32t2.dat
1 2 3 [code for tie]
6 9 [code for missing]
vrnd32t4.dat
1 2 3 [code for tie]
6 9 [code for missing]
friendship
@4[files with constant actor covariates:]
vars.dat
3 [number of covariates in this file; names follow:]
99 [code for missing]
```

```
gender
99 [code for missing]
program
99 [code for missing]
smoke
```

The basic data input is carried out by executing Siena01.exe. This programs reads the basic information file. Some preliminary output is given in the files *pname.out* and *pname.log*.

# **19.2** Definition files

The program writes and reads for internal use the following two definition files:

- pname.MO defines model specification and options for model estimation,
- pname.SI defines statistics and number of runs for simulation.

These definition files are read in a format where certain lines are skipped entirely and other lines are skipped after reading a certain number. These skipped parts are between square brackets [...]. Their purpose is to give information to the human reader about the meaning of the lines. Note, however, that SIENA does not check for the brackets, but skips information on the basis of line numbers and reading numerical information.

The three files *pname*.IN, *pname*.MO and *pname*.SI must be compatible (as they contain some overlapping information) for successfully running SIENA.

#### **19.2.1** Model specification through the MO file

The model specification options were already discussed in Section 12. These can be changed outside the StOCNET shell, by changing the *pname*.MO file by a text editor. In this way you may also used advanced SIENA options which are not yet available through StOCNET; whether such options exist, will depend on the versions of SIENA and StOCNET.

When looking at the *pname*.MO file, you can see that this file by and large contains the same information as the screen opening up in StOCNET when clicking the Model specification button. More precisely, the *pname*.MO file consists of several sections, marked by @-symbols, as follows:

- Section **@1** contains general information about the project, such as data format, numbers and names of variables. The information given in this section must be compatible to the information provided in the file *pname*.IN.
- Sections starting **@2.x** contain the model specification for dependent network variables (indexed by x). There are two subsections:
  - Subsections @2.x.1 contain the specification of the network rate function, which looks like this:

```
@2.1.1 [rate function effects for dependent network variable (#1):]
11 [number of these effects :]
[each effect is given in two rows]
[first row contains label of the effect,]
[second row contains flags for inclusion, fixing and testing,]
[the starting value and potential extra parameters for effect calculation.]
basic network rate parameter
0 0 0 5.851107 0
outdegrees effect on rate
```

0 0 0 0.000000 0 [...]

The section must contain first a row with the number of effects, further down for each of these effects two rows, one containing the effect name, the other containing a sequence of numbers. These have the following meaning:

- \* a 0/1 entry denoting whether the effect is included (1) or excluded (0);
- $\ast$  a 0/1 entry denoting whether the effect is fixed (1) or not fixed (0) during the estimation process;
- \* a 0/1 entry indicating whether a fixed effect shall be included (1) or excluded (0) in goodness-of-fit calculations (see Section 9);
- \* the starting value of the parameter for the estimation procedure;
- \* an internal effect parameter (for modeling the constants c in the mathematical definition of the effects, see Section 15.1.1.
- Subsections @2.x.2 contain the specification of the network decision rule (including the evaluation endowment functions), and which has the following shape:

```
02.1.2 [objective function effects for dependent network variable \langle \#1 
angle:]
46 [number of such effects :]
[first row contains label of the effect,]
[next three rows: functions in which effect can be included:]
[row 1: evaluation function, row 2: endowment function, row 3: reinforcement function,]
[columns correspond to: (a) inclusion, (b) random effects, (c) fixing, (d) testing,]
[(e) starting value, (f) potential extra parameters for effect calculation.]
density (outdegree)
1 0 0 0 -1.143698 0
0 0 0 0
          0.000000 0
0 0 0 0
          0.000000 0
reciprocity
1 0 0 0
          1.467572 0
0 0 0 0
          0.000000 0
0 0 0 0
          0.000000 0
[...]
```

The section must contain first a row with the number of effects, further down for each of these effects four rows, one containing the effect name, the other containing sequences of numbers. These have the following meaning:

- \* a 0/1 entry denoting whether the effect is included;
- \* a 0/1 entry denoting whether a corresponding random effect effect is included;
- \* a 0/1 entry denoting whether the effect is fixed;
- \* a 0/1 entry indicating whether a fixed effect is included in goodness-of-fit calculations;
- \* the starting value of the parameter for the estimation procedure;
- $\ast\,$  an internal effect parameter.

Of the three rows that follow this pattern, the first row corresponds to the evaluation function, the second row corresponds to the endowment function, and the third row is reserved for future model extensions that allow the modeling of adaptive learning behavior.

In the current version of SIENA, but one dependent network variable can be analyzed at a time, so there will only be sections @2.1.y.

- Sections starting @3.x contain the model specification for dependent action variables (again indexed by x). as for the network variables, there are two subsections:
  - Subsections @3.x.1 contain the specification of the behavioral rate function,
  - Subsections @3.x.2 contain the specification of the behavioral decision rule (including the evaluation endowment functions).

These subsections have the same format as the corresponding subsections for the network evolution model.

- The final Section @4 contains specifications of various estimation options. Most of these are accessible in StOCNET, e.g., through the model options mentioned in Section 12. The consecutive options are the following. Some of these may not be accessible from StOCNET, and be possible only when working with SIENA outside of StOCNET.
