Manual for SIENA version 3.2

Provisional version

Tom A.B. Snijders Christian E.G. Steglich Michael Schweinberger Mark Huisman

University of Groningen: ICS, Department of Sociology Grote Rozenstraat 31, 9712 TG Groningen, The Netherlands

University of Oxford: Department of Statistics

April 18, 2010



Abstract

SIENA (for Simulation Investigation for Empirical Network Analysis) is a computer program that carries out the statistical estimation of models for the evolution of social networks according to the dynamic actor-oriented model of Snijders (2001, 2005) and Snijders, Steglich, and Schweinberger (2007). It also carries out MCMC estimation for the exponential random graph model according to the procedures described in Snijders (2002) and Snijders, Pattison, Robins, and Handcock (2006).

Contents

1	General information	5								
Ι	Minimal Intro	7								
2 General remarks for StOCNET. 2.1 Operating StOCNET.										
3	Using SIENA 3.1 Steps for estimation: Choosing SIENA in StOCNET. 3.2 Steps for looking at results: Executing SIENA. 3.3 Giving references	8 8 9 10								
Π	User's manual	11								
4	Program parts	11								
5	Input data5.1Digraph data files5.1.1Structurally determined values5.2Dyadic covariates5.3Individual covariates5.4Interactions and dyadic transformations of covariates5.5Dependent action variables5.6Missing data5.7Composition change5.8Centering	12 13 14 15 16 16 16 17 19								
6	Model specification 6.1 Important structural effects for network dynamics 6.2 Effects for network dynamics associated with covariates 6.3 Effects on behavior evolution 6.4 Exponential Random Graph Models 6.4.1 Effects associated with covariates for ERGMs 6.5 Model Type 6.5.1 Model Type: directed networks 6.5.2 Model Type: non-directed networks 6.6.4 Additional interaction effects 6.6.1 Interaction effects for network dynamics 6.6.2 Interaction effects in the ERGM case 6.7 Random effects models: unobserved actor heterogeneity	 20 21 23 24 25 26 26 26 27 28 32 32 32 32 								
7	Estimation 7.1 Algorithm 7.2 Output 7.3 Maximum Likelihood and Bayesian estimation 7.4 Supplementing R functions 7.5 Other remarks about the estimation algorithm 7.5.1 Changing initial parameter values for estimation 7.5.2 Fixing parameters 7.5.3 Automatic fixing of parameters	34 35 38 39 39 39 40 40								

		7.5.4 7.5.5	Conditional and unconditional estimation	40 41				
8	Standard errors 4:							
9	Test	\mathbf{ts}		41				
	9.1	Score-	type tests	42				
	92	Exam	nle: one-sided tests, two-sided tests, and one-step estimates	42				
	0.2	0.9.1	Multi nanomotor tosta	12				
		9.2.1	Multi-parameter tests	45				
		9.2.2	Testing homogeneity assumptions	45				
	9.3	Altern	ative application: convergence problems	45				
10	Sim	ulation		16				
10	10.1			40				
	10.1	Condi	tional and unconditional simulation	46				
11	Exp	onenti	ial random graph models	48				
10	- -	iona fe	an model turns, estimation and simulation	50				
14	Opt	Jons IC	or model type, estimation and simulation	9 0				
10	C -4	+ =====================================	control	۲ 0				
13	Get	ung st		52				
	13.1	Model	choice	52				
		13.1.1	Exploring which effects to include	53				
	13.2	Conve	rgence problems	53				
	13.3	Comp	osition change	54				
14	ъ /	14 * 1 1		50				
14	. IVIU	Itilevel	i network analysis	50				
	14.1	Multı-	group Siena analysis	57				
	14.2	Meta-	analysis of Siena results	58				
15	For	mulas	for effects	60				
	15.1	Netwo	rk evolution	60				
		$15 \ 1 \ 1$	Network evaluation function	60				
		15 1 9	Notwork and amount function	65				
		15.1.2		00				
		15.1.3	Network rate function	66				
		15.1.4	Network rate function for Model Type 2	67				
	15.2	Behav	ioral evolution	67				
		$15 \ 2 \ 1$	Behavioral evaluation function	67				
		15.2.1		70				
		15.2.2	Behavioral endowment function	10				
		15.2.3	Behavioral rate function	70				
	15.3	Expor	nential random graph model	70				
16	Par	ameter	r interpretation	74				
	16.1	Longit	tudinal models	74				
	10.1	16 1 1		75				
		10.1.1	Ego – after selection tables	75				
		16.1.2	Ego – alter influence tables	79				
	16.2	Expor	nential random graph models	80				
				a -				
17	Rur	ning S	Siena outside of StOCNET	82				
	17.1	Siena	04: implementing internal effect parameters	83				
	17.2	Siena(04: copying model and option definitions	84				
				-				
18	18 Command line changes in parameters and options							
19	19 Limitations and time use							
-								
20	Cha	anges o	compared to earlier versions	88				

III Programmer's manual

21	SIENA files	91
	21.1 Basic information file	91
	21.2 Definition files	94
	21.2.1 Model specification through the MO file	94
	21.2.2 Specification of simulations through the SI file	98
	21.3 Data files	99
	21.4 Output files	99
	21.5 Other information files	100
22	Units and executable files	101
	22.1 Executable files	102
	22.2 Running SIENA in console mode	102
23	Starting to look at the source code	102
	23.1 Sketch of the simulation algorithm	104
	23.2 Remarks about the likelihood algorithm	106
24	Parameters and effects	107
	24.1 Effect definition	108
	24.2 Changing or adding definitions of effects	110
25	Statistical Monte Carlo Studies	112
26	Constants	113
27	References	114

1 General information

SIENA¹, shorthand for Simulation Investigation for Empirical Network Analysis, is a computer program that carries out the statistical estimation of models for repeated measures of social networks according to the dynamic actor-oriented model of Snijders and van Duijn (1997), Snijders (2001), and Snijders, Steglich, and Schweinberger (2007); also see Steglich, Snijders, and Pearson (2010). A tutorial for this method is given in Snijders, van de Bunt and Steglich (2010). Some examples are presented, e.g., in van de Bunt (1999); van de Bunt, van Duijn, and Snijders (1999); and van Duijn, Zeggelink, Stokman, and Wasseur (2003); and Steglich, Snijders, and West (2006). A website for SIENA is maintained at http://stat.gamma.rug.nl/snijders/siena.html . Introductions in French and Spanish are given in de Federico de la Rúa (2004, 2005) and Jariego and de Federico de la Rúa (2006).

The program also carries out MCMC estimation for the exponential random graph model (abbreviated to *ERGM* or *ERG model*), also called p^* model, of Frank and Strauss (1986), Frank (1991), Wasserman and Pattison (1996), and Snijders, Pattison, Robins, and Handcock (2006). The algorithm is described in Snijders (2002). A good introduction is Robins, Snijders, Wang, Handcock, and Pattison (2007).

This is a provisional manual for SIENA version 3.2 (which has not been released yet, but of which beta versions are obtainable from the SIENA website). Changes of this version compared to earlier versions are in Section 20. Official releases of the program and the manual can be downloaded from the web sites, http://stat.gamma.rug.nl/stocnet/ and http://www.stats.ox.ac.uk/siena/. One way to run SIENA is as part of the StOCNET program collection (Boer, Huisman, Snijders, Steglich, Wichers & Zeggelink, 2006), which can be downloaded from the same website. For the operation of StOCNET, the reader is referred to the corresponding manual. If desired, SIENA can be operated also independently of StOCNET, as is explained in Section 17.

This manual consists of two parts: the user's manual and the programmer's manual. It can be viewed and printed with the Adobe Acrobat reader. The manual is updated rather frequently, and it may be worthwhile to check now and then for updates.

The manual focuses on the use of SIENA for analysing the dynamics of directed networks. The case of non-directed networks is very similar, and at various points this case is described more in particular. Sections on data requirements, general operation, etc., apply as well to parameter estimation in the ERGM. Some sections are devoted specifically to this model.

For getting started, there are various options:

- 1. One excellent option is to read the User's Manual from start to finish (leaving aside the Programmer's Manual).
- 2. A second option is to read the Minimal Introduction contained in Sections 2 3, together with the table of contents to have an idea of what can be looked up later.
- 3. Another option is first to read the Minimal Introduction and further to focus on Sections 6 for the model specification, 7 to get a basic insight in what happens in the parameter estimation, 7.2 to understand the output file (which is meant to be as self-explanatory as possible), and 13 for the basis of getting started.

¹This program was first presented at the International Conference for Computer Simulation and the Social Sciences, Cortona (Italy), September 1997, which originally was scheduled to be held in Siena. See Snijders & van Duijn (1997).

We are grateful to Peter Boer, Bert Straatman, Minne Oostra, Rob de Negro, all (now or formerly) of SciencePlus, and Evelien Zeggelink, for their cooperation in the development of the StOCNET program and its alignment with SIENA. We also are grateful to NWO (Netherlands Organisation for Scientific Research) for their support to the integrated research program *The dynamics of networks and behavior* (project number 401-01-550), the project *Statistical methods for the joint development of individual behavior and peer networks* (project number 575-28-012), the project *An open software system for the statistical analysis of social networks* (project number 405-20-20), and to the foundation ProGAMMA, which all contributed to the work on SIENA and StOCNET.

Part I Minimal Intro

The following is a minimal cookbook-style introduction for getting started with SIENA as operated from within StOCNET.

2 General remarks for StOCNET.

- 1. Ensure that the directories in **Options Directories** are existing, and that these are the directories where your data are stored, and where the output is to be stored.
- 2. Always keep in mind that, when the green Apply sign is visible, StOCNET expects you to press this button in order to confirm the most recent commands and to continue.

(You can choose to Cancel if you do not wish to confirm.)

3. The output file which you see in Results is the file, with extension .out, that is stored in the directory specified in Options - Directories as the Directory of session files.

2.1 Operating StOCNET.

- 1. Start by choosing to enter a new session or open a previous session.
- You have to go sequentially through the various steps:
 Data Transformation (optional) Selection (optional) Model Results.
- 3. When starting a new session, you must select one or more network data sets as dependent variable(s), and optionally one or more network data sets as dyadic covariates (independent variables).

In addition, you can optionally select one or more files with actor-level covariates ('actor attributes') (as independent variables). If you do this, StOCNET will determined the number of variables in the data set and it is advisable to edit the names of the variables (which have the not very helpful default names of Attribute1, etc.).

- 4. After selecting the data files and clicking Apply, you are requested to save the session, and give it a name which serves later to identify this session.
- 5. If necessary, transform the data and indicate missing data values. This is self-explanatory (consult the StOCNETmanual if you need help). You have to note yourself how you transformed the variables. But it is recorded also in the session-tree on the right hand side of the StOCNETscreen.

It is also advisable to save the session (Session - Save session) after having transformed the data.

3 Using SIENA

3.1 Steps for estimation: Choosing SIENA in StOCNET.

1. In the Model step, select SIENA.

Then select Data Specification, where the dependent network variable(s) must go to Digraphs in seq. order and the dyadic covariates (if any) to the box with that name. If you specify one file as dependent network variable, then the ERGM (p*) model is applied. If you specify more than one file as dependent network variables, then the (longitudinal) actor-oriented model is applied, and the ordering of the files in the Digraphs in seq. order box

must be the correct order in time.

- 2. If you are analyzing only a network as the dependent variable, then the actor covariates (if any) must go to the box Constant covariates or Changing covariates; the 'changing' refers to change over time, and can be used only for the longitudinal option.
- 3. Next go to the Model specification and select the effects you wish to include in the model. When starting, choose a small number (e.g., 1 to 4) effects.
- 4. After clicking OK, you can then continue by estimating parameters: the Estimation option must be selected (which contrasts with Simulation), and the estimation algorithm then is started by clicking the Run button.
- 5. It will depend on the size of the data set and the number of parameters in the model, how long the estimation takes. The output file opens automatically in the **Results** step.
- 6. Below you see some points about how to evaluate the reliability of the results. If the convergence of the algorithm is not quite satisfactory but not extremely poor, then you can continue just by Running the estimation algorithm again.
- 7. If the parameter estimates obtained are very poor (not in a reasonable range), then it usually is best to start again, with a simpler model, and from a standardized starting value. The latter option must be selected in the Model specification Options screen.

SIENA estimates parameters by the following procedure:

- Certain statistics are chosen that should reflect the parameter values; the finally obtained parameters should be such, that the *expected values* of the statistics are equal to the *observed values*. Expected values are approximated as the averages over a lot of simulated networks. Observed values are calculated from the data set. These are also called the *target values*.
- 2. To find these parameter values, an *iterative stochastic simulation algorithm* is applied. This works as follows:
 - (a) In Phase 1, the sensitivity of the statistics to the parameters is roughly determined.
 - (b) In Phase 2, provisional parameter values are updated: this is done by simulating a network according to the provisional parameter values, calculating the statistics and the deviations between these simulated statistics and the *target values*, and making a little change (the 'update') in the parameter values that hopefully goes into the right direction. (Only a 'hopefully' good update is possible, because the simulated network is only a random draw from the distribution of networks, and not the expected value itself.)

(c) In Phase 3, the final result of Phase 2 is used, and it is checked if the average statistics of many simulated networks are indeed close to the target values. This is reflected in the so-called t statistics for deviations from targets.

3.2 Steps for looking at results: Executing SIENA.

- 1. Look at the start of the output file for general data description (degrees, etc.), to check your data input.
- 2. When parameters have been estimated, first look at the t ratios for deviations from targets. These are good if they are all smaller than 0.1 in absolute value, and reasonably good if they are all smaller than 0.2.

We say that the algorithm has converged if they are all smaller than 0.1 in absolute value, and that it has nearly converged if they are all smaller than 0.2.

These bounds are indications only, and may be taken with a grain of salt.

For well converged estimates, you can interpret the estimates and standard errors as explained later in this manual.

If the estimates are not good enough, just estimate once again, using the previously obtained values as the new starting values; this use of previous values is the default. However, if the estimates you obtained are way off, then it is better to start from clean slate and use so-called standard initial values – which is one of the estimation options. Turn this option off after one estimation job.

Items 3–4 apply only to the ERGM (non-longitudinal) case and to estimation for longitudinal data using the ML (Maximum Likelihood) method.

- 3. In the ERGM (non-longitudinal) case and when using the ML (Maximum Likelihood) method for longitudinal data, it is often harder to obtain good convergence. This means that it may take several runs of the estimation algorithm, and that it may be necessary to fiddle with two parameters in the Model Specification Options: the Multiplication factor and the Initial value of gain parameter.
- 4. (This item is of interest only for the ERGM and ML cases).

The Multiplication factor determines for these cases the number of Metropolis-Hastings steps taken for simulating each new network. When this is too low, the sequentially simulated networks are too similar, which will lead to high autocorrelation in the generated statistics. This leads to poor performance of the algorithm. These autocorrelations are given in the output file. When some autocorrelations are more than 0.1, it is good to increase the Multiplication factor.

When the Multiplication factor is unnecessarily high, computing time will be unnecessarily high.

5. (This item also is of interest mainly for the ERGM and ML cases).

The **Initial value of gain parameter** determines the step sizes in the parameter updates in the iterative algorithm. A too low value implies that it takes very long to attain a reasonable parameter estimate when starting from an initial parameter value that is far from the 'true' parameter estimate. A too high value implies that the algorithm will be unstable, and may be thrown off course into a region of unreasonable (e.g., hopelessly large) parameter values. In the longitudinal case using the Method of Moments (the default estimation procedure), it usually is unnecessary to change this. In the ERGM case, when the autocorrelations are

smaller than 0.1 but the t statistics for deviations from targets are relatively small (less than, say, 0.3) but do not all become less than 0.1 in absolute value in repeated runs of the estimation algorithm, then it will be good to decrease the Initial value of gain parameter. Do this by dividing it by, e.g., a factor 2 or a factor 5, and then try again a few estimation runs.

- 6. If all this is of no avail, then the conclusion may be that the model specification is incorrect for the given data set.
- 7. Further help in interpreting output is in Section 7.2 of this manual.

3.3 Giving references

When using SIENA, it is appreciated that you refer to this manual and to one or more relevant references of the methods implemented in the program. The reference to this manual is the following.

Snijders, Tom A.B., Christian E.G. Steglich, Michael Schweinberger, and Mark Huisman. 2008. Manual for SIENA version 3.2. Groningen: University of Groningen, ICS. Oxford: University of Oxford, Department of Statistics. http://www.stats.ox.ac.uk/siena/

A basic reference for the network dynamics model is Snijders (2001) or Snijders (2005). Basic references for the model of network-behavior co-evolution are Snijders, Steglich, and Schweinberger (2007) and Steglich, Snijders, and Pearson (2010). A tutorial is Snijders, van de Bunt, and Steglich (2010).