  - 1. Estimation method:
    - code 0 for unconditional estimation;
    - code 1 or code 21 for estimation conditional on observed changes in the network;
    - codes 21 + k for estimation conditional on observed changes on the dependent action variable k;
    - code 2 for maximum likelihood estimation;
    - code 3 for Bayesian estimation.

For exponential random graphs, another available option is to include incidental vertex parameters:

- code 10 for unconditional estimation with incidental vertex parameters;
- $-\,$  code 11 for conditional estimation with incidental vertex parameters.
- 2. For estimation method 2 and 3, an indicator of whether initial estimates should be computed:
  - code 0: no;
  - code 1: yes: unconditional estimates are computed and used as initial estimates.
- 3. For Bayesian estimation: Metropolis Hastings (M-H) algorithm for the fixed effects in the evaluation and endowment functions:
  - code 1: random walk M-H;
  - code 2: independence sampler;
  - code 3: first-order autoregressive M-H.
- 4. For Bayesian estimation: scale factor of the proposal distribution:
  - code 0: scale factor is calibrated during burn-in;
  - code c>0 (positive, real number): c is used as scale factor.
- 5. For models with actor-dependent random coefficients: Metropolis Hastings (M-H) algorithm for the random effects:
  - code 1: random walk M-H;
  - code 2: independence sampler;
  - code 3: first-order autoregressive M-H.
- 6. For models with actor-dependent random coefficients: scale factor of the proposal distribution:

- code 0: scale factor is calibrated during burn-in;
- $\operatorname{code} c > 0$  (positive, real number): c is used as scale factor.
- 7. A code for the type of model.

For longitudinal data this is the Model Code described in the section on model options. For exponential random graph models, this code defines the steps used in the oneobservation case for simulating a random (di)graph (see also the description above):

- code 1: Gibbs steps for single tie variables ;
- code 2: Gibbs steps for dyads;
- code 3: Gibbs steps for triplets;
- code 4: Metropolis Hastings steps for single tie variables, version A; this is the default for directed networks;
- code 5: Metropolis Hastings steps for single tie variables, version B;
- code 6: Metropolis Hastings steps for single tie variables, version A, for non-directed graphs;
- code 7: Metropolis Hastings steps for single tie variables for antisymmetric graphs ('tournaments');
- code 8: Metropolis Hastings steps keeping the in-degrees and out-degrees fixed.

To each of these values, the number 10 may be added (so the values become 11-18). In that case a *continuous* chain is used: i.e., the last generated graph is used as the initial value in the MCMC sequence for simulating the next graph. Otherwise (i.e., for the values 1-8), the initial value is an independently generated random graph. For practical purposes, one should *always* use a continuous chain – i.e., only use codes 11-18.

- 8. The number of subphases in phase 2 of the estimation algorithm (advice: 4).
- 9. The number of phase 3 iterations for the estimation algorithm (advice: 1000).
- 10. A number r proportional to the number of steps used for generating one graph in the one-observation case. The number of steps is  $r n^2/2d(1-d)$  where n is the number of actors and d is the observed density of the graph; if the observed density is less than .05 or more than .95, the value d = .05 is used.
- 11. The initial value of the gain parameter in the estimation algorithm (advice: 0.2 for longitudinal MoM, smaller for ERGM).
- 12. An indicator for the initial value used in the estimation algorithm:
  - code 0: current value as specified in the preceding sections,
  - code 1: standardized starting value, meaning that a good starting value is chosen for the density effect and in the one-observation (exponential random graph) case also for the reciprocity effect. For still other configurations, also starting values for the rate parameters and the tendency effect for dependent action variables are internally determined. All other effects then have a 0.0 starting value.
- 13. An indicator allowing for tests of temporal (between-period) homogeneity of parameters:
  - code 0: no test,
  - code m > 0: test for period m
- 14. An indicator allowing for tests of actor homogeneity of parameters:
  - code 0: no test,
  - code 1: test of the density (out-degree) effect,
  - code 2: test of the reciprocity effect.

- 15. A space-separated list giving the row numbers of the actors for such an actor homogeneity test.
- 16. A value determining the use of random numbers during the estimation process:
  - code 0: randomize seed,
  - $\operatorname{code} c > 0$  (positive integer): c is used as random seed.
- 17. An indicator for the method of estimating derivatives of expected values with respect to parameters:
  - code 0: Finite Differences (the older method of Snijders 2001);
  - code 1: Score Function method I, see Schweinberger and Snijders (2007); (not available for all models);

If you change anything in the *pname*.MO file, you must run Siena04.exe to let SIENA check (and if necessary: censor or re-define) the model specification.

## 19.2.2 Specification of simulations through the SI file

For running simulations, SIENA needs to be told which statistics it should simulate. This is done by manipulating the *pname*.SI file by an ASCII text editor before running the simulations. This file consists of two sections, again marked by **@**-symbols, as follows:

• Section @1 contains the list of possible statistics that can be simulated. It looks as follows:

```
@1 [statistics that can be generated]
60 [number of these statistics :]
[each statistic is given in two rows]
[first row contains label of the statistic,]
[second row contains flag for inclusion]
Amount of change
1
Number of ties
1
Number of reciprocated ties
0
[...]
```

As can be seen, for each effect, there are two rows:

- a row containing the statistic's name;
- a row contining an indicator whether the statistic shall be simulated.
- Section **@**2 contains options for simulation. Currently, there is but one option:
  - the default number of simulations for straight simulation (advice: 1000).

The file *pname*.SI by and large contains the same information as the screen opening up in StOCNET when clicking the Statistics specification button (which is possible as soon as Simulation is selected as the Run model).

If the number of simulations is specified as 1, then one complete data set is stored and written both as adjacency matrices and in Pajek format. For larger number of simulations, only aggregate information (means, standard deviations and covariance matrices of statistics) is reported.

## 19.3 Data files

After the initial project definition the original data files are not used any more, but the project data files are used. These are the following.

- pname.d01 Network data file time 1.
- pname.d02, etc. Network data file time 2, etc.
- pname.m01, etc. Network missings file time 1, etc.
- pname.s01, etc. Fixed part of network structure period 1, etc.
- pname.dav Data file constant actor-dependent variables (centered).
- *pname*.miv Missings file actor-dependent variables.
- *pname.dac* Data file with changing actor-dependent variables.
- *pname*.mac Missings file changing actor-dependent variables.
- pname.z01, etc. Data files non-changing dyadic covariates.
- pname.c01, etc. Data files changing dyadic covariates.
- pname.n01, etc. Missings files dyadic covariates.
- *pname*.dex Data file times of composition change.

The user does not need to care about these data files (but should not delete them either).

## **19.4** Output files

The output for the user goes to *pname.out*. Extra output is written to *pname.log*, which in the first place is a log file of what the program did. The estimation procedure also writes a file *pname.cck*, containing a more detailed report of the estimation algorithm. The latter two files are for diagnostic purposes only. The *pname.cck* file is overwritten with each new estimation procedure.

# 20 Units and executable files

The basic computational parts of SIENA are contained in the following units. First, there are six basic units.

- 1. S\_DAT contains the basic data structures together with some basics of the model definition; it uses unit EIGHT for some lower-level data storage, and also DIGRAPH for definition of network data and model ingredients.
- 2. S\_CONSTANTS contains definitions of maximal values of numbers of actors, numbers of variables, etc.
- 3. S\_BASE contains the basic model definition and the basic simulation procedures.
- 4. S\_SIM is the unit for carrying out straight simulations.
- 5. S\_EST contains the procedure for estimation, using S\_TEST for goodness-of-fit tests.
- 6. S\_DESC contains procedures for data description.

Then there are some special-purpose units, of which the most important ones are:

- S\_Start contains the procedure ReadWriteData to start a project by reading the *pname*.IN file and the initial data files, and writing the internally used files. It uses only S\_DAT. Procedure ReadWriteData from S\_START must be followed always by procedure BeforeFirst-ModelDefinition from S\_BASE.
- 8. S\_ML contains procedures for maximum likelihood estimation.
- 9. S\_RE contains procedures for random effect models (as yet undocumented).
- 10. S\_signed contains procedures for models for signed digraphs (as yet undocumented).

Further there are five units containing specific kinds of utilities. Their names do not start with S<sub>-</sub> because they do not use the other units (except perhaps S<sub>-</sub>CONSTANTS).

- 11. EIGHT is a unit for storing the data. Its name was chosen for historical reasons (in SIENA version 1 a byte was used to store eight booleans). This unit also connects the procedures in DIGRAPH to those in  $S_BASE$ ; this is necessary because DIGRAPH is defined independent of the data.
- 12. DIGRAPH is a unit for defining network data types and effect data types.
- 13. CHAINS defines data types for use in data augmentation procedures employed in maximum likelihood estimation.
- 14. SLIB is a library of various computational and input/output utilities.
- 15. RANGEN is a library for generation of random variables. It uses the URNS suite for random numbers generation.

#### 20.1 Executable files

The basic data input is carried out by executing Siena01.exe. This program executes ReadWriteData and BeforeFirstModelDefinition, thereby reading the basic information file.

Data description is carried out by Siena02.exe which executes Describe.

In the StOCNET operation the model specification is carried out by StOCNET changing the .MO file and then running Siena04.exe.

Siena04.exe is used for checking admissibility of a model specification, and for keeping consistency between the different session files. It reads and updates the .MO file, and also writes the corresponding .SI file. If you change *pname*.MO by an text editor outside of StOCNET, it is advisable to run Siena04.exe before proceeding.

Siena04.exe can also be used to copy a model definition from one .mo file to another.

The simulation is carried out by Siena05.exe which executes Simulate.

The estimation is carried out by executing Siena07.exe which executes Estimate.

The multilevel combination of estimation runs for several data sets according to the same basic model specification is done by Siena08.exe (see Section 14). This program is independent of the other programs, and reads the output files produced by Siena07.exe.

# 21 Starting to look at the source code

If you wish to start with understanding the structure of the source code, it may be helpful to take the following tour of some essential ingredients.

- 1. Unit DIGRAPH defines network data types:
  - (a) TValuedDigraph, storing a valued directed graph, used as the basic network data structure;
  - (b) **TDigraph**, storing a valued directed graph, used for storing indicators of missing data and structurally determined data.

Both data types store valued digraphs as doubly linked arc lists, allowing to search arcs both from the sender node and from the receiver node, and using less memory space than the adjacency matrices used in versions 1 and 2 of SIENA.