Basic references for the non-longitudinal (Exponential Random Graph) model are Frank and Strauss (1986); Wasserman and Pattison (1996); Snijders (2002); and Snijders, Pattison, Robins, and Handcock (2006). A more didactic reference here is Robins, Snijders, Wang, Handcock, and Pattison (2007).

More specific references are Schweinberger (2005) for the score-type goodness of fit tests; Schweinberger and Snijders (2007) for the calculation of standard errors of the Method of Moments estimators; and Snijders, Koskinen and Schweinberger (2010) for maximum likelihood estimation.

Part II User's manual

The user's manual gives the information for using SIENA. It is advisable also to consult the user's manual of StOCNET because normally, the user will operate SIENA from within StOCNET.

4 Parts of the program

The operation of the SIENA program is comprised of four main parts:

- 1. input of basic data description,
- 2. model specification,
- 3. estimation of parameter values using stochastic simulation,
- 4. simulation of the model with given and fixed parameter values.

The normal operation is to start with data input, then specify a model and estimate its parameters, and then continue with new model specifications followed by estimation or simulation. For the comparison of (nested) models, statistical tests can be carried out.

The program is organized in the form of *projects*. A project consists of data and the current model specification. All files internally used in a given project have the same root name, which is called the project name, and indicated in this manual by *pname*. (In view of its use for special simulation purposes, it is advised not to use the project name *sisim*; also avoid to use the name *siena*, which is too general for this purpose.)

The main output is written to the text file *pname.out*, auxiliary output is contained in the text files *pname.log* and *pname.eff*.

5 Input data

The main statistical method implemented in SIENA is for the analysis of repeated measures of social networks, and requires network data collected at two or more time points. It is possible to include changing actor variables (representing behavior, attitudes, outcomes, etc.) which also develop in a dynamic process, together with the social networks. As repeated measures data on social networks, at the very least, two or more data files with digraphs are required: the observed networks, one for each time point. The number of time points is denoted M.

The other statistical method implemented in SIENA if the parameter estimation for the exponential random graph model ('ERGM'). For this method, one observed network data set is required.

In addition, various kinds of variables are allowed:

- 1. actor-bound or individual variables, also called actor attributes, which can be symbolized as v_i for each actor *i*; these can be constant over time or changing; the changing individual variables can be dependent variables (changing dynamically in mutual dependence with the changing network) or independent variables (exogenously changing variables; then they are also called individual covariates).
- 2. dyadic covariates, which can be symbolized as w_{ij} for each ordered pair of actors (i, j); they are allowed only to have integer values.

All variables must be available in ASCII ('raw text') data files, described in detail below. It is best to use the 'classical' type of filenames, without embedded blanks and not containing special characters. These files, the names of the corresponding variables, and the coding of missing data, must be made available to SIENA. In the StOCNET environment, files and variable names are entered in the Data dialog window, while missing data are identified in the Transformation dialog window. In the Model dialog window, network data and additional variables subsequently can be selected for SIENA analyses. This is done by first choosing SIENA from the list of available statistical methods, and then pushing the Data specification button.

Names of variables must be composed of at most 11 characters. This is because they are used as parts of the names of effects which can be included in the model, and the effect names should not be too long. The use of the default variable and file names proposed by **StOCNET** is not recommended.

5.1 Digraph data files

Each digraph must be contained in a separate input file. Two data formats are allowed. For large number of nodes (say, larger than 100), the Pajek format is preferable to the adjacency matrix format. For more than a few hundred nodes,

1. Adjacency matrices.

The first is an adjacency matrix, i.e., n lines each with n integer numbers, separated by blanks or tabs, each line ended by a hard return. The diagonal values are meaningless but must be present.

Although this section talks only about digraphs (directed graphs), it is also possible that all observed adjacency matrices are symmetric. This will be automatically detected by SIENA, and the program will then utilize methods for non-directed networks.

The data matrices for the digraphs must be coded in the sense that their values are converted by the program to the 0 and 1 entries in the adjacency matrix. A set of code numbers is required for each digraph data matrix; these codes are regarded as the numbers representing a present arc in the digraph, i.e., a 1 entry in the adjacency matrix; all other numbers will be regarded as 0 entries in the adjacency matrix. Of course, there must be at least one such code number. All code numbers must be in the range from 0 to 9, except for structurally determined values (see below).

This implies that if the data are already in 0-1 format, the single code number 1 must be given. As another example, if the data matrix contains values 1 to 5 and only the values 4 and 5 are to be interpreted as present arcs, then the code numbers 4 and 5 must be given.

2. Pajek format.

If the digraph data file has extension name .net, then the program assumes that the data file has Pajek format. This can not yet be done when using StOCNET, and therefore this option is available only when running SIENA outside of StOCNET, as described in Section 17.

The keywords Arcs*, Edges*, Arcslist*, and Edgeslist* are allowed, followed by data lines according to the Pajek rules. These keywords must be in lines that contain no further characters. All of these keywords may be used in one input file. The Edges* and Edgeslist* keywords announce that mutual ties are following. Note that the Arcslist* and Edgeslist* formats produce binary data. Tie values different from 1, which are used to indicate missings, structurally determined values, or valued data, can only be input in the Pajek format by using the keywords Arcs* and Edges*.

An example of such input files is given in the s50 data set that is distributed on the SIENA website.

Code numbers for missing numbers also must be indicated – in the case of either input data format. These codes must, of course, be different from the code numbers representing present arcs.

Although this section talks only about digraphs (directed graphs), it is also possible that all observed ties (for all time points) are mutual. This will be automatically detected by SIENA, and the program will then utilize methods for non-directed networks.

If the data set is such that it is never observed that ties are terminated, then the network dynamics is automatically specified internally in such a way that termination of ties is impossible. (In other words, in the simulations of the actor-based model the actors have only the option to create new ties or to retain the status quo, not to delete existing ties.)

5.1.1 Structurally determined values

It is allowed that some of the values in the digraph are structurally determined, i.e., deterministic rather than random. This is analogous to the phenomenon of 'structural zeros' in contingency tables, but in SIENA not only structural zeros but also structural ones are allowed. A structural zero means that it is certain that there is no tie from actor i to actor j; a structural one means that it is certain that there is a tie. This can be, e.g., because the tie is impossible or formally imposed, respectively.

Structural zeros provide an easy way to deal with actors leaving or joining the network between the start and the end of the observations. Another way (more complicated but it gives possibilities to represent actors entering or leaving at specified moments between observations) is described in Section 5.7.

Structurally determined values are defined by reserved codes in the input data: the value 10 indicates a structural zero, the value 11 indicates a structural one. Structurally determined values can be different for the different time points. (The diagonal of the data matrix always is composed of structural zeros, but this does not have to be indicated in the data matrix by special codes.) The correct definition of the structurally determined values can be checked from the brief report of this in the output file, and by looking at the file *pname*.s01 (for the first time point), *pname*.s02

(second time point), etc. In these files, the structurally determined positions (structural zeros as well as structural ones) are indicated by the value 1, all others (i.e., the positions where ties are random) by the value 0.

Structural zeros offer the possibility of analyzing several networks simultaneously under the assumption that the parameters are identical. Another option to do this is given in Section 14. E.g., if there are three networks with 12, 20 and 15 actors, respectively, then these can be integrated into one network of 12 + 20 + 15 = 47 actors, by specifying that ties between actors in different networks are structurally impossible. This means that the three adjacency matrices are combined in one 47×47 data file, with values 10 for all entries that refer to the tie from an actor in one network to an actor in a different network. In other words, the adjacency matrices will be composed of three diagonal blocks, and the off-diagonal blocks will have all entries equal to 10. In this example, the number of actors per network (12 to 20) is rather small to obtain good parameter estimates, but if the additional assumption of identical parameter values for the three networks is reasonable, then the combined analysis may give good estimates.

In such a case where K networks (in the preceding paragraph, the example had K = 3) are combined artificially into one bigger network, it will often be helpful to define K - 1 dummy variables at the actor level to distinguish between the K components. These dummy variables can be given effects in the rate function and in the evaluation function (for "ego"), which then will represent that the rate of change and the out-degree effect are different between the components, while all other parameters are the same.

It will be automatically discovered by SIENA when functions depend only on these components defined by structural zeros, between which tie values are not allowed. For such variables, only the ego effects are defined and not the other effects defined for the regular actor covariates and described in Section 6.2. This is because the other effects then are meaningless. If at least one case is missing (i.e., has the missing value data code for this covariate), then the other covariate effects are made available.

When SIENA simulates networks including some structurally determined values, if these values are constant across all observations then the simulated tie values are likewise constant. If the structural fixation varies over time, the situation is more complicated. Consider the case of two consecutive observations m and m + 1, and let X_{ij}^{sim} be the simulated value at the end of the period from t_m to t_{m+1} . If the tie variable X_{ij} is structurally fixed at time t_m at a value $x_{ij}(t_m)$, then X_{ij}^{sim} also is equal to $x_{ij}(t_m)$, independently of whether this tie variable is structurally fixed at time t_{m+1} at the same or a different value or not at all. This is the direct consequence of the structural fixation. From SIENA version 3.17s onward, the following rule is also used. If X_{ij} is not structurally fixed at time t_m but it is structurally fixed at time t_{m+1} at some value $x_{ij}(t_{m+1})$, then in the course of the simulation process from t_m to t_{m+1} this tie variable can be changed as part of the process in the usual way, but after the simulation is over and before the statistics are calculated it will be fixed to the value $x_{ij}(t_{m+1})$.

The target values for the algorithm of the Method of Moments estimation procedure are calculated for all observed digraphs $x(t_{m+1})$. However, starting from SIENA version 3.17s, for tie variables X_{ij} that are structurally fixed at time t_m , the observed value $x_{ij}(t_{m+1})$ is replaced by the structurally fixed value $x_{ij}(t_m)$. This gives the best possible correspondence between target values and simulated values in the case of changing structural fixation.

5.2 Dyadic covariates

As the digraph data, also each measurement of a dyadic covariate must be contained in a separate input file with a square data matrix, i.e., n lines each with n integer numbers, separated by blanks or tabs, each line ended by a hard return. The diagonal values are meaningless but must be present. Pajek input format is currently not possible for dyadic covariates.

A distinction is made between constant and changing dyadic covariates, where change refers to changes over time. Each constant covariate has one value for each pair of actors, which is valid for all observation moments, and has the role of an independent variable. Changing covariates, on the other hand, have one such value for each period between measurement points. If there are M waves of network data, this covers M - 1 periods, and accordingly, for specifying a single changing dyadic covariate, M - 1 data files with covariate matrices are needed.

The StOCNET interface requires the user to enter these in blocks of M - 1, and within each block in sequential order. This is done in the Data specification menu of the SIENA model page. For each such block, also a name must be provided to identify the changing dyadic covariate. For data sets with only two waves, the specification of changing dyadic covariates is meaningless, because there is only one period, hence there is no change over periods possible. Constant dyadic covariates can be selected in the respective section of the Data specification menu. They are identified by the name given to them in the initial Data step in StOCNET.

The mean is always subtracted from the covariates. See the section on *Centering*.

5.3 Individual covariates

Individual (i.e., actor-bound) variables can be combined in one or more files. If there are k variables in one file, then this data file must contain n lines, with on each line k numbers which all are read as real numbers (i.e., a decimal point is allowed). The numbers in the file must be separated by blanks and each line must be ended by a hard return. There must not be blank lines after the last data line.

Also here, a distinction is made between constant and changing actor variables. Each constant actor covariate has one value per actor valid for all observation moments, and has the role of an independent variable.

Changing variables can change between observation moments. They can have the role of dependent variables (changing dynamically in mutual dependence with the changing network) or of independent variables; in the latter case, they are also called 'changing individual covariates'. Dependent variables are treated in the section below, this section is about individual variables in the role of independent variables – then they are also called individual covariates.

When changing individual variables have the role of independent variables, they are assumed to have constant values from one observation moment to the next. If observation moments for the network are $t_1, t_2, ..., t_M$, then the changing covariates should refer to the M - 1 moments t_1 through t_{M-1} , and the *m*-th value of the changing covariates is assumed to be valid for the period from moment t_m to moment t_{m+1} . The value at t_M , the last moment, does not play a role. Changing covariates, as independent variables, are meaningful only if there are 3 or more observation moments, because for 2 observation moments the distinction between constant and changing covariates is not meaningful.

Each changing individual covariate must be given in one file, containing k = M - 1 columns that correspond to the M - 1 periods between observations. It is not a problem if there is an M'th column in the file, but it will not be read.

The mean is always subtracted from the covariates. See the section on *Centering*.

When an actor covariate is constant within waves, or constant within components separated by structural zeros (which means that ties between such components are not allowed), then only the ego effect of the actor covariate is made available. This is because the other effects then are meaningless. This may cause problems for combining several data sets in a meta-analysis (see Section 14). If at least one case is missing (i.e., has the missing value data code), then the other covariate effects are made available. When analysing multiple data sets in parallel, for which the same set of effects is desired to be included in the .MO file, it is therefore advisable to give data sets in which a given covariate has the same value for all actors one missing value in this covariate; purely to make the total list of effects in the MO file independent of the observed data.

5.4 Interactions and dyadic transformations of covariates

For actor covariates, two kinds of transformations to dyadic covariates are made internally in SIENA. Denote the actor covariate by v_i , and the two actors in the dyad by i and j. Suppose that the range of v_i (i.e., the difference between the highest and the lowest values) is given by r_V . The two transformations are the following:

- 1. dyadic similarity, defined by $1 (|v_i v_j|/r_V)$, and centered so the the mean of this similarity variable becomes 0; note that before centering, the similarity variable is 1 if the two actors have the same value, and 0 if one has the highest and the other the lowest possible value;
- 2. same V, defined by 1 if $v_i = v_j$, and 0 otherwise (not centered) (V is the name of the variable). This can also be referred to as dyadic identity with respect to V.

Dyadic similarity is relevant for variables that can be treated as interval-level variables; dyadic identity is relevant for categorical variables.

In addition, SIENA offers the possibility of user-defined two- and three-variable interactions between covariates; see Section 6.6.

5.5 Dependent action variables

SIENA also allows dependent action variables, also called dependent behavior variables. This can be used in studies of the co-evolution of networks and behavior, as described in Snijders, Steglich, and Schweinberger (2007) and Steglich, Snijders, and Pearson (2010). These action variables represent the actors' behavior, attitudes, beliefs, etc. The difference between dependent action variables and changing actor covariates is that the latter change exogenously, i.e., according to mechanisms not included in the model, while the dependent action variables change endogenously, i.e., depending on their own values and on the changing network. In the current implementation only one dependent network variable is allowed, but the number of dependent action variables are not assumed to be constant between observations.

Dependent action variables must have nonnegative integer values; e.g., 0 and 1, or a range of integers like 0,1,2 or 1,2,3,4,5. Each dependent action variable must be given in one file, containing k = M columns, corresponding to the M observation moments.

5.6 Missing data

SIENA allows that there are some missing data on network variables, on covariates, and on dependent action variables. Missing data in changing dyadic covariates are not yet implemented. Missing data must be indicated by missing data codes (this can be specified in StOCNET, if SIENA is operated through StOCNET), *not* by blanks in the data set.

Missingness of data is treated as non-informative. One should be aware that having many missing data can seriously impair the analyses: technically, because estimation will be less stable; substantively, because the assumption of non-informative missingness often is not quite justified. Up to 10% missing data will usually not give many difficulties or distortions, provided missingness is indeed non-informative. When one has more than 20% missing data on any variable, however, one may expect problems in getting good estimates.

In the current implementation of SIENA, missing data are treated in a simple way, trying to minimize their influence on the estimation results. This method is further explained in Huisman and Steglich (2008), where comparisons are also made with other ways of dealings with the missing information.

The basic idea is the following. A brief sketch of the procedure is that missing values are imputed to allow meaningful simulations; for the calculation of the target statistics in the Method of Moments, tie variables and actor variables with missings are not used. More in detail, the procedure is as follows.

The simulations are carried out over all variables, as if they were complete. To enable this, missing data are imputed. In the initial observation, missing entries in the adjacency matrix are set to 0, i.e., it is assumed that there is *no* tie; this is done because normally data are sparse, so 'no tie' is the modal value of the tie variable. In the further observations, for any variable, if there is an earlier observed value of this variable then the last observed value is used to impute the current value (the 'last observation carry forward' option, cf. Lepkowski, 1989); if there is no earlier observed value, the value 0 is imputed. For the dependent behavior variables the same principle is used: if there is a previous observation of the same variable then this value is imputed, if there is none then the observationwise mode of the variable is imputed. Missing covariate data are replaced by the variable's average score at this observation moment. In the course of the simulations, however, the adjusted values of the dependent action variables and of the network variables are allowed to change.