In addition, unit DIGRAPH defines types that allow to define the model components:

- (a) **TEffect**, defining a component in the objective function for the longitudinal case, and a component in the log-odds for the non-longitudinal (ERGM) case;
- (b) TEffects, defining an array of TEffect and many operations on such arrays.
- 2. Unit EIGHT contains the fundamental data structures, using the types defined in DIGRAPH.(i) Functions starting with y represent dependent variables.
  - (a) yn is the adjacency matrix of the dependent network variable (current value): yn(i,j) is the tie variable from i to j;
  - (b) ya is the vector of dependent individual variables (current value):ya(i,h) is the h'th individual variable for actor i;
  - (c) ynm gives the adjacency matrices of the dependent network variable (all values): ynm(i,j,m) is the tie variable from i to j at observation moment m.
- (d) mis is the indicator matrix for missing values:
  - mis(i,j,m) = 0 if ynm(i,j,m) is an observed value and 1 if it is missing.

Note that yn and ya change dynamically during the simulation process; the variables containing the data from which ynm and mis are calculated, are read in the input phase and then are left unaltered.

For the sake of flexibility, the variables in the list above are implemented as functions, not as arrays.

(ii) Variables starting with a single z represent individual variables.

- (a) Array z contains the constant actor variables; the number of such variables is nz.
- (b) Array zc contains the changing actor variables; their number is nzc.
- (c) The number of dependent actor variables is nza; these variables are the first nza among the nzc changing actor variables.
- (iii) Variables starting with zz represent individual variables.
- (a) Array zz contains the constant dyadic covariates; their number is nzz.
- (b) Array zzc contains the changing dyadic covariates; their number is nzzc.

Further, unit EIGHT contains procedures linking the methods in DIGRAPH to the data structures available in EIGHT.

- 3. Unit S\_DAT contains the defining ingredients for the network models. The intended demarcation between S\_DAT and S\_BASE, is that the former contains the model definition and the latter the operation of the model dynamics. This is not realized completely because of various conflicting constraints.
- 4. Unit S\_BASE contains the simulation procedures. The main procedures in this unit are the following.
  - (a) Function SimStats generates the required statistics and is called by procedure Simulate in S\_SIM and (indirectly, through FRAN in unit S\_ML) by Estimate in S\_EST. The procedure SimStats first simulates the network and behavior by the procedure Runepoch and then calculates statistics by the procedures NetworkStatistics and Action-Statistics; the values of these statistics are output arguments of SimStats.
  - (b) Procedures NetworkStatistics and ActionStatistics calculate the statistics from the generated network (or adjacency matrix) and behavior variables, and is called by SimStats.
  - (c) Procedure Runepoch generates a stochastic network and action variables for given parameter values and a given initial situation by simulating the dynamic model for one period between two observations. This procedure is called by procedure SimStats. If there are M observations ( $M \ge 2$ ), Runepoch is called M 1 times.
  - (d) Procedure Runstep makes one stochastic step according to the actor-oriented evolution model, i.e., it either changes one entry (i, j) of the adjacency matrix to be changed, or it changes the value of one action variable for one. The time variable time is also increased by an amount depending stochastically on the rate functions. This procedure is called by procedure Runepoch.
  - (e) Procedure ChangeTie is called at the end of procedure RunStep if a change in the network is made, and carries out the required change of the network and the associated updates of various statistics.

- (f) Procedure ChangeBehavior is called at the end of procedure RunStep if a change in behavior is made, and carries out the required change of the dependent action variable and the associated updates of various statistics.
- (g) Function NetworkLambda, which is the rate function for the network changes made by each actor, and is used in procedure Runstep. For Model Type 2, it uses functions ξ and ν.
- (h) Function ActionLambda, which is the rate function for the behavior changes made by each actor, and is used in procedure Runstep.
- (i) Procedure ChoiceProbabilities, which defines the probabilities with which a given actor i chooses to change the tie variable to actor j, for each of j = 1, ..., n.
- (j) Function contr\_fa, which defines the contribution  $s_{ih}^Z(x)$  of each given effect h to the evaluation function for the behavior, and is used in procedure Runstep.
- (k) Function contr\_ga, which defines the contribution of each given effect to the endowment function for the behavior, and is used in procedure Runstep.
- 5. Unit S\_ML contains procedures for maximum likelihood estimation, including the 'chain' data structure for the augmented data. The main function computed here is Scores, the score function for the augmented data. This unit includes the function FRAN ('*Function Random*'), which is the basic function in the equation solved by procedure POLRUP in unit S\_EST. Depending on the type of estimation requested, FRAN calls function SimStats in unit S\_BASE, or function Scores in unit S\_ML.
- 6. In unit S\_EST, the Robbins-Monro algorithm is contained in the procedure POLRUP (for Polyak-Ruppert, see Snijders, 2001). When parameters are to be tested by Neyman-Rao tests, S\_EST calls the procedure TestStatistic in the unit S\_TEST.
- 7. Unit S\_SIM is used for simulations, and calls the function SimStats in S\_BASE.
- 8. The unit S\_ERGM\_EST and the file S\_ERGM.PAS which is an include file used in S\_BASE contain procedures used only for the ERGM (non-longitudinal / 1 observation) case.