In order to ensure a minimal impact of missing data treatment on the results of parameter estimation (method of moments estimation) and/or simulation runs, the calculation of the target statistics used for these procedures uses only non-missing data. When for an actor in a given period, any variable is missing that is required for calculating a contribution to such a statistic, this actor in this period does not contribute to the statistic in question. For network and dependent action variables, the tie variable or the actor variable, respectively, must provide valid data both at the beginning and at the end of a period for being counted in the respective target statistics.

5.7 Composition change

SIENA can also be used to analyze networks of which the composition changes over time, because actors join or leave the network between the observations. This can be done in two ways: using the method of Huisman and Snijders (2003), or using structural zeros. (For the maximum likelihood estimation and multi-group options, the Huisman-Snijders method is not implemented, and only the structural zeros method can be used.) Structural zeros can specified for all elements of the tie variables toward and from actors who are absent at a given observation moment. How to do this is described in subsection 5.1.1. This is straightforward and not further explained here.² This subsection explains the method of Huisman and Snijders (2003), which uses the information about composition change in a somewhat more efficient way.

For this case, a data file is needed in which the *times of composition change* are given. For networks with constant composition (no entering or leaving actors), this file is omitted and the current subsection can be disregarded.

Network composition change, due to actors joining or leaving the network, is handled separately from the treatment of missing data. The digraph data files must contain all actors who are part of the network at any observation time (denoted by n) and each actor must be given a separate (and fixed) line in these files, even for observation times where the actor is not a part of the network

 $^{^{2}}$ In the Siena01 program there is an option to automatically convert data with a file of composition change into a SIENA project with the composition change treated as structural zeros. This is option 4 in the basic information file; see the end of this subsection.

(e.g., when the actor did not yet join or the actor already left the network). In other words, the adjacency matrix for each observation time has dimensions $n \times n$.

At these times, where the actor is not in the network, the entries of the adjacency matrix can be specified in two ways. First as missing values using missing value code(s). In the estimation procedure, these missing values of the joiners before they joined the network are regarded as 0 entries, and the missing entries of the leavers after they left the network are fixed at the last observed values. This is different from the regular missing data treatment. Note that in the initial data description the missing values of the joiners and leavers are treated as regular missing observations. This will increase the fractions of missing data and influence the initial values of the density parameter.

A second way is by giving the entries a regular observed code, representing the absence or presence of an arc in the digraph (as if the actor was a part of the network). In this case, additional information on relations between joiners and other actors in the network before joining, or leavers and other actors after leaving can be used if available. Note that this second option of specifying entries always supersedes the first specification: if a valid code number is specified this will always be used.

For joiners and leavers, crucial information is contained in the times they join or leave the network (i.e., the times of composition change), which must be presented in a separate input file. This data file must contain n lines, each line representing the corresponding actor in the digraph files, with on each line four numbers. The first two concern joiners, the last two concern leavers: 1) the last observation moment at which the actor is not yet observed, 2) the time of joining (expressed as a fraction of the length of the period), 3) the last observation moment at which the actor is observation moment at which the network at all observation moments must be given values in this file. In the following example, the number of observation moments is considered to be M = 5, which means there are four periods; period m starts at observation moment m and ends at m + 1 for m = 1, 2, ..., 4 = M - 1.

Example	of file	with	times	of	composition	change	

Present at all five observation times	0	1.0	5	0.0
Joining in period 2 at fraction 0.6 of length of period	2	0.6	5	0.0
Leaving in period 3 at fraction 0.4 of length of period	0	1.0	3	0.4
Joining in per. 1 (0.7) and leaving in per. 4 (0.2)	1	0.7	4	0.2
Leaving in per. 2 (0.6) and joining in per. 3 (0.8)	3	0.8	2	0.6

Note that for joining, the numbers 0 1.0 have a different meaning than the numbers 1 0.0. The former numbers indicate that an actor is observed at time 1 (he/she joined the network right before the first time point), the latter indicate that an actor is not observed at observation time 1 (he/she joined just after the first time point). The same holds for leavers: 5 0.0 indicates that an actor is observed at time point 5, whereas 4 1.0 indicates that an actor left right before he/she was observed at time point 5.

From the example it follows that an actor is only allowed to join, leave, join and then leave, or leave and then join the network. The time that the actor is part of the network must be an uninterrupted period. It is not allowed that an actor joins twice or leaves twice. When there is no extra information about the time at which an actor joins or leaves (in some known period), there are three options: set the fraction equal to 0.0, 0.5, or 1.0. The second option is thought to be least restrictive.

The following special options are available for treatment of composition change by indicating this in the corresponding line in the basic information file (see Section 21.1):

- 2. The values of the joiners before joining are replaced by the value 0 (no ties), and the values of the leavers after leaving are treated as regular missing data.
- 3. The values of the joiners before joining and the values of the leavers after leaving are treated as regular missing data.
- 4. Before joining and after leaving, actors are treated as structural zeros.

Option 4 has the same effect as specifying the data for the absent actors as structural zeros; this option is useful for users who have a data set ready with joiners and leavers and wish to transform it automatically to a data set with structural zeros, e.g., because they wish to use the maximum likelihood estimation procedure.

5.8 Centering

Individual as well as dyadic covariates are centered by the program in the following way.

For individual covariates, the mean value is subtracted immediately after reading the variables. For the changing covariates, this is the global mean (averaged over all periods). The values of these subtracted means are reported in the output.

For the dyadic covariates and the similarity variables derived from the individual covariates, the grand mean is calculated, stored, and subtracted during the program calculations. (Thus, dyadic covariates are treated by the program differently than individual covariates in the sense that the mean is subtracted at a different moment, but the effect is exactly the same.)

The formula for balance is a kind of dissimilarity between rows of the adjacency matrix. The mean dissimilarity is subtracted in this formula and also reported in the output. This mean dissimilarity is calculated by a formula given in Section 15.

The dependent network variable is not centered.

6 Model specification

After defining the data, the next step is to specify a model. In the StOCNET environment, this is done by clicking the Model specification button that is activated after a successful Data specification in StOCNET 's Model menu, provided that SIENA was selected from the list of available models.

The model specification consists of a selection of 'effects' for the evolution of each dependent variable (network or behavior). A list of all available effects for a given SIENA project is given in the secondary output file *pname.log*. A list of all effects in the objective function is given in the file *pname.eff*.

For the longitudinal case, three types of effects are distinguished (see Snijders, 2001; Steglich, Snijders and Pearson, 2010):

• rate function effects

The rate function models the speed by which the dependent variable changes; more precisely: the speed by which each network actor gets an opportunity for changing her score on the dependent variable.

Advice: in most cases, start modeling with a constant rate function without additional rate function effects. Constant rate functions are selected by exclusively checking the 'basic rate parameter' (for network evolution) and the main rate effects (for behavioral evolution) on the **model specification** screen. (When there are important size or activity differences between actors, it is possible that different advice must be given, and it may be necessary to let the rate function depend on the individual covariate that indicates this size; or on the out-degree.)

• evaluation function effects

The evaluation function³ models the network actors' satisfaction with their local network neighborhood configuration. It is assumed that actors change their scores on the dependent variable such that they improve their total satisfaction – with a random element to represent the limited predictability of behavior. In contrast to the endowment function (described below), the evaluation function evaluates only the local network neighborhood configuration that results from the change under consideration. In most applications, the evaluation function will be the main focus of model selection.

The network evaluation function normally should always contain the 'density', or 'out-degree' effect, to account for the observed density. For directed networks, it mostly is also advisable to include the reciprocity effect, this being one of the most fundamental network effects. Likewise, behavior evaluation functions should normally always contain the shape parameter, to account for the observed prevalence of the behavior, and (unless the behavior is dichotomous) the quadratic shape effect, to account more precisely for the distribution of the behavior.

• endowment function effects

The endowment function⁴ is an extension of the evaluation function that allows to distinguish between new and old network ties (when evaluating possible network changes) and between increasing or decreasing behavioral scores (when evaluating possible behavioral changes). The function models the loss of satisfaction incurred when existing network ties are dissolved or when behavioral scores are decreased to a lower value (hence the label 'endowment').

For a number of effects, the endowment function is implemented not for the Method of Moments estimation method, but only for Maximum Likelihood and Bayesian estimation. This is indicated in Section 15.

Advice: start modeling without any endowment effects, and add them at a later stage. Do not use endowment effects for behavior unless the behavior variable is dichotomous.

³The evaluation function was called *objective function* in Snijders, 2001.

⁴The endowment function is similar to the *gratification function* in Snijders, 2001.

The estimation and simulation procedures of SIENA operate on the basis of the model specification which comprises the set of effects included in the model as described above, together with the current parameter values and the Model Type (see Section 6.5). After data input, the constant rate parameters and the density effect in the network evaluation function have default initial values, depending on the data. All other parameter values initially are 0. The estimation process changes the current value of the parameters to the estimated values. Values of effects not included in the model are not changed by the estimation process. It is possible for the user to change parameter values and to request that some of the parameters are fixed in the estimation process at their current value.

6.1 Important structural effects for network dynamics

For the structural part of the model for network dynamics, the most important effects are as follows. The mathematical formulae for these and other effects are given in Section 15. Here we give a more qualitative description.

- 1. The *out-degree effect* which always must be included.
- 2. The *reciprocity effect* which practically always must be included.
- 3. There is a choice of four network closure effects. Usually it will be sufficient to express the tendency to network closure by including one or two of these. They can be selected by theoretical considerations and/or by their empirical statistical significance. Some researchers may find the last effect (distances two) less appealing because it expresses network closure inversely.
 - a. The transitive triplets effect, which is the classical representation of network closure by the number of transitive triplets. For this effect the contribution of the tie $i \rightarrow j$ is proportional to the total number of transitive triplets that it forms – which can be transitive triplets of the type $\{i \rightarrow j \rightarrow h; i \rightarrow h\}$ as well as $\{i \rightarrow h \rightarrow j; i \rightarrow j\}$;



- b. The balance effect, which may also be called structural equivalence with respect to outgoing ties. This expresses a preference of actors to have ties to those other actors who have a similar set of outgoing ties as themselves. Whereas the transitive triplets effect focuses on how many same choices are made by ego (the focal actor) and alter (the other actor) the number of h for which $i \to h$ and $j \to h$, i.e., $x_{ih} = x_{jh} = 1$ where i is ego and j is alter , the balance effect considers in addition how many the same non-choices are made $x_{ih} = x_{jh} = 0$.
- c. The *transitive ties effect* is similar to the transitive triplets effect, but instead of considering for each other actor j how many two-paths $i \rightarrow h \rightarrow j$ there are, it is only considered whether there is at least one such indirect connection. Thus, one indirect tie suffices for the network embeddedness.
- d. The *number of actors at distance two effect* expresses network closure inversely: stronger network closure (when the total number of ties is fixed) will lead to fewer geodesic distances equal to 2. When this effect has a negative parameter, actors will have a preference for having few others at a geodesic distance of 2 (given their out-degree, which is the number of others at distance 1); this is one of the ways for expressing network closure.

4. The *three-cycles effect*, which can be regarded as generalized reciprocity (in an exchange interpretation of the network) but also as the opposite of hierarchy (in a partial order interpretation of the network). A negative three-cycles effect, together with a positive transitive triplets or transitive ties effect, may be interpreted as a tendency toward local hierarchy. The three-cycles effect also contributes to network closure.



In a non-directed network, the three-cycles effect is identical to the transitive triplets effect.

- 5. Another triadic effect is the *betweenness effect*, which represents brokerage: the tendency for actors to position themselves between not directly connected others, i.e., a preference of i for ties $i \rightarrow j$ to those j for which there are many h with $h \rightarrow i$ and $h \not\rightarrow j$.
- ⊙ Degree-related effects may be important especially for networks where degrees are theoretically important and represent social status or other features important for network dynamics; and/or for networks with high dispersion in in- or out-degrees (which may be an empirical reflection of the theoretical importance of the degrees). Include them if there are theoretical reasons for doing so, but only in such cases.
- 6. The *in-degree popularity effect* (again, with or without 'sqrt', with the same considerations applying) reflects tendencies to dispersion in in-degrees of the actors; or, tendencies to actors with high in-degrees attracting extra incoming ties 'because' of their high current in-degrees.
- 7. The *out-degree popularity effect* (again, with or without 'sqrt', with the same considerations applying) reflects tendencies to actors with high out-degrees attracting extra incoming ties 'because' of their high current out-degrees. This leads to a higher correlation between indegrees and out-degrees.
- 8. The *in-degree activity effect* (with or without 'sqrt') reflects tendencies to actors with high in-degrees to send out extra outgoing ties 'because' of their high current in-degrees. This leads to a higher correlation between in-degrees and out-degrees. The in-degree popularity and out-degree activity effects are not distinguishable in Method of Moments estimation; then the choice between them must be made on theoretical grounds.
- 9. The *out-degree activity effect* (with or without 'sqrt') reflects tendencies to actors with high out-degrees to send out extra outgoing ties 'because' of their high current out-degrees. This also leads to dispersion in out-degrees of the actors.
- 10. The *in-in degree assortativity effect* (where parameter 2 is the same as the sqrt version, while parameter 1 is the non-sqrt version) reflects tendencies to actors with high in-degrees to preferably be tied to other actors with high in-degrees.
- 11. The *in-out degree assortativity effect* (with parameters 2 or 1 in similar roles) reflects tendencies to actors with high in-degrees to preferably be tied to other actors with high out-degrees.
- 12. The *out-in degree assortativity effect* (with parameters 2 or 1 in similar roles) reflects tendencies to actors with high out-degrees to preferably be tied to other actors with high in-degrees.
- 13. The *out-out degree assortativity effect* (with parameters 2 or 1 in similar roles) reflects tendencies to actors with high out-degrees to preferably be tied to other actors with high out-degrees.

6.2 Effects for network dynamics associated with covariates

For each individual covariate, there are several effects which can be included in a model specification, both in the network evolution part and in the behavioral evolution part (should there be dependent behavior variables in the data).

- network rate function
 - 1. the covariate's effect on the rate of network change of the actor;
- network evaluation and endowment functions
 - 1. the covariate-similarity effect; a positive parameter implies that actors prefer ties to others with similar values on this variable thus contributing to the network-autocorrelation of this variable not by changing the variable but by changing the network;
 - 2. the effect on the actor's activity (covariate-ego); a positive parameter will imply the tendency that actors with higher values on this covariate increase their out-degrees more rapidly;
 - 3. the effect on the actor's popularity to other actors (covariate-alter); a positive parameter will imply the tendency that the in-degrees of actors with higher values on this covariate increase more rapidly;
 - 4. the effect of the squared variable on the actor's popularity to other actors (squared covariate-alter) (included only if the range of the variable is at least 2). This normally makes sense only if the covariate-alter effect itself also is included in the model. A negative parameter implies a unimodal preference function with respect to alters' values on this covariate;
 - 5. the interaction between the value of the covariate of ego and of the other actor (covariate ego × covariate alter); a positive effect here means, just like a positive similarity effect, that actors with a higher value on the covariate will prefer ties to others who likewise have a relatively high value; when used together with the alter effect of the squared variable this effect is quite analogous to the similarity effect, and for dichotomous covariates, in models where the ego and alter effects are also included, it even is equivalent to the similarity effect (although expressed differently), and then the squared alter effect is superfluous;
 - 6. the 'same covariate', or covariate identity, effect, which expresses the tendency of the actors to be tied to others with exactly the same value on the covariate; whereas the preceding four effects are appropriate for interval scaled covariates (and mostly also for ordinal variables), the identity effect is suitable for categorical variables;
 - 7. the interaction effect of covariate-similarity with reciprocity;
 - 8. the effect of the covariate of those to whom the actor is indirectly connected, i.e., through one intermediary but not with a direct tie; this value-at-a-distance can represent effects of indirectly accessed social capital.

The usual order of importance of these covariate effects on network evolution is: evaluation effects are most important, followed by endowment and rate effects. Inside the group of evaluation effects, it is the covariate-similarity effect that is most important, followed by the effects of covariate-ego and covariate-alter.

When the network dynamics is not smooth over the observation waves — meaning that the pattern of ties created and terminated, as reported in the initial part of the output file under the

heading Initial data description – Change in networks – Tie changes between subsequent observations, is very irregular across the observation periods — it can be important to include effects of time variables on the network. Time variables are changing actor covariates that depend only on the observation number and not on the actors. E.g., they could be dummy variables, being 1 for one or some observations, and 0 for the other observations.

For actor covariates that are constant within observation waves, or - in the case that there are structurally determined values - constant within connected components, only the ego effects are defined, because only those effects are meaningful. This exclusion of the alter, similarity and other effects for such actor variables applies only to variables without any missing values.

For each dyadic covariate, the following network evaluation effects can be included in the model for network evolution:

- network evaluation and endowment functions
 - 1. main effect of the dyadic covariate;
 - 2. the interaction effect of the dyadic covariate with reciprocity.