#### 21.1 Sketch of the simulation algorithm

The simulation algorithm, used in the Method of Moments estimation as well as for straight simulation, is explained here using the notation of Snijders (2005) and Snijders, Steglich and Schweinberger (2007). It is formulated here only for the case of two observation moments  $t_1$ and  $t_2$ , and no dependent behavior variables. The distinction between evaluation function and endowment function is obscured here; both are jointly referred to as objective function.

The following notation is used; in typewriter font are the symbols in the source code.

number of actors)	n
dependent network (digraph)	matyn
indicator variable of the from $i$ to $j$ in digraph $X$	yn(i,j)
digraph x in which $x_{ij}$ is replaced by $1 - x_{ij}$ ,	
and the other elements of $x$ remain unchanged	
rate function for network change	networklambda
number of terms in the objective function	
parameters in the objective function	<pre>alpa_f[k], alpa_g[k], theta[k]</pre>
components of the objective function	
exponential distribution with parameter $\lambda$	
	number of actors) dependent network (digraph) indicator variable of tie from <i>i</i> to <i>j</i> in digraph <i>X</i> digraph <i>x</i> in which $x_{ij}$ is replaced by $1 - x_{ij}$ , and the other elements of <i>x</i> remain unchanged rate function for network change number of terms in the objective function parameters in the objective function components of the objective function exponential distribution with parameter $\lambda$

The schematic outline of the algorithm is as follows. In typewriter font the procedures and variables are mentioned that are most important for this step in the algorithm; the outline is followed by an indication of where these procedures and variables are defined. The algorithm as a whole is implemented in the procedure SimStats in unit S\_Base.

Replacing a variable by the sign + means summation over this index.

"generate  $\sim$ " means to generate a random variable with the indicated distribution.

"choose random  $\sim$ " means to generate a discrete random variable with probabilities *proportional* to the indicated values.

1.	Initial conditions:	network $X = x(t_1);$	Initialise_y, matyn
		time $t = 0$ .	time
2.	Repeat		RunEpoch, RunStep
		$\begin{bmatrix} 3. & \text{generate } \Delta t \sim \mathcal{E}(\lambda_+); \end{bmatrix}$	rs_tau
		4. choose random $i \sim \lambda_i$ ;	NewRanp, i
		5. for all $k = 1,, K$ and $j = 1,, n$ ,	UtilityComponents
		calculate $s_{ik}(x(i \rightsquigarrow j));$	or
		see $(4), (5)$ in Section 22.1;	Contrib_n, Contrib_fn
		6. for all $j = 1, \ldots, n$ , calculate	ChoiceProbabilities
		$f_i(x(i \rightsquigarrow j)) = \sum_{k=1}^K \beta_k s_{ik}(x(i \rightsquigarrow j));$	
		7. choose random $\overline{j} \sim \exp(f_i(x(i \rightsquigarrow j));)$	NewRanp, j
		8. Set $t := t + \Delta t$ ;	
		9. Set $x_{ij} := 1 - x_{ij};$	ChangeTie
	until		
		[ 10. a. if estimation method is conditional: ]	
		$\sum_{i,j}  x_{ij} - x_{ij}(t_1) $	Distance
		$= \sum_{i,j}  x_{ij}(t_2) - x_{ij}(t_1) ;$	ObservedNetworkDistance
		b. if est. method is unconditional:	
		$t \ge 1;$	time
11.	Calculate	statistics $u(X)$ .	NetworkStatistics

The procedures and variables in this outline are defined in the following data types, procedures, and units. The list is in the order of their occurrence in the outline.

Defined in type or procedure	in unit
ModelSpecification	S_Base Eight
ModelSpecification	$S\_Base$
ModelSpecification	$S\_Base$
ModelSpecification	$S\_Base$
RunStep	$S\_Base$
	RanGen
RunStep	$S\_Base$
TEffects	Digraph
	Defined in type or procedure ModelSpecification ModelSpecification ModelSpecification RunStep RunStep TEffects

TEffects	Digraph
TEffects	Digraph
ModelSpecification	$S_Base$
	RanGen
RunStep	$S\_Base$
ModelSpecification	$S_Base$
ModelSpecification	$S_Base$
DataSpecification	$S_Dat$
ModelSpecification	$S_Base$
ModelSpecification	$S_Base$
	TEffects TEffects ModelSpecification RunStep ModelSpecification DataSpecification ModelSpecification ModelSpecification

# 22 Parameters and effects

In the source code there are two kinds of parameters: alpha ( $\alpha$ ; in the source code: alpa\_l for the rate function, alpa\_f for the evaluation function, alpa\_g for the endowment function) and theta ( $\theta$ ). The alpha parameters are used in the stochastic model, and each alpha parameter corresponds to one effect, independently of whether this effect is included in the current model specification. Their values are stored in the *pname*.MO file, which also indicates (by 0-1 codes) whether these variables are included in the model, and whether they are fixed at their current value in the estimation process. The theta parameters are the statistical parameters that correspond to the effects in the current model specification. Thus, the theta parameters are those elements of the alpha parameters that are currently active.

Procedures SetAlpha and SetTheta in unit S\_BASE define the correspondence between the alpha and theta parameters. This correspondence sets the order of the theta parameters as follows:

1. First the parameters for network dynamics:

- (a) Rate parameters;
- (b) evaluation function parameters;
- (c) endowment function parameters;
- 2. Then the parameters for behavior dynamics (if there is a dependent behavior variable):
  - (a) Rate parameters;
  - (b) evaluation function parameters;
  - (c) endowment function parameters.

If there is more than one dependent behavior variable, then within these three categories the parameters are ordered according to the dependent variable whose dynamics they influence.

The distinction between the theta and alpha parameters in principle also allows linear (or other) restrictions between the alphas. In the present version, the possibility of such restrictions is not implemented, but this possibility may be elaborated in a later version.

The effects for the evolution model, distinguishing between effects for the network dynamics and effects for the behavior dynamics, are defined in several procedures, which of course must correspond. To each effect corresponds one statistic used for estimating the parameter for this effect. This is spelled out in Sections 22.1 and 22.2.

Originally unit S\_DAT contained the data definition and S\_BASE the model definition. This distinction has been a bit blurred in version 3.0 by the construction work on the program, but the plan is to reorganize the units again to make the distinction between the units more helpful.

In unit S\_DAT, the numbers of the various types of effects are defined:

- 1. NetworkEffects\_f, the number of effects in the evaluation function for the network dynamics;
- 2. NetworkEffects\_g, the number of effects in the endowment function for the network dynamics;
- 3. NetworkEffects\_I, the number of effects in the rate function for the network dynamics;
- 4. ActionEffects\_f, the number of effects in the evaluation function for the behavior dynamics;
- 5. ActionEffects\_g, the number of effects in the endowment function for the behavior dynamics;
- 6. ActionEffects\_I, the number of effects in the rate function for the behavior dynamics;

- NetworkFunctions, the number of statistics that can be calculated and which can be used if there are no dependent behavior variables; this number is equal to NetworkEffects\_f + NetworkEffects\_g + NetworkEffects\_l.
- 8. ActionFunctions, the number of statistics that can be used additionally to the statistics mentioned before, in case that there are dependent behavior variables.

In units  $S_DAT$  and  $S_BASE$ , the definitions of the effects and statistics are given. These use the basic data and effect structures defined in unit DIGRAPH (which has a name not starting with  $S_b$  because it is itself independent of the other SIENA units), which is described below.

- A. For the rate function:
  - 1. Procedure DefineModel\_Inames defines the names of the effects and the index numbers of the corresponding statistics;
  - 2. function NetworkLambda defines the network change rate function;
  - procedure Transform\_I calculates some variables for more efficient calculations in NetworkLambda;
  - 4. for Model Type 2, the rate function for network change also depends on the functions  $\xi$  and  $\nu$ ;
  - 5. function ActionLambda defines the behavior change rate function.
- B. For the evaluation and endowment functions of the network dynamics:
  - 1. Unit S\_BASE has an include file S\_EFFECTS which contains the basic definitions of the available effects, i.e., components of the evaluation and endowment functions. The effects are defined in the procedure DefineNetworkEffects. The effects in the evaluation and endowment functions are defined conjointly (see below).
- C. For the evaluation and endowment functions of the behavior dynamics:
  - 1. Procedure DefineModel\_fnames and DefineModel\_gnames define the names of the effects in the evaluation and endowment functions, respectively;
  - 2. function Contr\_fa and Contr\_ga give the contribution of each effect to the evaluation and endowment functions, respectively.
- D. For the statistics:
  - 1. Procedure DefineFunctionnames defines the names of the statistics; for the network statistics, this uses the names defined in procedure DefineNetworkEffects in file S\_EFFECTS;
  - 2. procedures NetworkStatistics and ActionStatistics calculate the statistics.

### 22.1 Effect definition

Unit DIGRAPH is used not only to define data types but also for defining effects:

- 1. type **TEffect** defines an effect, which is a term in the evaluation function, and possibly a corresponding term in the endowment function;
- 2. type **TEffects** defines an array of effects.

The effects are defined by means of the arrays

- 1. ContributionWeight, giving weights for the contribution of this effect to the evaluation function;
- 2. ConfigWeight, giving weights for the definition of the corresponding statistic;

and the functions

- 3. flijc, defining a contribution of tie  $x_{ij} = 1$  to the evaluation function; this function has arguments (dg, i, j, m, t, par), where dg is the valued digraph, (i, j) indicates the tie variable, m defines the period (observation number), t is shorthand for the current value of tie variable  $x_{ij}$  of digraph dg (avoiding its unnecessary calculation), and par is a fixed parameter incorporated in the effect;
- 4. flij, defining a contribution of tie  $x_{ij} = 1$  to the statistic, with arguments (dg, i, j, m, par);
- 5. fli, defining an extra contribution of actor i to the statistic, with arguments (dg, i, m, par).

The parameter par is a parameter that can be used to modify the definition of the effect (e.g., par could be the order k of a k-star), and which is given in the MO file as the last element of each line corresponding to an effect.

In many cases ContributionWeight = ConfigWeight,  $f_{1ij}(dg, i, j, m, par) = f_{1ijc}(dg, i, j, m, 1, par)$ , and  $f_{1i} = 0$ , but flexibility and possibilities for handling missing data are gained by this way of specifying; which contains some redundancies for data sets without any missings.