The main evaluation effect is usually the most important. In the current version of SIENA, there are no effects of dyadic covariates on behavioral evolution.

6.3 Effects on behavior evolution

For models with one or more dependent behavior variables, i.e., models for the co-evolution of networks and behavior, the most important effects for the behavior dynamics are the following. In these descriptions, with the 'alters' of an actor we refer to the other actors to whom the focal actor has an outgoing tie. The dependent behavior variable is referred to as Z.

- 1. The shape effect, expressing the basic drive toward high values on Z. A zero value for the shape will imply a drift toward the midpoint of the range of the behavior variable.
- 2. The effect of the behavior Z on itself, or quadratic shape effect, which is relevant only if the number of behavioral categories is 3 or more. This can be interpreted as giving a quadratic preference function for the behavior. When the coefficient for the shape effect is β_1^Z and for the effect of Z on itself, or quadratic shape effect, is β_2^Z , then the contributions of these two effects are jointly $\beta_1^Z (z_i \bar{z}) + \beta_2^Z (z_i \bar{z})^2$. With a negative coefficient β_2^Z , this is a unimodal preference function, with the maximum attained for $z_i = \bar{z} 2\beta_1^Z/\beta_2^Z$. (Of course additional effects will lead to a different picture; but as long as the additional effects are linear in z_i which is not the case for similarity effects! –, this will change the location of the maximum but not the unimodal shape of the function.) This can also be regarded as negative feedback, or a self-correcting mechanism: when z_i increases, the further push toward higher values of z_i will become smaller and when z_i decreases, the further push toward lower values of z_i will become smaller. On the other hand, when the coefficient β_2^Z is positive, the feedback will be positive, so that changes in z_i are self-reinforcing. This can be an indication of addictive behavior.
- 3. The average similarity effect, expressing the preference of actors to being similar with respect to Z to their alters, where the total influence of the alters is the same regardless of the number of alters.
- 4. The total similarity effect, expressing the preference of actors to being similar to their alters, where the total influence of the alters is proportional to the number of alters.

- 5. The average alter effect, expressing that actors whose alters have a higher average value of the behavior Z, also have themselves a stronger tendency toward high values on the behavior.
- 6. The indegree effect, expressing that actors with a higher indegree (more 'popular' actors) have a stronger tendency toward high values on the behavior.
- 7. The outdegree effect, expressing that actors with a higher outdegree (more 'active' actors) have a stronger tendency toward high values on the behavior.

Effects 1 and 2 will practically always have to be included as control variables. (For dependent behavior variables with 2 categories, this applies only to effect 1.) When the behavior dynamics is not smooth over the observation waves — meaning that the pattern of steps up and down, as reported in the initial part of the output file under the heading *Initial data description – Dependent actor variables – Changes*, is very irregular across the observation periods — it can be important to include effects of time variables on the behavior. Time variables are changing actor covariates that depend only on the observation number and not on the actors. E.g., they could be dummy variables, being 1 for one or some observations, and 0 for the other observations.

The average similarity, total similarity, and average alter effects are different specifications of social influence. The choice between them will be made on theoretical grounds and/or on the basis of statistical significance.

For each actor-dependent covariate as well as for each of the other dependent behavior variables, the effects on Z which can be included are the following.

- 1. The main effect: a positive value implies that actors with a higher value on the covariate will have a stronger tendency toward high Z values.
- 2. An interaction effect, which is a choice among three, dependent on the internal parameter for this effect:

value 1: interaction of actor variable with average similarity; value 2: interaction of actor variable with total similarity; value 3: interaction of actor variable with average alter. See Section 15.2.1.

3. Interactions between two or three actor variables, see Section 6.6.

6.4 Exponential Random Graph Models

For the non-longitudinal ('ERGM') case, default advice is given in Snijders et al. (2006) and in Robins et al. (2007). The basic structural part of the model is comprised, for directed networks, by the following effects. For their mathematical definition, see Section 15.3.

- 1. The reciprocity effect.
- 2. The alternating k-out-stars effect to represent the distribution of the out-degrees.
- 3. The alternating k-in-stars effect to represent the distribution of the in-degrees.
- 4. For alternating transitive k-triangles effect to represent the tendency to transitivity.
- 5. The alternating independent two-paths effect to represent the preconditions for transitivity, or alternatively, the association between in-degrees and out-degrees.
- 6. The number of three-cycles to represent cyclicity, or generalized reciprocity, or the converse of hierarchy.

For nondirected networks. the basic structural part is comprised by the following, smaller, set.

- 1. The alternating k-stars effect to represent the distribution of the degrees.
- 2. For alternating transitive k-triangles effect to represent the tendency to transitivity.
- 3. The alternating independent two-paths effect to represent the preconditions for transitivity, or alternatively, the association between in-degrees and out-degrees.

Other effects can be added to improve the fit.

To obtain good convergence results for the ERGM case, it will usually be necessary to increase the default value of the multiplication factor; see Sections 3.2 and 11.

6.4.1 Effects associated with covariates for ERGMs

For each individual covariate, there are several effects which can be included in a model specification for an Exponential Random Graph Model. The main of these are the following.

- 1. the covariate-similarity effect; a positive parameter implies that actors prefer ties to others with similar values on this variable thus contributing to the network-autocorrelation of this variable;
- 2. the effect on the actor's activity (covariate-ego); a positive parameter will imply the tendency that actors with higher values on this covariate have higher out-degrees;
- 3. the effect on the actor's popularity to other actors (covariate-alter); a positive parameter will imply the tendency that actors with higher values on this covariate have higher in-degrees;
- 4. the 'same covariate', or covariate identity, effect, which expresses the tendency of the actors to be tied to others with exactly the same value on the covariate; whereas the preceding four effects are appropriate for interval scaled covariates (and mostly also for ordinal variables), the identity effect is suitable for categorical variables.

For each dyadic covariate, the following network evaluation effects can be included in the ERG model:

1. main effect of the dyadic covariate.

6.5 Model Type

When the data is perfectly symmetric, this will be detected by SIENA. Then the analysis options for nondirected networks will be followed.

6.5.1 Model Type: directed networks

For directed networks, the Model Type distinguishes between the model of Snijders (2001) (Model Type 1), that of Snijders (2003) (Model Type 2), and the tie-based model described in Snijders (2006) (Model Type 3). Model Type 1 is the default model and is described in the basic publications on Stochastic Actor-Oriented Models for network dynamics.

Model type 2 is at this moment not implemented in SIENA version 3.

In Model Type 2, the 'decisions' by the actors consist of two steps: first a step to increase or decrease their out-degree; when this step has been taken, the selection of the other actor towards whom a new tie is extended (if the out-degree rises) or from a an existing tie is withdrawn (if the out-degree drops). The decision by an actor to increase or decrease the number of outgoing ties

is determined on the basis of only the current degree; the probabilities of increasing or decreasing the out-degree are expressed by the distributional tendency function ξ (indicated in the output as xi) and the volatility function ν (indicated as nu). Which new tie to create, or which existing tie to withdraw, depends in the usual way on the evaluation and endowment functions. Thus, the outdegree distribution is governed by parameters that are not connected to the parameters for the structural dynamics. The use of such an approach in statistical modeling minimizes the influence of the observed degrees on the conclusions about the structural aspects of the network dynamics. This is further explained in Snijders (2003).

For Model Type 2, in the rate function, effects connected to these functions ξ and ν are included. On the other hand, effects in the evaluation function that depend only on the outdegrees are canceled from the model specification, because they are not meaningful in Model Type 2. To evaluate whether Model Type 1 or Model Type 2 gives a better fit to the observed degree distribution, the output gives a comparison between the observed out-degrees and the fitted distribution of the out-degrees (as exhibited by the simulated out-degrees). For Model Type 2 this comparison is always given. For Model Type 1, this comparison is given by adding 10 to the Model Code in the advanced options. (For LATEX users: the log file contains code that can be used to make a graph of the type given in Snijders, 2003).

For using Model Type 2, it is advised to first estimate some model according to Model Type 1 (this may be a simple model containing a reciprocity effect only, but it could also include more effects), and then – using the parameters estimated under Model Type 1 – change the specification to Model Type 2, and use the unconditional estimation method (see Section 7.5.4) (instead of the conditional method which is the default). It is likely that the very first model estimated under Model Type 2 will have results with poor convergence properties, but in such cases it is advised just to estimate the same model another time, now using the parameter values obtained under the previous Model Type 2 run as the initial values for the estimation.

To obtain a good model specification with respect to the rates of change in dependence of the out-degrees, three effects can be included:

- 1. the out-degrees effect
- 2. the factorial out-degree effect
- 3. the logarithmic out-degree effect.

These are the effects defined in formula (18) of Snijders (2003b) and indicated with the parameters α_1 , α_2 , and α_3 , respectively. The user has to see from the estimation results which, or which two, out of these effects should be included to yield a good fit for the out-degrees.

In addition these types, there is Model Type 6 which implements the reciprocity model of Wasserman (1979) and Leenders (1995) (also see Snijders, 1999, 2005) — provided that no other effects are chosen than the outdegree effect, the reciprocity effect and perhaps the reciprocity endowment effect, and possible also effects of actor covariates or dyadic covariates. This model is meaningful only as a "straw man" model to provide a test of the null hypothesis that the dynamics of the dyads are mutually independent, against the alternative hypothesis that there do exist network effects (which make the dyad processes mutually dependent). For this purpose, Model Type 6 can be chosen, while for one or more network effects such as the effects representing transitivity, the null hypothesis is tested that their coefficients are zero (see Section 9).

The Model Type is specified in the model options as (part of) the Model Code.

6.5.2 Model Type: non-directed networks

Non-directed networks are an undocumented option (there currently only is the presentation Snijders 2007), and therefore mentioned here reluctantly for those users who want to use this option

anyway.

SIENA detects automatically when the networks all are non-directed, and then employs a model for this special case. For non-directed networks, the Model Type has seven possible values, as described in Snijders (2007).

1. Forcing model:

one actor takes the initiative and unilaterally imposes that a tie is created or dissolved.

2. Unilateral initiative and reciprocal confirmation:

one actor takes the initiative and proposes a new tie or dissolves an existing tie; if the actor proposes a new tie, the other has to confirm, otherwise the tie is not created; for dissolution, confirmation is not required.

3. Tie-based model:

a random pair of actors is chosen (actor-specific rate functions are not used here), and the average change in objective function (1) for toggling (i, j) and (j, i) is the log-odds of the probability of changing the tie variable.

- 4. Pairwise conjunctive model: a pair of actors is chosen and reconsider whether a tie will exist between them; the tie will exist if both agree, it will not exist if at least one does not choose for it.
- 5. Pairwise disjunctive (forcing) model: a pair of actors is chosen and reconsider whether a tie will exist between them; the tie will exist if at least one of them chooses for the tie, it will not exist if both do not want it.
- 6. Pairwise compensatory (additive) model: a pair of actors is chosen and reconsider whether a tie will exist between them; this is based on the sum of their utilities for the existence of this tie.

In Models 1-2, where the initiative is one-sided, the rate function is comparable to the rate function in directed models. In Models 4-6, however, the pair of actors is chosen at a rate which is the *product* of the rate functions λ_i and λ_j for the two actors. This means that opportunities for change of the single tie variable x_{ij} occur at the rate $\lambda_i \times \lambda_j$. The numerical interpretation is different from that in Models 1-2.

6.6 Additional interaction effects

It is possible for the user to define additional interaction effects for the network and the behavior. This applies both to longitudinal and non-longitudinal (ERG) modeling. The basis is provided by the initial definition, by SIENA, of "unspecified interaction effects". Modifying the internal effect parameters of these effects allows the definition of two-way or three-way interactions.

6.6.1 Interaction effects for network dynamics

The following kinds of user-defined interactions are possible for the network dynamics.

- For *longitudinal models*:
 - a. Ego effects of actor variables can interact with all effects.

b. Further, interaction effects are permitted which are combinations of actor variables, dyadic variables, and reciprocity.

c. The transitive triplets effect can interact with the reciprocity effect in two ways, in four ways with similarity between actor variables, and in four ways with dyadic covariates.

d. The transitive triplets, 3-cycles, and transitive ties effects can be restricted to triplets having the same value on an actor covariate; or triplets in which all pairs have the value 1 on a dyadic covariate.

• For *non-longitudinal* (ERG) models:

Actor covariates and dyadic covariates can interact with each other and with reciprocity.

The specification is made by changing the *internal effect parameter* for the interaction effects. The values of these internal parameters can be changed in the *pname.mo* file described in Section 21.2.1. In StOCNET, they are accessible as the "par." column in the Advanced Options screen of the Model Specification.

For interaction effects, this parameter represents a code for the two or three interacting effects. Each effect is represented by its index number in three digits (including leading zeros) as reported in the file called *pname*.eff (recall that *pname* stands for your project name). Thus, two-way interactions are represented by twice three digits: e.g., the code 020003 refers to the interaction between the effects numbered 20 and 3, where the numbers are the rank numbers in this list of all effects. Leading zeros of the total parameter can be skipped, so that the code 020003 can also be represented by 20003 (but not by 203!). The order does not matter, so that the codes 020003, 003020, 20003, and 3020 all are equivalent. Three-way interactions similarly are represented by thrice three digits. For example, the code 020028003 represents the interaction effects between the effects numbered 20, 28, and 3. E.g., to implement the interaction effect between the effects numbered 20 and 3, in the *pname*.mo file the lines that initially are

```
unspecified interaction effect
```

0	0	0	0	0.000000 0
0	0	0	0	0.000000 0
0	0	0	0	0.000000 0

must be changed into

unspecified interaction effect 0 0 0 0 0 0.000000 020003 0 0 0 0 0 0.000000 0 0 0 0 0 0 0.000000 0

After this is done, SIENA will automatically replace the name by the suitable interaction effect name. (This is done by Siena04; and also when successfully exiting Siena03 and Siena07.)

One of the uses of interaction effects is non-homogeneity in time: interactions with an actor variable that depends only on time (the observation number).

For example, if there are four observations, two cumulative dummy variables could be used for the periods, defined by one data file with all rows equal to

0 1 1 1

and another data file with all rows equal to

0 0 1 1

where these data files are used as changing actor covariates, called, e.g., dum1 and dum2. For the detailed interpretation, it is important to realize that all covariates are centered internally in SIENA. To avoid misinterpretation, one can look up in the file *pname.dac* (see Section 21.3) what are the values used by SIENA. If there are no other time-changing actor covariates, then in this example all lines in the .dac files are

-0.6667 0.3333 0.3333 -0.3333 -0.3333 0.6667

Now suppose that the model specification includes a parameter β_r for reciprocity, β_{r1} for the

interaction of reciprocity with dummy variable dum1, and β_{r2} for the interaction of reciprocity with dum2. Then the total effect of reciprocity is given by

 $\beta_r + \beta_{r1} \operatorname{dum1} + \beta_{r2} \operatorname{dum2}$;

this is then equal to $\beta_r - 0.6667\beta_{r1} - 0.3333\beta_{r2}$ for period 1 (from observation 1 to observation 2), $\beta_r + 0.3333\beta_{r1} - 0.3333\beta_{r2}$ for period 2, and $\beta_r + 0.3333\beta_{r1} + 0.6667\beta_{r2}$ for period 3.

The *.dac file made internally by Siena stores the values of time-changing covariates (see Siena manual section 21.3). Here the variables have been centered, and they are as used internally by Siena. For four observations / three periods, the number of elements of each row is three times the number of changing actor covariates. The order of the variables is the same as the order in the output file (initial section). This means that, if for the network process, the interaction with dum1, and the interaction with dum2, I get parameter estimates bn, bnd1, bnd2, then the resulting values for the network process are bn - 0.6667*bnd1 - 0.3333*bnd2 for period 1 bn + 0.3333*bnd1 - 0.3333*bnd2 for period 2 bn + 0.3333*bnd1 + 0.6667*bnd2 for period 3.

Interactions are also possible between reciprocity and transitive triplets. Here it must be taken into account that several ways are possible for such an interaction. The two following interactions are available.



To interpret these interactions, keep in mind that the existence of the tie $i \rightarrow j$ is the 'dependent variable'. The usual condition for this tie in the transitive triplets effect is the number of two-paths $i \rightarrow h \rightarrow j$. For the type 1 interaction, this condition is extended with the extra requirement that the 'dependent' tie is already reciprocated, i.e., there already is the tie $j \rightarrow i$. For the type 2 interaction, the condition on each two-path is extended with the extra requirement that the two-path is reciprocated, i.e., the two-path is extended with the extra requirement that the two-path is reciprocated, i.e., the two-path $j \rightarrow h \rightarrow i$ also exists. To specify these interaction effects, simply change the internal effect parameter into 1002003 or 2002003, respectively.