The contribution of the effect to the *evaluation function* is defined as follows: if the effect has weight  $\alpha$  (a statistical parameter, to be distinguished from the preset parameter indicated by *par*), then increasing tie variable  $x_{ij}$  from 0 to 1 will increase the evaluation function by  $\alpha$  times

$$c_{1} + c_{2} x_{ji} + c_{3} x_{i+} + c_{4} x_{+i} + c_{5} x_{j+} + c_{6} x_{+j} + c_{7} T P_{ij} + c_{8} O S_{ij} + c_{9} I S_{ij} + c_{10} T P_{ji} + f 1 i j c (dg, i, j, m, 0, par) ,$$

$$(4)$$

where  $c_h = \text{ContributionWeight}[h]$ ; decreasing tie variable  $x_{ij}$  from 1 to 0 will decrease the evaluation function by  $\alpha$  times

$$c_{0} + c_{1} + c_{2} x_{ji} + c_{3} x_{i+} + c_{4} x_{+i} + c_{5} x_{j+} + c_{6} x_{+j} + c_{7} T P_{ij} + c_{8} O S_{ij} + c_{9} I S_{ij} + c_{10} T P_{ji} + f 1 i j c (dg, i, j, m, 1, par).$$
(5)

Here TP, OS, and IS are the numbers of twopaths, outstars, and instars, respectively, defined by

$$TP_{ij} = \sum_{h} x_{ih} x_{hj}$$
  

$$OS_{ij} = \sum_{h} x_{hi} x_{hj}$$
  

$$IS_{ij} = \sum_{h} x_{ih} x_{jh} ;$$

dg refers to the current network x, and m to the current period. Equations (4), (5) are (except for the sign in the second case) equal to  $s_{ik}(x(i \rightarrow j)) - s_{ik}(x)$  in Snijders (2001, 2005) and Snijders, Steglich and Schweinberger (2007), where k is the index of the effect.

Equations (4), (5) are calculated in either of two ways: in procedure UtilityComponents, and in procedures Contrib\_n and Contrib\_fn, all in unit Digraph. (The procedures in Digraph are linked to unit S\_Base by means of procedures in unit Eight.) Procedure UtilityComponents calculates (4) and

(5) and makes them available directly. Procedures Contrib\_n and Contrib\_fn compute the change in objective function, which is

$$\sum_{k} \alpha_k \{ s_{ik} (x(i \rightsquigarrow j)) - s_{ik}(x) \} .$$
(6)

This is calculated for changing tie variables from 0 to 1, using (4) and  $\alpha_k = alpa_f[k]$ , in procedures Contrib\_n and Contrib\_n\_alpha in unit Digraph; for changing tie variables from 1 to 0 it is calculated, using (5) and  $\alpha_k = alpa_f[k] + alpa_g[k]$ , in procedures Contrib\_fn and Contrib\_fn and Contrib\_fn\_alpha in unit Digraph. How these procedures are placed in the simulation algorithm is indicated in Section 21.1.

The *statistic* used for estimating the weight  $\alpha$  of the evaluation effect is given by

$$\sum_{m} \sum_{i} f_{1i}(dg^{m+1}, m, par) + \sum_{i \neq j} x_{ij}^{m+1} \left( c_1 + c_2 x_{ji}^{m+1} + c_3 x_{i+}^{m+1} + c_4 x_{+i}^{m+1} + c_5 x_{j+}^{m+1} + c_6 x_{+j}^{m+1} + c_7 T P_{ij}^{m+1} + c_8 O S_{ij}^{m+1} + c_9 I S_{ij}^{m+1} + c_{10} T P_{ji}^{m+1} + f_{1ij}(dg^{m+1}, i, j, m, par) \right),$$
(7)

where  $c_h = \text{ConfigWeight}[h]$  and the superscript  $^{m+1}$  refers to the observation moment. This statistic is calculated by procedure CalcFunctions\_f in unit DIGRAPH.

The endowment function is defined only if  $f_{1i} = 0$ . The contribution of the effect to the endowment function (i.e., an extra component of the loss incurred when changing tie variable  $x_{ij}$  from 1 to 0) is given by the analogue of (5) given that the the corresponding statistical parameter multiplied by (5). The *statistic* used for estimating the weight  $\alpha$  of the endowment effect is given by

$$\sum_{i \neq j} (1 - x_{ij}^{m+1}) x_{ij}^m \left( c_1 + c_2 x_{ji}^m + c_3 x_{i+}^m + c_4 x_{+i}^m + c_5 x_{j+}^m + c_6 x_{+j}^m \right. \\ \left. + c_7 T P_{ij}^m + c_8 O S_{ij}^m + c_9 I S_{ij}^m + c_{10} T P_{ji}^m + f_{1ij} (dg^m, i, j, m, par) \right), \qquad (8)$$

where  $c_h = \text{ConfigWeight}[h]$  and the superscripts m and m+1 refer to the observation moments. Note that the factor  $(1 - x_{ij}^{m+1})x_{ij}^m$  means that the summation extends only over (i, j) for which there was a tie at observation m which had disappeared at moment m + 1, while the subscripts mbetween the parentheses imply that the "quantity lost" is calculated by reference to moment m. This statistic is calculated by procedure CalcFunctions\_g in unit DIGRAPH.

#### 22.2 Changing or adding definitions of effects

Objective function effects for the network dynamics are defined by the procedure AddEffect defined in unit DIGRAPH, and called in procedure DefineNetworkEffects in include file S\_Effects which is part of unit S\_DAT. Procedure AddEffect\* defines the name, the arrays ContributionWeight and ConfigWeight, and the functions flijc, flij, and fli, all described in Section 22.1. Note that various versions AddEffect1, AddEffect2, etc., are available for procedure AddEffect\*, where omitted arguments are 0 or nil. Usually when a new effect is defined, also new functions will have to be defined that are then used in the roles of flijc, flij, and/or fli. Many examples can be found in file S\_Effects.

If new effects are added to the rate function for the network dynamics, these additions must be made in a coherent way to each of the following procedures.

- 1. Procedure DefineFunctionNames in unit  $S_DAT$ , which contains the names of all the statistics calculated from each simulation; item Procedure DefineModel\_Inames in unit  $S_DAT$ , which contains the names of all effects in the network change rate function;
- 2. Function NetworkLambda in unit S\_BASE, the network change rate function itself;
- 3. Procedure NetworkStatistics in unit S\_BASE, for the statistics used to estimate the parameters by the Method of Moments.

(For the maximum likelihood estimation procedure, non-constant rate functions are not yet implemented.)

If new effects are added to the model for the behavior dynamics, these additions must be made coherently to each of the following procedures.

- For each kind of effect:
  - 1. Procedure DefineFunctionNames in unit S\_DAT, which contains the names of all the statistics calculated from each simulation;
  - 2. Procedure ActionStatistics in unit S\_BASE, which calculates these statistics.
- For effects in the rate function for behavior change, the analogous procedures have to be changed as those for the rate function for network change: function ActionEffects\_l in unit S\_DAT, procedure DefineModel\_Inames in unit S\_BASE, function ActionLambda in unit S\_BASE, and procedure ActionStatistics in unit S\_BASE.
- For effects in the evaluation and endowment functions for behavior change, the following procedures have to be changed: function ActionEffects\_f in unit S\_DAT, procedure DefineModel\_fnames in unit S\_BASE, and function Contr\_fa in unit S\_BASE; the latter function must be coordinated with procedure CalcComponents\_fa in unit EIGHT.

The functions AddNoTies\_yn, SubtractTies\_yn, Contr\_fa, Contr\_ga, NetworkLambda, and Action-Lambda are evaluated very frequently by the algorithm. Therefore these have been written so that very few calculations are needed to evaluate them. Such calculations for a large part are replaced by updating and storing the basic numerical information needed to compute them. These updates are contained in the procedure ChangeTie in unit S\_BASE, and the initialisation is contained in the procedure Initialise\_Running\_Statistics.

# 23 Statistical Monte Carlo Studies

According to Sir Ronald A. Fisher, there are three main statistical problems, model specification, model estimation, and problems of distribution. The last one concerns the distribution of statistics, such as the distribution of parameter estimates around the true (data-generating) parameter value or the distribution of test statistics, and can be studied by SIENA in various ways. One way is to use Siena05 and Siena07 repeatedly in batch files. It can be useful to know that if Siena05 is called with only one run, then one data set is simulated and also stored in the internal SIENA format under the project name *sisim*. Further, Siena07 gives brief estimation reports in the files *pname*.bof and *pname*.bos, which can be used more easily as summaries of repeated runs than the normal output file.

Another way to do simulation studies using SIENA, if one has access to a Delphi compiler, is as follows. Open the unit Siena\_7 and go to the procedure TEstForm.FormActivate. The first statement in the procedure is simulate := false. Set the global variable simulate to true. SIENA will then simulate data sets according to the probability model specified in the *pname*.MO file.

Then, manipulate the global constant sequences declared in unit Siena\_7 by setting it to some positive integer value k (the default is 1). The constant sequences gives the number of runs (sequences).

The result is that running SIENA will generate k data sets according to the probability model specified *pname*.MO file. From each data set, the parameters are estimated and test statistics are evaluated.

Some Matlab source files are (by default) generated by SIENA. The source code, when interpreted by Matlab, produces histograms of some statistics, in particular histograms of the parameter estimates and the test statistics.

It should be noted that SIENA generates networks with desired properties, but (by default) no covariates. If covariates are desired, suitable code must be added at the beginning of the procedure SimulateData in the unit S\_EST. Please note that both internal and external storage (see Section 19.3) of generated covariates is required. Internal storage is difficult unless one knows SIENA -it is advisable to contact the authors in such cases.

### 24 Constants

The program contains the following constants. Trying to use a basic information file that implies a data set going beyond these constants leads to an error message in the output file and stops the further operation of SIENA.

name	meaning	in unit
pmax	maximum number $p$ of included effects	S_Constants
ccmax	maximum number of possible statistics	$S_Constants$
nzmax	maximum number $nz$ of individual variables	EIGHT
nzzmax	maximum number $nzz$ of dyadic covariates	EIGHT

Reasonable values for these constants are the following:

pmax = 70;

ccmax = 500; the maximum number of statistics depends on the number of available effects, the number of dependent behavior variables, and the number of observations M, and is given by MaxFunctions in unit S\_Dat; this should not be more than ccmax;

nzmax = 30;

nzzmax = 20.

The number M of observations may not be higher than 99. Since the number of observations is dealt with by a dynamic array, this is not reflected by some constant. The only reason for the upper bound of 99 is that the index number of the observation is used in the internal data file extension names and may not have more than two digits. But 99 seems quite a high upper bound for practical data sets.

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