In addition, interaction effects can be specified between transitive triplets and (1) similarity between actor variables and (2) dyadic covariates. Here the variables are represented by the number 001 for the first variable, 002 for the second variable, etc., so the numbers are not from the *pname.eff* file but just the order in which the variables occur in all of the files. Generally, def stands for the 3-digit representation (with leading zeros) of the number of the actor variable and also the number of the dyadic covariate; these sets of variables are numbered separately, so the first dyadic covariate is represented also by 001.

The parameter 1003def, 2003def, 3003def, and 4003def, specifies transitive triplet effects where the transitive triplet is weighted by the similarity between two actors on actor variable number def: for code 1003def between actors i and j, for 2003def between actors i and h, for 3003def between actors j and h, and for 4003def by the product of the similarity between actors iand h and the similarity between actors j and h. Analogously, the parameter 11003def, 12003def, 13003def, and 14003def, specifies transitive triplet effects where the transitive triplet is weighted by dyadic variable number def: for code 11003def for actors i and j, for 12003def for actors i and h, and for 13003def for actors j and h. The dyadic variables here are not centered!! For example, for the first actor variable, code 1003001 will define the transitive triplets effect weighted by the similarity between actors i and j on the first actor variable.

Further, three triadic effects: transitive triplets, 3-cycles, and transitive ties, can be restricted to triplets which all have the same value of an actor variable; or triplets in which all pairs have the value 1 on a dyadic covariate. This is achieved by the following codes:

- 8003def : transitive triplets restricted to triplets of actors having the same value on actor covariate number def;
- 8005def : 3-cycles restricted to triplets of actors having the same value on actor covariate number def;
- 8006def : transitive ties restricted to triplets of actors having the same value on actor covariate
 number def;
- 18003def : transitive triplets restricted to triplets of actors where all pairs have the value 1 on dyadic covariate number def;
- 18005def : 3-cycles restricted to triplets of actors where all pairs have the value 1 on dyadic covariate number def;
- 18006def : transitive ties restricted to triplets of actors where all pairs have the value 1 on dyadic covariate number def.

The calculation of user-defined effects is slightly more time-consuming than the calculation of internally defined effects. Therefore, when there is the choice between two equivalent effects – e.g., in longitudinal modeling, interactions of actor covariates with reciprocity – it is advisable to use the predefined interaction effects.

6.6.2 Interaction effects for behavior dynamics

For behavior dynamics, interaction effects can be defined by the user, for each dependent behavior variable separately, as interactions of two or three actor variables. The actor variables (changing and non-changing) are numbered in the order in which they appear in the *pname.mo* file: first the dependent variables, then the non-changing actor variables, then the other changing actor variables. For the internal effect parameter defining the interaction effect, each actor variable is represented by its index number in this order, in two digits (including leading zero if the number is less than 10). E.g., the parameter 0203 represents the interaction between variables number 2 and number 3, and parameter 010404 represents the interaction between variables 1, 4 and 4. The interactions are represented by products of the centered variables.

In addition, there are interactions available between actor variables and influence, as described in Section 15.2.1.

6.6.3 Interaction effects in the ERGM case

In the ERGM case, covariates can interact with each other and also with reciprocity. In the case that there are structurally determined values, variables that are constant within connected components can interact with any other effects. Other interaction effects are currently not supported.

Interaction effects in the ERGM case are defined simply by the change statistic given by the product of the change statistics of the two or three components. This does not work for interactions between two or more structural effects, and only to a limited extent for interactions involving covariates. The reason is that the product of two change statistics is not necessarily a change statistic again.

In cases where this simple rule does not apply, the program will run but an error message will be given only at the end of the estimations or simulations. The error message includes the diagnosis 'An incorrect change statistic has been used'. The results then are meaningless.

6.7 Random effects models: unobserved actor heterogeneity

The network (and behavior) evolution may be affected by the fact that actors are heterogeneous. If all relevant actor heterogeneity is observed in the form of actor covariates, then actor heterogeneity can be taken into account by including covariates in the model. If not all relevant actor heterogeneity is observed, then more complex models are required, such as random effects models. Random effects models (see Schweinberger and Snijders, 2007a) allow to take unobserved actor heterogeneity into account by assuming that the network (and behavior) evolution is affected by unobserved outcomes of actor-dependent random variables (random effects), which represent the combined effect of the unobserved actor heterogeneity on the network (and behavior) evolution.

These models can be estimated only using the maximum likelihood estimation option, see Sections 7 and 12. The maximum likelihood estimation of random effects models requires MCMCbased data imputation of the unobserved random effects (which can be regarded as missing data). SIENA supplies three alternative MCMC algorithms for the MCMC-based data imputation of the random effects:

(1) random walk M-H,

- (2) autoregressive M-H,
- (3) independence sampler (default).

The algorithms require the determination of the scale factor of the so-called proposal distribution, which may affect the efficiency of the algorithms and the accuracy of the results (for a given number of iterations). It is recommended to choose the default algorithm (3); and to choose as scale factor of the proposal distribution 0—which would be a pointless scale factor, but which communicates to SIENA that the user wishes to leave the determination of the scale factor to the defaults within the algorithm, which features an adaptive method for determining suitable scale factors. In the ideal case, the choice of algorithm does not affect the parameter estimates—though the efficiency of the algorithms and the accuracy of the results (for a given number of iterations) may be affected.

The estimation of random effects models (i.e., the estimation of the parameters, including the variances of the random effects) may be done by either Maximum Likelihood or Bayesian estimation (see Section 7.3).

The interpretation of the parameter estimates is straightforward; the estimates of the variances of the random effects indicate the magnitude of the unobserved actor heterogeneity.

7 Estimation

The model parameters are estimated under the specification given during the model specification part, using a stochastic approximation algorithm. Three estimation procedures are implemented: the Method of Moments (MoM) (Snijders, 2001; Snijders, Steglich, and Schweinberger, 2007); the Method of Maximum Likelihood (ML) (Snijders, Koskinen and Schweinberger, 2010); and a Bayesian method (Koskinen, 2005; Koskinen and Snijders, 2007; Schweinberger and Snijders, 2007). For non-constant rate functions, currently only MoM estimation is available. The Method of Moments is the default; the other two methods require much more computing time. Given the greater efficiency but longer required computing time for the ML and Bayesian methods, these can be useful especially for smaller data sets and relatively complicated models (networks and behavior; endowment effects).

In the following, the number of parameters is denoted by p. The algorithms are based on repeated (and repeated, and repeated...) simulation of the evolution process of the network. These repetitions are called 'runs' in the following. The MoM estimation algorithm is based on comparing the observed network (obtained from the data files) to the hypothetical networks generated in the simulations.

Note that the estimation algorithm is of a stochastic nature, so the results can vary! This is of course not what you would like. For well-fitting combinations of data set and model, the estimation results obtained in different trials will be very similar. It is good to repeat the estimation process at least once for the models that are to be reported in papers or presentations, to confirm that what you report is a stable result of the algorithm.

The initial value of the parameters normally is the current value (that is, the value that the parameters have immediately before you start the estimation process); as an alternative, it is possible to start instead with a standard initial value. Usually, a sequence of models can be fitted without problems, each using the previously obtained estimate as the starting point for the new estimation procedure. Sometimes, however, problems may occur during the estimation process, which will be indicated by some kind of warning in the output file or by parameter estimates being outside a reasonably expected range. In such cases the current parameter estimates may be unsatisfactory, and using them as initial values for the new estimation process might again lead to difficulties in estimation. Therefore, when the current parameter values are unlikely and also when they were obtained after a divergent estimation algorithm, it is advisable to start the estimation algorithm with a *standard initial value*. The use of standard initial values is one of the model options. If this has successfully led to a model with convergent parameter estimates and model fitting is continued, then the option can be reset to the current initial values.

7.1 Algorithm

During the estimation process, StOCNET transfers control to the SIENA program. The estimation algorithm is an implementation of the Robbins-Monro (1951) algorithm, described in Snijders (2001, 2002), and has for both the MoM and ML method three phases:

- 1. In phase 1, the parameter vector is held constant at its initial value. This phase is for having a first rough estimate of the matrix of derivatives.
- 2. Phase 2 consists of several subphases. More subphases means a greater precision. The default number of subphases is 4. The parameter values change from run to run, reflecting the deviations between generated and observed values of the statistics. The changes in the parameter values are smaller in the later subphases.

The program searches for parameter values where these deviations average out to 0. This is reflected by what is called the 'quasi-autocorrelations' in the output screen. These are averages of products of successively generated deviations between generated and observed statistics. It is a good sign for the convergence of the process when the quasi-autocorrelations are negative (or positive but close to 0), because this means the generated values are jumping around the observed values.

3. In phase 3, the parameter vector is held constant again, now at its final value. This phase is for estimating the covariance matrix and the matrix of derivatives used for the computation of standard errors.

The default number of runs in phase 3 is 1000. This requires a lot of computing time, but when the number of phase 3 runs is too low, the standard errors computed are rather unreliable.

The number of subphases in phase 2, and the number of runs in phase 3, can be changed in the model options.

There is an additional possibility (since version 3.181) of requesting extra estimation runs if the maximum *t*-ratio for convergence is higher than 0.15. This is obtained formally by specifying more than 10 subphases; the maximum additional number of repeated estimation runs then is the rounded value of number of subphases divided by 10; the real number of subphases is the remainder in this division.

The user can break in and modify the estimation process in three ways:

- 1. it is possible to terminate the estimation;
- 2. in phase 2, it is possible to terminate phase 2 and continue with phase 3;
- 3. in addition, it is possible to change the current parameter values and restart the whole estimation process.

For the ML estimation option and for the non-longitudinal case, tuning the 'multiplicaton factor' and the 'initial gain parameter' can be important for getting good results; for Bayesian estimation the 'multiplicaton factor' can likewise be important; this is briefly described in Section 3.2.

7.2 Output

There are three output files. All are ASCII ('text') files which can be read by any text editor. The main output is given in the *pname.out* file (recall that **pname** is the project name defined by the user). A brief history of what the program does is written to the file *pname.log*. The latter file also contains some supplementary output that usually is disregarded but sometimes is helpful. Some diagnostic output containing a history of the estimation algorithm which may be informative when there are convergence problems is written to the file *pname.cck* (for 'check'). This file is overwritten for each new estimation. Normally, you only need to look at *pname.out*.

The output is divided into sections indicated by a line @1, subsections indicated by a line @2, subsubsections indicated by @3, etc. For getting the main structure of the output, it is convenient to have a look at the @1 marks first.

The primary information in the output of the estimation process consists of the following three parts. Results are presented here which correspond to Table 2, column " t_1 , t_3 " of Snijders (2001). The results were obtained in an independent repetition of the estimation for this data set and this model specification; since the repetition was independent, the results are slightly different, illustrating the stochastic nature of the estimation algorithm.

1. Convergence check

In the first place, a convergence check is given, based on Phase 3 of the algorithm. This check considers the deviations between simulated values of the statistics and their observed values (the latter are called the 'targets'). Ideally, these deviations should be 0. Because of the stochastic nature of the algorithm, when the process has properly converged the deviations are small but not exactly equal to 0. The program calculates the averages and standard deviations of the deviations and combines these in a *t*-ratios (in this case, average divided by standard deviation). For longitudinal modeling, convergence is excellent when these *t*-ratios are less than 0.1 in absolute value, good when they are less than 0.2, and moderate when they are less than 0.3. For published results, it is suggested that estimates presented come from runs in which all *t*-ratios for convergence are less than 0.1 in absolute value – or nearly so. (These bounds are indications only, and are not meant as exact limitations.) The corresponding part of the output is the following.

Total of 1954 iterations. Parameter estimates based on 954 iterations, basic rate parameter as well as convergence diagnostics, covariance and derivative matrices based on 1000 iterations. Information for convergence diagnosis. Averages, standard deviations, and t-ratios for deviations from targets: 1. -0.236 7.006 -0.0342. 0.204 7.059 0.029 22.242 3. -1.592-0.072

Good convergence is indicated by the t-ratios being close to zero.

In this case, the t-ratios are -0.034, -0.029, and -0.072, which is less than 0.1 in absolute value, so the convergence is excellent. In data exploration, if one or more of these t-ratios are larger in absolute value than 0.3, it is advisable to restart the estimation process. For results that are to be reported, it is advisable to carry out a new estimation when one or more of the t-ratios are larger in absolute value than 0.1. Large values of the averages and standard deviations are in themselves not at all a reason for concern.

For the exponential random graph (or p^*) model, the convergence of the algorithm is more problematic than for longitudinal modeling. A sharper value of the *t*-ratios must be found before the user may be convinced of good convergence. It is advisable to try and obtain *t*-values which are less than 0.15. If, even with repeated trials, the algorithm does not succeed in producing *t*-values less than 0.15, then the estimation results are of doubtful value.

2. Parameter values and standard errors

The next crucial part of the output is the list of estimates and standard errors. For this data set and model specification, the following result was obtained.

©3 Estimates and standard errors

0. Rat	e parameter	5.4292	(0.6920)
Other p	arameters:			
1. eva	1: outdegree (density)	-0.7648	(0.2957)
2. eva	1: reciprocity	2.3071	(0.5319)
3. eva	1: number of actors at distance 2	-0.5923	(0.1407)

The rate parameter is the parameter called ρ in section 15.1.3 below. The value 5.4292 indicates that the estimated number of changes per actor (i.e., changes in the choices made by this actor, as reflected in the row for this actor in the adjacency matrix) between the two observations is 5.43 (rounded in view of the standard error 0.69). Note that this refers to unobserved changes, and that some of these changes may cancel (make a new choice and then withdraw it again), so the average
observed number of differences per actor will be somewhat smaller than this estimated number of unobserved changes.

The other three parameters are the weights in the evaluation function. The terms in the evaluation function in this model specification are the out-degree effect defined as s_{i1} in Section 15.1.1, the reciprocity effect s_{i2} , and the number of distances 2 (indirect relations) effect, defined as s_{i5} . Therefore the estimated evaluation function here is

$$-0.76 s_{i1}(x) + 2.31 s_{i2}(x) - 0.59 s_{i5}(x) .$$

The standard errors can be used to test the parameters. For the rate parameter, testing the hypothesis that it is 0 is meaningless because the fact that there are differences between the two observed networks implies that the rate of change must be positive. The weights in the evaluation function can be tested by t-statistics, defined as estimate divided by its standard error. (Do not confuse this t-test with the t-ratio for checking convergence; these are completely different although both are t ratios!) Here the t-values are, respectively, -0.7648/0.2957 = -2.59, 2.3071/0.5319 = 4.34, and -0.5923/0.1407 = -4.21. Since these are larger than 2 in absolute value, all are significant at the 0.05 significance level. It follows that there is evidence that the actors have a preference for reciprocal relations and for networks with a small number of other actors at a distance 2. The value of the density parameter is not very important; it is important that this parameter is included to control for the density in the network, but as all other statistics are correlated with the density, the density is difficult to interpret by itself.

When for some effects the parameter estimate as well as the standard error are quite large, say, when both are more than 2, and certainly when both are more than 5, then it is possible that this indicates poor convergence of the algorithm: in particular, it is possible that the effect in question does have to be included in the model to have a good fit, but the precise parameter value is poorly defined (hence the large standard error) and the significance of the effect cannot be tested with the *t*-ratio. This can be explored by estimating the model without this parameter, and also with this parameter fixed at some large value (see section 13.1) – whether the value is large positive or large negative depends on the direction of the effect. For the results of both model fits, it is advisable to check the fit by simulating the resulting model and considering the statistic corresponding to this particular parameter. (The indicative sizes of 2 and 5 result from experience with network effects and with effects of covariates on usual scales with standard deviations ranging between, say, 0.4 and 2. These numbers have to be modified for covariates with different standard errors.)

3. Collinearity check

After the parameter estimates, the covariance matrix of the estimates is presented. In this case it is

Covariance matrix of estimates (correlations below diagonal): 0.087 -0.036 0.003 -0.230 0.283 -0.033

0.078	-0.440	0.020

The diagonal values are the variances, i.e., the squares of the standard errors (e.g., 0.087 is the square of 0.2957). Below the diagonal are the correlations. E.g., the correlation between the estimated density effect and the estimated reciprocity effect is -0.230. These correlations can be used to see whether there is an important degree of collinearity between the effects. Collinearity means that several different combinations of parameter values could represent the same data pattern, in this case, the same values of the network statistics. When one or more of the correlations are very close to -1.0 or +1.0, this is a sign of near collinearity. This will also lead to large standard errors of those parameters. It is then advisable to omit one of the corresponding effects from the model,

because it may be redundant given the other (strongly correlated) effect. It is possible that the standard error of the retained effect becomes much smaller by omitting the other effect, which can also mean a change of the *t*-test from non-significance to significance.

However, correlations between parameter estimates close to -1.0 or +1.0 should not be used too soon in themselves as reasons to exclude effects from a model. This is for two reasons. In the first place, network statistics often are highly correlated (for example, total number of ties and number of transitive triplets) and these correlations just are one of the properties of networks. Second, near collinearity is not a problem in itself, but the problem (if any) arises when standard errors are high, which may occur because the value of the parameters of highly correlated variables is very hard to estimate with any precision. The problem resides in the large standard errors, not in itself in the strong correlation between the parameter estimates. If for both parameters the ratio of parameter estimate to standard error, i.e., the *t*-ratio, is larger than 2 in absolute value, in spite of the high correlations between the parameter estimates, then the significance of the *t*-test is evidence anyway that both effects merit to be included in the model. In other words, in terms of the 'signal-to-noise ratio': the random noise is high but the signal is strong enough that it overcomes the noise.

As a rule of thumb for parameter correlations, usually for correlations of estimated structural network effects there is no reason for concern even when these correlations are as strong as .9.

7.3 Maximum Likelihood and Bayesian estimation

SIENA can estimate models by three estimation methods: the (unconditional or conditional) Method of Moments ('MoM', the default; Snijders, 2001; Snijders, Steglich, and Schweinberger 2007), the Maximum Likelihood method ('ML', see Snijders, Koskinen, and Schweinberger, 2010), and Bayesian methods (see Koskinen, 2005; Koskinen and Snijders, 2007; Schweinberger and Snijders, 2007b). In nice situations (relatively small and large network data sets, and large network and behavior data sets), the three methods tend to agree and there seems not to be no reason to use the more time-consuming ML or Bayesian methods. In not-so-nice situations (very small network data sets, small network and behavior data sets in combination with complex models), however, ML and Bayesian methods tend to produce more accurate results than MoM. Statistical theory suggests that ML is a more efficient estimation method than MoM in the sense of producing estimates with smaller standard errors. But in the 'nice situations' the efficiency advantage of ML is very small. Bayesian estimation is based on a different statistical paradigm, and assumes and requires that the uncertainty about parameters is expressed itself in a probability distribution.

SIENA supplies three alternative MCMC algorithms for the Bayesian estimation of the objective function parameters:

- (1) random walk M-H (default),
- (2) autoregressive M-H,
- (3) independence sampler.

The algorithms require the determination of the scale factor of the so-called proposal distribution, which may affect the efficiency of the algorithms and the accuracy of the results (for a given number of iterations). It is recommended to make a short run with the default algorithm (1), and then to make a longer run with algorithm (3); and to choose as scale factor of the proposal distribution 0—which would be a pointless scale factor, but which communicates to SIENA that the user wishes to leave the determination of the scale factor to the defaults provided in the algorithm, which features an adaptive method for determining suitable scale factors. In the ideal case, the choice of algorithm does not affect the results of primary interest, the parameter estimates—though the efficiency of the algorithms and the accuracy of the results (for a given number of iterations) may be affected.

Bayesian estimation gives rise to more results than the parameter estimates printed in the output file. The additional results can be best inspected by using R and the R function siena_bayes written by Michael Schweinberger (see Section 7.4).

7.4 Supplementing R functions

To examine the MCMC output of SIENA for Maximum Likelihood (ML) and Bayesian estimation, the R functions siena_mle and siena_bayes can be used, respectively, which were programmed by Michael Schweinberger. The R functions input files generated by Siena and output, among other things, trace plots and MCMC lag 1,..., 100 autocorrelations of sampled entities (see Schweinberger and Snijders, 2007a,b), and, in the Bayesian case, in addition 95% posterior intervals, histograms, and Gaussian kernel density estimates of the marginal posterior densities of the parameters.

The R functions siena_mle and siena_bayes can be downloaded from the website http://stat.gamma.rug.nl/stocnet, and can be used in R as follows:

- (1) Load the **R** function:
 - ML estimation: source("siena_mle.r").
 - Bayesian estimation: source("siena_bayes.r").

(2) Call the R function:

- ML estimation: siena_mle(project_name, full_output, no_random_effects, no_actors).
- Bayesian estimation: siena_bayes(project_name, full_output, no_random_effects, no_actors).

The arguments are:

- project_name (string): the name of the Siena project that is to be examined; note that calling siena_mle or siena_bayes presumes that Siena carried out ML or Bayesian estimation of the project project_name, respectively.
- full_output (0 or 1): 1 indicates that the full output is desired, while 0 indicates that selected output is desired.
- no_random_effects (non-negative integer): the number of actor-dependent weights (parameters) in the model.
- no_actors (positive integer): the number of actors.

Examples are provided by siena_mle("alcohol", 1, 3, 50) and siena_bayes("alcohol", 1, 3, 50).

7.5 Other remarks about the estimation algorithm

7.5.1 Changing initial parameter values for estimation

When you wish to change initial parameter values for running a new estimation procedure, this can be done in StOCNET as one of the model options. It can also be done by 'breaking in' into the SIENA program.

7.5.2 Fixing parameters

Sometimes an effect must be present in the model, but its precise numerical value is not welldetermined. E.g., if the network at time t_2 would contain only reciprocated choices, then the model should contain a large positive reciprocity effect but whether it has the value 3 or 5 or 10 does not make a difference. This will be reflected in the estimation process by a large estimated value and a large standard error, a derivative which is close to 0, and sometimes also by lack of convergence of the algorithm. (This type of problem also occurs in maximum likelihood estimation for logistic regression and certain other generalized linear models; see Geyer and Thompson (1992, Section 1.6), Albert and Anderson (1984), Hauck and Donner (1978).) In such cases this effect should be fixed to some large value and not left free to be estimated. This can be specified in the model specification under the Advanced button. As another example, when the network observations are such that ties are formed but not dissolved (some entries of the adjacency matrix change from 0 to 1, but none or hardly any change from 1 to 0), then it is possible that the density parameter must be fixed at some high positive value.

7.5.3 Automatic fixing of parameters

If the algorithm encounters computational problems, sometimes it tries to solve them automatically by fixing one (or more) of the parameters. This will be noticeable because a parameter is reported in the output as being fixed without your having requested this. This automatic fixing procedure is used, when in phase 1 one of the generated statistics seems to be insensitive to changes in the corresponding parameter.

This is a sign that there is little information in the data about the precise value of this parameter, when considering the neighborhood of the initial parameter values. However, it is possible that the problem is not in the parameter that is being fixed, but is caused by an incorrect starting value of this parameter or one of the other parameters.

When the warning is given that the program automatically fixed one of the parameter, try to find out what is wrong.

In the first place, check that your data were entered correctly and the coding was given correctly, and then re-specify the model or restart the estimation with other (e.g., 0) parameter values. Sometimes starting from different parameter values (e.g., the default values implied by the model option of "standard initial values") will lead to a good result. Sometimes, however, it works better to delete this effect altogether from the model.

It is also possible that the parameter does need to be included in the model but its precise value is not well-determined. Then it is best to give the parameter a large (or strongly negative) value and indeed require it to be fixed (see Section 13.1).

7.5.4 Conditional and unconditional estimation

SIENA has two methods for MoM estimation and simulation: conditional and unconditional. They differ in the *stopping rule* for the simulations of the network evolution. In unconditional estimation, the simulations of the network evolution in each time period (and the co-evolution of the behavioral dimensions, if any are included) carry on until the predetermined time length (chosen as 1.0 for each time period between consecutive observation moments) has elapsed.

In conditional estimation, in each period the simulations run on until a stopping criterion is reached that is calculated from the observed data. Conditioning is possible for each of the dependent variables (network, or behavior), where 'conditional' means 'conditional on the observed number of changes on this dependent variable'.

Conditioning on the network variable means running simulations until the number of different entries between the initially observed network of this period and the simulated network is equal to the number of entries in the adjacency matrix that differ between the initially and the finally observed networks of this period.

Conditioning on a behavioral variable means running simulations until the sum of absolute score differences on the behavioral variable between the initially observed behavior of this period and the simulated behavior is equal to the sum of absolute score differences between the initially and the finally observed behavior of this period.

Conditional estimation is slightly more stable and efficient, because the corresponding rate parameters are not estimated by the Robbins Monro algorithm, so this method decreases the number of parameters estimated by this algorithm. Therefore, it is the default for models that do not include any dependent behavior variables. For models including dependent behavior variables, the default estimation type is unconditional (because in most applications, there will be no straightforward choice for the conditioning variable). The possibility to choose between unconditional and the different types of conditional estimation is one of the model options.

If there are changes in network composition (see Section 5.7), only the unconditional estimation procedure is available.

7.5.5 Required changes from conditional to unconditional estimation

Even though conditional estimation is slightly more efficient than unconditional estimation, there is one kind of problem that sometimes occurs with conditional estimation and which is not encountered by unconditional estimation.

It is possible (but luckily rare) that the initial parameter values were chosen in an unfortunate way such that the conditional simulation does not succeed in ever attaining the condition required by its stopping rule (see Section 7.5.4). The solution is either to use standard initial values or to to unconditional estimation.

8 Standard errors

The estimation of standard errors of the MoM estimates requires the estimation of derivatives, which indicate how sensitive the expected values of the statistics (see Section 7.1) are with respect to the parameters. The derivatives can be estimated by three methods:

- (0) finite differences method with common random numbers,
- (1) score function method 1 (default),
- (2) score function method 2.

Schweinberger and Snijders (2006) point out that the finite differences method is associated with a bias-variance dilemma, and proposed the unbiased and consistent score function methods. These methods demand less computation time than method (0). It is recommended to use at least 1000 iterations (default) in phase 3. For published results, it is recommended to have 2000 or 4000 iterations in phase 3.

9 Tests

Three types of tests are available in SIENA.

1. *t*-type tests of single parameters can be carried out by dividing the parameter estimate by its standard error. Under the null hypothesis that the parameter is 0, these tests have approximately a standard normal distribution.

- 2. Score-type tests of single and multiple parameters are described in the following section.
- 3. In the maximum likelihood estimation methods (both the ERGM case and the longitudinal case provided, for the latter, that the maximum likelihood option has been chosen) it is possible to request likelihood ratio tests. The log likelihood ratio is computed by bridge sampling (Gelman and Meng, 1998; Handcock and Hunter, 2006). This can be requested (a bit deviously) by the number of runs in phase 3 (defined in the specification options):
 - (a) If the number of phase 3 runs is a multiple of 100 plus 1 (e.g., 101, 501, etc.), then the log likelihood ratio is calculated comparing the estimates obtained with the standard initial values.
 - (b) If the number of phase 3 runs is a multiple of 100 plus 2 (e.g., 102, 502, etc.), then the log likelihood ratio is calculated comparing the estimates obtained with the initial values used in the current estimation procedure.

The first option will be the most frequently useful, because it yields log likelihood ratios which, for different models fitted to a given data set, all are comparable.

9.1 Score-type tests

A generalized Neyman-Rao score test is implemented for the MoM estimation method in SIENA (see Schweinberger, 2005). For the ML estimation method (including the ERGM case), following the same steps produces the Rao (1947) efficient score test.

Most goodness-of-fit tests will have the following form: some model is specified and one or more parameters are restricted to some constant, in most cases 0. Such restrictions on parameters can be imposed in the StOCNET program collection by pressing the Model specifications button on the main SIENA interface, selecting the parameter of interest, pressing the Advanced button, checking the box in the column with label t corresponding to the parameter of interest, and specifying the value to which the parameter is restricted. Outside the StOCNET program collection, parameters can be restricted by opening the *pname*.MO file, going to the parameters of interest and setting the values in the fourth column equal to 1. The goodness-of-fit test proceeds by simply estimating the restricted model (not the unrestricted model, with unrestricted parameters) by the standard SIENA estimation algorithm. No more information needs to be communicated. When the model is restricted, SIENA by default assumes that the restricted model is to be tested against the unrestricted model, and by default SIENA evaluates the generalized Neyman-Rao score test statistic.

9.2 Example: one-sided tests, two-sided tests, and one-step estimates

Suppose that it is desired to test the goodness-of-fit of the model restricted by the null hypothesis that the reciprocity parameter is zero. The following output may be obtained:

@2 Generalised score test <c> ------Testing the goodness-of-fit of the model restricted by (1) eval: reciprocity = 0.0000

42

c = 3.9982 d.f. = 1 p-value = 0.0455
one-sided (normal variate): 1.9996

One-step estimates:

1: constant network rate (period 1)	6.3840
1: constant network rate (period 2)	6.4112
eval: outdegree (density)	0.9404
eval: reciprocity	1.2567

To understand what test statistic $\langle c \rangle$ is about, consider the case where the network is observed at two time points, and let R be the number of reciprocated ties at the second time point. Then it can be shown that the test statistic is some function of

Expected R under the restricted model - observed R.

Thus, the test statistic has some appealing interpretation in terms of goodness-of-fit: when reciprocated ties do have added value for the firms—which means that the reciprocity parameter is not 0, other than the model assumes—then the deviation of the observed R from the R that is expected under the model will be large (large misfit), and so will be the value of the test statistic. Large values of the test statistic imply low p-values, which, in turn, suggests to abandon the model in favor of models incorporating reciprocity.

The null distribution of the test statistic c tends, as the number of observations increases, to the chi-square distribution, with degrees of freedom equal to the number of restricted parameters. The corresponding p-value is given in the output file.

In the present case, one parameter is restricted (reciprocity), hence there is one degree of freedom d.f. = 1. The value of the test statistic c = 3.9982 at one degree of freedom gives p = 0.0455. That is, it seems that reciprocity should be included into the model and estimated as the other parameters.

The one-sided test statistic, which can be regarded as normal variate, equals 1.9996 indicating that the value of the transitivity parameter is positive.

The one-step estimates are approximations of the unrestricted estimates (that is, the estimates that would be obtained if the model were estimated once again, but without restricting the reciprocity parameter). The one-step estimate of reciprocity, 1.2567, hints that this parameter is positive, which agrees with the one-sided test.

9.2.1 Multi-parameter tests

In the case where K > 1 model parameters are restricted, SIENA evaluates the test statistic with K degrees of freedom. A low p-value of the joint test would indicate that the goodness-of-fit of the model is intolerable. However, the joint test with K degrees of freedom gives no clue as to what parameters should be included into the model: the poor goodness-of-fit could be due to only one of the K restricted parameters, it could be due to two of the K restricted parameters, or due to all of them. Hence SIENA carries out, in addition to the joint test with K degrees of freedom, additional tests with one degree of freedom that test the single parameters one-by-one. The goodness-of-fit table looks as follows:

@2
Generalised score test <c>

Testing the goodness-of-fit of the model restricted by (1) eval: covariate_ij (centered) = 0.0000 (2)= 0.0000 eval: covariate_i alter = 0.0000 (3) eval: covariate_i similarity _____ Joint test: c = 92.5111 d.f. = 3 p-value [0.0001 (1) tested separately: _____ - two-sided: c = 62.5964 d.f. = 1 p-value [0.0001 - one-sided (normal variate): 7.9118 (2) tested separately: _____ - two-sided: c = 16.3001 d.f. = 1 p-value [0.0001 - one-sided (normal variate): 4.0373 (3) tested separately: _____ - two-sided: c = 23.4879 d.f. = 1 p-value [0.0001 - one-sided (normal variate): 4.8464 _____ One-step estimates: 1: constant network rate (period 1) 7.4022 1: constant network rate (period 2) 6.4681 eval: outdegree (density) -0.4439 eval: reciprocity 1.1826 eval: transitive triplets 0.1183 eval: covariate_ij (centered) 0.4529 eval: covariate_i alter 0.1632 eval: covariate_i similarity 0.4147

In the example output, three parameters are restricted. The joint test has test statistic c, which has under the null hypothesis a chi-squared distribution with d.f. = 3. The *p*-value corresponding to the joint test indicates that the restricted model is not tenable. Looking at the separate tests, it seems that the misfit is due to all three parameters. Thus, it is sensible to improve the goodness-of-fit of the baseline model by including all of these parameters, and estimate them.

9.2.2 Testing homogeneity assumptions

SIENA by default assumes that the parameter values are constant across actors and periods. Such assumptions are sometimes hardly credible in the light of substantive insight and empirical data, and it may be desired to test them by including suitable dummy variables. See Schweinberger (2005) for examples.

9.3 Alternative application: convergence problems

An alternative use of the score test statistic is as follows. When convergence of the estimation algorithm is doubtful, it is sensible to restrict the model to be estimated. Either "problematic" or "non-problematic" parameters can be kept constant at preliminary estimates (estimated parameters values). Though such strategies may be doubtful in at least some cases, it may be, in other cases, the only viable option besides simply abandoning "problematic" models. The test statistic can be exploited as a guide in the process of restricting and estimating models, as small values of the test statistic indicate that the imposed restriction on the parameters is not problematic.

10 Simulation

The simulation option simulates the network evolution for fixed parameter values. This is meaningful mainly at the point that you have already estimated parameters, and then either want to check again whether the statistics used for estimation have expected values very close to their observed values, or want to compute expected values of other statistics. The statistics to be simulated can be specified in a special screen in StOCNET. For running the simulations using Siena05.exe outside of StOCNET, they are specified in the file *pname*.si, as documented in Section 21.2.2.

The number of runs is set at a default value of 1,000, and can be changed in the simulation options. The user can break in and terminate the simulations early. When only 1 run is requested, an entire data set is generated and written to file in SIENA format and also in Pajek format. When exactly 10 runs are requested and the maximum likelihood option is chosen, then the sequence of changes from each observation to the next is written to file *pname*.cha in the format described in Section 23.2.

The output file contains means, variances, covariances, and correlations of the selected statistics. The output file also contains *t*-statistics for the various statistics; these can be regarded as tests for the simple null hypothesis that the model specification with the current parameter values is correct.

For simulating networks and behavior, the output includes the autocorrelation statistics known as Moran's I and Geary's c. For formulae and interpretation see, e.g., Ripley (1981, 98–99). These measure the extent to which the value of the variable in question is similar between tied actors. This similarity is expressed by relatively high values for Moran's I and by relatively low values for Geary's c. The null values, which are the expected values for variables independent of the network, are given by -1/(n-1) for Moran's I and by 1 for Geary's c.

(The output of the descriptive statistics, which can be obtained from the 'Descriptives' in StOCNET and from Siena02 outside of StOCNET, also contains Moran's I and Geary's c, computed for the observed data, together with their null means and standard deviations.)

The simulation feature can be used in the following way. Specify a model and estimate the parameters. After this estimation (supposing that it converged properly), add a number of potential effects. This number might be too large for the estimation algorithm. Therefore, do not Estimate but choose Simulate instead. The results will indicate which are the statistics for which the largest deviations (as measured by the *t*-statistics) occurred between simulated and observed values. Now go back to the model specification, and return to the specification for which the parameters were estimated earlier. The effects corresponding to the statistics with large *t*-values are candidates for now being added to the model. One should be aware, however, that such a data-driven approach leads to capitalization on chance. Since the selected effects were chosen on the basis of the large deviation between observed and expected values, the *t*-tests, based on the same data set, will tend to give significant results too easily. The tests described in Section 9 do not have this problem of chance capitalization.

The generated statistics for each run are also written to the file *pname.sdt* ('sdt' for 'simulation data'), so you can inspect them also more precisely. This file is overwritten each time you are simulating again. A brief history of what the program does is again written to the file *pname.log*.

10.1 Conditional and unconditional simulation

When doing simulations, use the same options as in the method of moments estimation (which is the default anyway), as this will produce a selection from among the statistics that are potentially used by the method of moments procedure.

The distinction between conditional and unconditional simulation also applies here; this is the same as for the estimation option of SIENA, described in Section 7.5.4. The unconditional option

simulates with a fixed time duration implying a random number of differences between consecutive observations; whereas the conditional option simulates until the same number of differences is simulated as observed in the data set.

If the conditional simulation option was chosen (which is the default) and the simulations do not succeed in achieving the condition required by its stopping rule (see Section 7.5.4), then the simulation is terminated with an error message, saying *This distance is not achieved for this parameter vector*. In this case, you are advised to change to unconditional simulation.

11 One observation: exponential random graph models

By choosing only one observation moment, the user specifies that not a model for network evolution is studied, but an exponential random graph model ('ERGM'), also called a p^* model (Frank & Strauss, 1986; Frank, 1991; Wasserman & Pattison, 1996; Snijders, Pattison, Robins, and Handcock, 2006; Snijders, 2002; Robins, Snijders, Wang, Handcock, and Pattison, 2007). In this model, the probability of observing the graph is given by (5). A good introduction to current knowledge about this model is Robins et al. (2007).

SIENA carries out Markov chain Monte Carlo (MCMC) estimation for this model, as described in Snijders (2002). This algorithm computes a Monte Carlo approximation of the maximum likelihood estimate. However, if the model specification is not well in accordance with the data set, then the algorithm will not converge properly. This is discussed in Snijders (2002) and Handcock (2002). How to specify the model is discussed in Snijders, Pattison, Robins and Handcock (2006), focusing on how to specify transitivity. To model this concept more complicated effects are required than the traditional transitive triplet count. Also when the model specification is good, however, it may require repeated SIENA runs, each using the previously obtained estimate as the new starting value, to obtain satisfactory convergence of the algorithm. This means in practice that (1) great care is required for the model specification, (2) the user may have to tune two constants for the algorithm, called the *multiplication factor* and the *initial gain parameter*, which are discussed below. In any case, it is advisable always to choose the conditional estimation/simulation option, which means here that the total number of ties is kept fixed. For unconditional estimation, the total number of ties is a random variable. The choice between these two is made in the advanced options.

If there are structural zeros (see Section 5.1.1) and the elements of the adjacency matrix that are *not* structurally determined split the network into two or more components (which will happen in the case where several smaller networks are artificially combined into one network with structural zeros between the original smaller networks), then the conditional estimation option keeps the total number of ties constant within each component.

The program recognizes automatically if the data set is symmetric (a non-directed graph, with $x_{ij} = x_{ji}$ for all i, j) or anti-symmetric (a tournament, with $x_{ij} \neq x_{ji}$ for all $i \neq j$). In such cases, this is respected by the Metropolis-Hastings algorithm (numbers 6 or 7 in the list below) chosen by SIENA for the MCMC estimation, and the exponential random graph model is considered only on the set of all symmetric or all antisymmetric graphs, respectively.

The program has the following possibilities for the definition of the steps in the MCMC procedure (cf. Snijders, 2002):

- 1. Gibbs steps for single tie variables x_{ij} ;
- 2. Gibbs steps for dyads (x_{ij}, x_{ji}) ;
- 3. Gibbs steps for triplets (x_{ij}, x_{jh}, x_{ih}) and (x_{ij}, x_{jh}, x_{hi}) ;
- 4. Metropolis Hastings steps for single tie variables x_{ij} , version A;
- 5. Metropolis Hastings steps for single tie variables x_{ij} , version B;
- 6. Metropolis Hastings steps for single tie variables x_{ij} , version A, for non-directed graphs;
- 7. Metropolis Hastings steps for single tie variables x_{ij} , for antisymmetric graphs ('tournaments');
- Metropolis Hastings steps keeping the in-degrees and out-degrees fixed; see Snijders and van Duijn (2002);

- 9. Metropolis Hastings steps for pairs of tie variables x_{ij}, x_{ik} , keeping the out-degrees fixed, version A;
- 10. Metropolis Hastings steps for pairs of tie variables x_{ij}, x_{ik} , keeping the out-degrees fixed, version B;
- 11. to all of these, the number 10 is added to represent a continuous chain (see below) which is more efficient and which is the default.

The choice between these types of steps is made in the model options. The default for directed networks is step type 4, which is represented by code 14 because of the necessity to use a continuous chain (see below). Some other options are available by modifying the *pname*.MO file as indicated in Section 21.2.1 below. When there are structurally determined positions, options 6 and 7 should be used only if these positions also are placed symmetrically.

Since version 3.181, the special option 26, available only for non-directed graphs, conditions additionally on the connectedness of the network, provided that the observed graph is connected.

The options with fixed out-degrees can be useful in cases where the observed out-degrees depart strongly from what would be observed in a random draw from the ERGM – which will occur when all out-degrees are equal or almost equal (as in the case of data collection using a fixed-choice design) and also when the out-degrees have a very skewed distribution. In the latter case, option 10 will usually be more efficient than option 9.

In the conditional option (where the number of arcs is fixed), options 1 and 4–6 exchange values of arcs x_{ij} and x_{hk} with $(i, j) \neq (h, k)$ with probabilities defined by the Gibbs and Metropolis-Hastings rules, respectively; option 2 changes values of dyads (x_{ij}, x_{ji}) and (x_{hk}, x_{kh}) with $(i, j) \neq$ (h, k), keeping $x_{ij} + x_{ji} + x_{hk} + x_{kh}$ constant; and option 3 changes the value of one triplet (x_{ij}, x_{jh}, x_{ih}) or (x_{ij}, x_{jh}, x_{hi}) , keeping the sum $x_{ij} + x_{jh} + x_{ih}$ or $x_{ij} + x_{jh} + x_{hi}$ constant.

The number of steps, or run length, for generating one exponential random graph is $r n^2/2d(1-d)$, where r is a constant which can be changed in the model options, and called *multiplication* factor in the model and estimation options screen; n is the number of actors; and d is the density of the graph, truncated to lie between 0.05 and 0.95. The default value of r can be increased when it is doubted that the run length is sufficient to achieve convergence of the MCMC algorithm. It can be helpful to start the estimation on a given data set by specifying a very simple model, with only the out-degree and the reciprocity effects – for non-directed networks, only the degree effect. Then by some trial and error determine the multiplication factor so that the autocorrelations reported in the SIENA output are less than .4. This will presumably be a suitable value of the multiplication factor also for other, more complicated models. If later on the largest reported autocorrelations become much smaller or larger, then tune the multiplication factor such that the largest reported autocorrelation is smaller than 0.4.

The algorithm uses by default a continuous chain to make successive draws from the ERGM, even for different parameter values. This is in order to improve convergence speed. For this purpose, the possibilities 1–8 mentioned above should be communicated in the model options (see p. 97) by the values 11–18.

When convergence (as evidenced by all *t*-ratios for convergence being less than 0.1 in absolute value) is not easy to obtain, then one can try to improve convergence in repeated runs of SIENA, with the following options, When some autocorrelations are markedly higher than 0.1, then it can help to increase the multiplication factor. When the provisional parameter estimate (used as initial value for the estimation algorithm) seems to be reasonably close to a satisfactory value, then decrease the initial gain parameter (see Section 12), e.g., to the value 0.001 or 0.0001. Some guidance for how to do this is also given in Section 3.2.

12 Options for model type, estimation and simulation

There are several options available in SIENA. The main options concern the model type and the estimation procedure used.

Options concerning model type and estimation procedure can be accessed in the StOCNET environment via the Model specification screen's 'option' page. More detailed information is given starting at page 96.

1. There is a choice between conditional (1) and unconditional (0) Method of Moments estimation. If there are dependent action variables, the default for conditional estimation is to condition on the observed distance for the network variable; but it then is possible also to condition on the distances observed for the dependent action variables.

In addition, there are options for maximum likelihood (2) and Bayesian (3) estimation; these are beginning to be documented.

2. The Model Code.

This defines the Model Type and an associated output option.

In the longitudinal case, the meaning of this code is as follows.

Model Codes 10 or more give extra output for evaluating the fit of the out-degree distribution and for the explained variation (Snijders, 2004);

the integer Model Code in the unit position (i.e., Model Code itself if it is less than 10, and Model Code - 10 if the code is more than 10) defines the Model Type defined in Section 6.5. In the ERGM (non-longitudinal) case, the Model Code defines the kind of steps made in the MCMC algorithm. It is advised to use one of the values 11-16, because these generate a continuous chain which yields much better convergence.

3. The number of subphases in phase 2 of the estimation algorithm.

This determines the precision of the estimate. Advice: 3 for quick preliminary investigations, 4 or 5 for serious estimations.

There is an additional possibility (since version 3.181) of requesting extra estimation runs if the maximum *t*-ratio for convergence is higher than 0.15. This is obtained formally by specifying more than 10 subphases; the maximum additional number of repeated estimation runs then is the rounded value of number of subphases divided by 10; the real number of subphases is the remainder in this division.

4. The number of runs in phase 3 of the estimation algorithm.

This determines the precision of the estimated standard errors (and covariance matrix of the estimates), and of the *t*-values reported as diagnostics of the convergence. Advice: 200 for preliminary investigations when precise standard errors and *t*-values are not important, 1000 for serious investigations, 2000 to 4000 for estimations of which results are to be reported in publications.

(These numbers can be twice as low if, instead of the new (from Version 2.3) default option of estimation by the Score Function method, the older method of Finite Differences is used. The latter method has runs that take more time, but needs fewer runs.)

- 5. A constant used in other estimation procedures. In the ERGM (non-longitudinal) case, this is the multiplication factor r for the run length used in the MCMC algorithm.
- 6. The initial gain value, which is the step size in the starting steps of the Robbins-Monro procedure, indicated in Snijders (2001) by a_1 .

- 7. The choice between standard initial values (suitable estimates for the density and reciprocity parameters and zero values for all other parameters) or the current parameter values as initial values for estimating new parameter values.
- 8. The selection of the period for which a goodness-of-fit on period homogeneity is to be carried out.
- 9. The selection of the effect for which a goodness-of-fit on actor homogeneity is to be carried out (1 for the out-degree effect, 2 for the reciprocity effect); if this is selected, a list of actors also has to be supplied.
- 10. A random number seed. If the value 0 is chosen, the program will randomly select a seed. This is advised to obtain truly random results. If results from an earlier run are to be exactly replicated, the random number seed from this earlier run can be used.
- 11. The method to estimate derivatives; 0 is the older finite differences method (this is the method used in SIENA versions 1 and 2, which has a bias); 1 and 2 are the more efficient and unbiased methods proposed by Schweinberger and Snijders (2007); the preferred method is number 1. See Section 8.

Options about the simulation runs can be accessed in the StOCNET environment via the simulation specification button on the SIENA model's main screen. This button only is activated when 'simulation' is chosen as the 'Run model'. There is one option for simulations that can be chosen here.

5. The number of runs in the straight simulations. Advice: the default of 1000 will usually be adequate.

Depending on the choice for conditional or unconditional estimation in the estimation options, also the simulations are run conditionally or unconditionally.

13 Getting started

For getting a first acquaintance with the model, one may use the data set collected by Gerhard van de Bunt, discussed extensively in van de Bunt (1999), van de Bunt, van Duijn, and Snijders (1999), and used as example also in Snijders (2001) and Snijders (2005). The data files are provided with the program and at the SIENA website. The digraph data files used are the two networks vrnd32t2.dat, vrnd32t4.dat. The networks are coded as 0 = unknown, 1 = best friend, 2 = friend, 3 = friendly relation, 4 = neutral, 5 = troubled relation, 6 = item non-response, 9 = actor non-response. In the Transformations screen of StOCNET, choose the values '1 2 3' as the values to be coded as 1 for the first as well as the second network. Choose '6 9' as missing data codes.

The actor attributes are in the file vars.dat. Variables are, respectively, gender (1 = F, 2 = M), program, and smoking (1 = yes, 2 = no). See the references mentioned above for further information about this network and the actor attributes.

Specify the data in StOCNET by using subsequently the Data and Transformation menus (do not forget to click Apply when finishing each of these parts), then select SIENA in the Model menu and click the Data specification button. Select the network data, in temporal sequence, on the left side of the Data specification screen, then click on OK. Back on the SIENA main screen, now click on the Model specification button.

You will be requested to make some choices for the specification, the meaning of which should be clear given what is explained above. On the left hand side of the **Model specification** screen, you can specify evaluation and endowment effects, indicated by two columns of checkboxes marked 'u' and 'e', respectively. In the specification of the evaluation function, choose the out-degree effect, the reciprocity effect, and one other effect. In the specification of the endowment function, choose no effects at all. On the right hand side of the screen, you can specify rate function effects. At first, leave the specification of the rate function as it is (see Section 6, in which it was advised to start modeling with a constant rate function).

Then let the program estimate the parameters. You will see a screen with intermediate results: current parameter values, the differences ('deviation values') between simulated and observed statistics (these should average out to 0 if the current parameters are close to the correct estimated value), and the quasi-autocorrelations discussed in Section 7.

It is possible to intervene in the algorithm by clicking on the appropriate buttons: the current parameter values may be altered or the algorithm may be restarted or terminated. In most cases this is not necessary.

Some patience is needed to let the machine complete its three phases. How this depends on the data set and the number of parameters in the model is indicated in Section 19. After having obtained the outcomes of the estimation process, the model can be respecified: non-significant effects may be excluded (but it is advised always to retain the out-degree and the reciprocity effects) and other effects may be included.

13.1 Model choice

For the selection of an appropriate model for a given data set it is best to start with a simple model (including, e.g., 2 or 3 effects), delete non-significant effects, and add further effects in groups of 1 to 3 effects. Like in regression analysis, it is possible that an effect that is non-significant in a given model may become significant when other effects are added or deleted!

When you start working with a new data set, it is often helpful first to investigate the main endogenous network effects (reciprocity, transitivity, etc.) to get an impression of what the network dynamics looks like, and later add effects of covariates. The most important effects are discussed in Section 6; the effects are defined mathematically in Section 15.

13.1.1 Exploring which effects to include

The present section describes an exploratory approach to model specification. A more advanced approach to testing model specifications is described in Section 9.

For an exploration of further effects to be included, the following steps may be followed:

- 1. Estimate a model which includes a number of basic effects;
- 2. Simulate the model for these parameter values but also include some other relevant statistics among the simulated statistics;
- 3. Look at the *t*-values for these other statistics; effects with large *t*-values are candidates for inclusion in a next model.

It should be kept in mind, however, that this exploratory approach may lead to capitalization on chance, and also that the *t*-value obtained as a result of the straight simulations is conditional on the fixed parameter values used, without taking into account the fact that these parameter values are estimated themselves.

It is possible that for some model specifications the data set will lead to divergence, e.g., because the data contains too little information about this effect, or because some effects are 'collinear' with each other. In such cases one must find out which are the effects causing problems, and leave these out of the model. Simulation can be helpful to distinguish between the effects which should be fixed at a high positive or negative value and the effects which should be left out because they are superfluous.

When the distribution of the out-degrees is fitted poorly (which can be investigated by the extra output requested by selecting Model Code larger than 10 in the model options), an improvement usually is possible either by including non-linear effects of the out-degrees in the evaluation function, or by changing to Model Type 2 (see Section 6.5).

13.2 Convergence problems

If there are convergence problems, this may have several reasons.

- The data specification was incorrect (e.g., because the coding was not given properly).
- The starting values were poor. Try restarting from the standard initial values (a certain non-zero value for the density parameter, and zero values for the other parameters); or from values obtained as the estimates for a simpler model that gave no problems. The initial default parameter values can be obtained by choosing the model option "standard initial values".

When starting estimations with Model Type 2 (see Section 6.5), there may be some problems to find suitable starting values. For Model Type 2, it is advised to start with unconditional estimation (see the model options) and a simple model, and to turn back to conditional estimation, using the current parameter values as initial estimates for new estimation runs, only when satisfactory estimates for a simple model have been found.

• The model does not fit well in the sense that even with well-chosen parameters it will not give a good representation of the data.

This can be the case, e.g., when there is a large heterogeneity between the actors which is not well represented by effects of covariates. The out-degrees and in-degrees are given in the begin of the SIENA output to be able to check whether there are outlying actors having very high in- or out-degrees, or a deviating dynamics in their degrees. Strong heterogeneity between the actors will have to be represented by suitable covariates; if these are not available, one may define one or a few dummy variables each representing an outlying actor, and give this dummy variable an ego effect in the case of deviant out-degrees, and an alter effect in the case of deviant in-degrees.

Another possibility is that there is time heterogeneity. Indications about this can be gathered also from the descriptives given in the start of the output file: the number of changes upward and downward, in the network and also – if any – in the dependent behavioral variable. If these do not show a smooth or similar pattern across the observations, then it may be useful to include actor variables representing time trends. These could be smooth – e.g., linear – but they also could be dummy variables representing one or more observational periods; these must be included as an ego effect to represent time trends in the tendency to make ties (or to display higher values of the behavior in question).

- Too many weak effects are included. Use a smaller number of effects, delete non-significant ones, and increase complexity step by step. Retain parameter estimates from the last (simpler) model as the initial values for the new estimation procedure, provided for this model the algorithm converged without difficulties.
- Two or more effects are included that are almost collinear in the sense that they can both explain the same observed structures. This will be seen in high absolute values of correlations between parameter estimates. In this case it may be better to exclude one of these effects from the model.
- An effect is included that is large but of which the precise value is not well-determined (see above: section on fixing parameters). This will be seen in estimates and standard errors both being large and often in divergence of the algorithm. Fix this parameter to some large value. (Note: large here means, e.g., more than 5 or less than -5; depending on the effect, of course.)

If the algorithm is unstable, with parameter values (the left hand list in the SIENA window) changing too wildly, or with the algorithm suddenly seeming stuck and not moving forward, the a solution may be to simplify the model (perhaps later on making it more complex again in forward parameter estimation steps); another solution may be to decrease the initial gain parameter (see Section 12).

If there are problems you don't understand, but you do know something about the operation of SIENA, you could take a look at the file *pname.log*; and, if the problems occur in the estimation algorithm, at the file *pname.cck*. These files give information about what the program did, which may be helpful in diagnosing the problem. E.g., you may look in the *pname.cck* file to see if some of the parameters are associated with positive values for the so-called quasi-autocorrelations. If this happens from subphase 2.2 onward for some parameters, these may be the parameters that led to problems in the estimation algorithm (e.g., because the corresponding effect is collinear with other effects; or because they started from unfortunate starting values; or because the data set contains too little information about their value).

13.3 Composition change

Example data files for a network of changing composition are also provided with the program. These files are called vtest2.dat, vtest3.dat, and vtest4.dat. They contain the same network data as the friendship data files of van de Bunt (for these three observation times and with the same coding), except that in these data some joiners and leavers were artificially created. These actors were given the code '9' for the observation moment at which they were not part of the network. The attribute file vtestexo.dat contains the times at which the network composition changes (see

also the example in Section 5.7). This file is necessary for the program to correctly include the times at which actors join or leave the network. For example, the first line of the file contains the values

1 0.7 3 0.0

which indicates that the first actor joins the network at fraction 0.7 of period 1 (the period between the first and second observation moments) and leaves the network right after the beginning of the third period, i.e., he/she does not leave the network before the last observation at the third time point. Thus, the first actor joins the network and then stays in during the whole period being analyzed.

14 Multilevel network analysis

For combining SIENA results of several independent networks, there are three options. ('Independent' networks here means that the sets of actors are disjoint, and it may be assumed that there are no direct influences from one network to another.) The first two options assume that the parameters of the actor-based models for the different networks are the same – except for the basic rate parameters and for those differences that are explicitly modeled by interactions with dummy variables indicating the different networks. The first and third options require that the number of observations is the same for the different networks. This is not required for the second option. These methods can be applied for two or more networks.

The three options are:

1. Combining the different networks in one large network, indicating by structural zeros that ties between the networks are not permitted. This is explained in Section 5.1.1. This can be executed inside or outside StOCNET.

The special effort to be made here is the construction of the data files for the large (combined) network.

2. Combining different sub-projects, each defined by its own .in file (see Section 21.1), into one *multi-group* project. The 'sub-projects' are the same as the 'different networks' mentioned here. This is explained in Section 14.1. This can be executed only outside StOCNET. If .in files are available, the only effort here is to run SIENA outside of StOCNET as explained in Section 17.

A difference between options 1 and 2 is that the use of structural zeros (option 1) will lead to a default specification where the rate parameters are equal across networks (this can be changed by making the rate dependent upon dummy actor variables that indicate the different networks) whereas the multi-group option yields rate parameters that are distinct across different networks.

3. Analyzing the different networks separately, without any assumption that parameters are the same but using the same model specification, and post-processing the output files by a meta-analysis using Siena08. This is explained in Section 14.2.

The first and second options will yield nearly the same results, with the differences depending on the basic rate (and perhaps other) parameters that are allowed to differ between the different networks, and of course also depending on the randomness of the estimation algorithm. The second option is more 'natural' given the design of SIENA and will normally run faster than the first. Therefore the second option seems preferable to the first.

The third option makes much less assumptions because parameters are not constrained at all across the different networks. Therefore the arguments usual in statistical modeling apply: as far as assumptions is concerned, option 3 is safer; but if the assumptions are satisfied (or if they are a good approximation), then options 1 and 2 have higher power and are simpler. However, option 3 requires that each of the different network data sets is informative enough to lead to well-converged estimates; this will not always be the case for small data sets, and then options 1 or 2 are preferable.

When the data sets for the different networks are not too small individually, then a middle ground might be found in the following way. Start with option 3. This will show for which parameters there are important differences between the networks. Next follow option 2, with interactions between the sub-project dummies and those parameters for which there were important between network differences. This procedure may work less easily when the number of different networks is relatively high, because it may then lead to too many interactions with dummy variables.

14.1 Multi-group Siena analysis

The multi-group option 'glues' several projects (further referred to as *sub-projects*) after each other into one larger multi-group project. These sub-projects must have the same sets of variables of all kinds: that is, the list of dependent networks, dependent behavioral variables, actor covariates, and dyadic covariates must be the same for the various sub-projects. The number of actors and the number of observations can be different, however. These sub-projects then are combined into one project where the number of actors is the largest of the number of actors of the sub-projects, and the number of observations is the sum of the observations of the sub-projects. As an example, suppose that three projects with names sub1, sub2, and sub3 are combined. Suppose sub1 has 21 actors and 2 observations, sub2 has 35 actors and 4 observations, and sub3 has 24 actors with 5 observations. Then the combined multi-group project has 35 actors and 11 observations. The step from observation 2 to 3 switches from sub-project sub1 to sub-project sub2, while the step from observation 6 to 7 switches from sub-project sub2 to sub3. These switching steps do not correspond to simulations of the actor-based model, because that would not be meaningful.

The different sub-projects are considered to be unrelated except that they have the same model specification and the same parameter values.

Given the potentially large number of periods that can be implied by the multi-group option, it probably is advisable, when using Method of Moments estimation, to use the conditional estimation option.

The multi-group option can be executed only outside of StOCNET. Execution of SIENA outside of StOCNET is generally explained in Section 17. The following is required for the multi-group option.

1. For each of the sub-projects, a correct .in file must be available, as described in Section 21.1. These .in files and the corresponding data files must all be together in one directory. The names of the .in files should not contain spaces or any of the following list of characters:

! " # \$ % & ' () * + ,

The total number of observations should not exceed 99.

- 2. This directory must also the data files (unless differently indicated by the pathnames in the .in files).
- 3. In addition, this directory must contain a file with extension name .min (for 'multi-project input'). The root name of this file will be the project name of the multi-group project. This file must contain the project names of the sub-projects, each one at a separate line.

If this file includes also a line with the symbol '+', then the multi-group project will also include dummy variables indicating the sub-projects. In contrast to other actor variables, these variables are not centered.

An example thus would be a file multip.min with contents

sub1 sub2 sub3 +

if projects with basic information files sub1.in, sub2.in, and sub3.in are available.

4. The multi-project is used just like any other projects by the programs Siena01, ..., Siena07 as indicated in Section 17.

Currently, Siena02 does not yet operate in the correct way for multi-group projects.

14.2 Meta-analysis of Siena results

The program Siena08.exe is a relatively simple multilevel extension to SIENA. This program must be run independently, i.e., not through StOCNET, after having obtained estimates for a common model estimated for several data sets. Siena08 combines the estimates in a meta-analysis or multilevel analysis according to the methods of Snijders and Baerveldt (2003), and according to a Fisher-type combination of one-sided *p*-values. This combination method of Fisher (1932) is described in Hedges and Olkin (1985) and (briefly) in Snijders and Bosker (1999, Chapter 3). Some more information is at the SIENA website.

All SIENA output files to be used must already exist, and the *last estimation results* in these output files will be used. It is required that all these last estimation runs have the same set of estimated parameters, and of parameters tested by score tests. The program does not check that the score tests (if any) in the output files refer to the same parameters. It is also required that the decimal separator is a point, not a comma. (This depends on your Windows settings; if your output files have commas, just change all commas into points using an editor.) The Siena08 project is the collection of output files to be combined, which is defined in the project .mli file.

An easy way to operate Siena08 is to make a batch file containing the single line

Siena08 ABC

where ABC is the projectname.

E.g., suppose the projectname is ABC. Then there must be a project file with the name ABC.mli (the root name "ABC" can be chosen by the user, the extension name "mli" is prescribed.) If the number of network evolution projects combined in this Siena08 run is given by K, e.g. the K = 3 projects with names A, B and C, then the file ABC.mli must give the project names on separate lines and in addition the options, as indicated in the following example file:

```
[This file contains specifications for the meta-analysis of Siena projects.]
[It serves as input for the Siena08 program.]
```

```
@1 [general information about the Siena project list ]
10 [number of projects, names follow:]
A
B
C
@2 [options for estimation of projects]
5 [upper bound for standard error in meta-analysis]
1 [code 0=estimate, 1=aggregate from .out-files, 2=generate .dsc-file]
1 [code 1=extra output]
0 [number of score tests]
```

Executing the batch file (e.g. by double clicking) will execute Siena08. To get started, try this out with a small data set. Some further explanation and example data are provided on the SIENA website.

When working with SIENA for many projects, it can be useful to operate with the executable programs in batch files, as explained in Section 17. The main programs then will be Siena01.exe

for starting up the projects and Siena07.exe for the estimation. The model specification can be changed by modifying the *pname*.MO file with an editor, or by using the siena03.exe program. Model and option specifications can be copied from one SIENA project to another by siena04.exe, see 17.2.

15 Mathematical definition of effects

Here, the mathematical formulae for the definition of the effects are given. In Snijders (2001, 2005) and Steglich, Snijders and Pearson, (2010), further background to these formulae can be found. The effects are grouped into effects for modelling network evolution and effects for modelling behavioral evolution (i.e., the dynamics of dependent actor variables). Within each group of effects, the effects are listed in the order in which they appear in SIENA.

Some of the effects contain a number which is denoted in this section by c, and called in this manual an *internal effect parameter*. (These are totally different from the statistical parameters which are the weights of the effects in the objective function.) These numbers can be determined by the user as the "par." column in the advanced model specification options of StOCNET, or by changing the *pname.mo* file described in Section 21.2.1.

15.1 Network evolution

The model of network evolution consists of the model of actors' decisions to establish new ties or dissolve existing ties (according to *evaluation* and *endowment functions*) and the model of the timing of these decisions (according to the *rate function*). The objective function of the actor is the sum of the network evaluation function and the network endowment function

$$u^{\text{net}}(x) = f^{\text{net}}(x) + g^{\text{net}}(x) , \qquad (1)$$

and a random term; where the evaluation function $f^{\text{net}}(x)$ and the endowment function $g^{\text{net}}(x)$ are as defined in the following subsections.

For some effects (those for which the function f1i in Section 24.1 is non-zero) the endowment function is implemented not for estimation by the Method of Moments but only by the Maximum Likelihood or Bayesian method; this is indicated below by "endowment effect only likelihoodbased".

(It may be noted that the network evaluation function was called objective function, and the endowment function was called gratification function, in Snijders, 2001.)

15.1.1 Network evaluation function

The network evaluation function for actor i is defined as

$$f^{\rm net}(x) = \sum_{k} \beta_k^{\rm net} s_{ik}^{\rm net}(x) \tag{2}$$

where β_k^{net} are parameters and $s_{ik}^{\text{net}}(x)$ are effects as defined below.

The potential effects in the network evaluation function are the following. Note that in all effects where a constants c occurs, this constant can be chosen and changed by the user; this is the internal effect parameter mentioned above. Also note that the evaluation effects which are a function only of the out-degree of actor i are excluded for Model Type 2. For non-directed networks, the same formulae are used, unless a different formula is given explicitly.

- 1. out-degree effect or density effect, defined by the out-degree $s_{i1}^{\text{net}}(x) = x_{i+} = \sum_j x_{ij}$, where $x_{ij} = 1$ indicates presence of a tie from *i* to *j* while $x_{ij} = 0$ indicates absence of this tie;
- 2. reciprocity effect, defined by the number of reciprocated ties $s_{i2}^{\text{net}}(x) = \sum_{j} x_{ij} x_{ji};$

3. transitive triplets effect, defined by the number of transitive patterns in i's relations (ordered pairs of actors (j, h) to both of whom i is tied, while also j is tied to h), for directed networks, $s_{i3}^{\text{net}}(x) = \sum_{j,h} x_{ij} x_{ih} x_{jh}$; and for non-directed networks, $s_{i3}^{\text{net}}(x) = \sum_{j < h} x_{ij} x_{ih} x_{jh}$; there was an error here until version 3.17p, which amounted to combining the transitive

triplets and transitive mediated triplets effects:

4. transitive mediated triplets effect, defined by the number of transitive patterns in i's relations where i has the mediating position (ordered pairs of actors (i, h) for which i is tied to i and i to h, while also j is tied to h), which is different from the transitive triplets effect only for directed networks,

$$s_{i4}^{\mathrm{net}}(x) = \sum_{i,h} x_{ji} x_{ih} x_{jh};$$

this cannot be used together with the transitive triplets effect in Method of Moments estimation, because of perfect collinearity of the fit statistics;

- 5. number of 3-cycles, $s_{i5}^{\text{net}}(x) = \sum_{j,h} x_{ij} x_{jh} x_{hi};$
- 6. transitive ties effect (earlier called *(direct and indirect ties)* effect), defined by the number of actors to whom i is directly as well as indirectly tied, $s_{i6}^{\mathrm{net}}(x) = \sum_{j} x_{ij} \, \max_h(x_{ih} \, x_{hj});$
- $7. \ betweenness \ count,$ $s_{i7}^{\text{net}}(x) = \sum_{i,h} x_{hi} x_{ij} (1 - x_{hj});$
- 8. balance, defined by the similarity between the outgoing ties of actor i and the outgoing ties of the other actors j to whom i is tied,

$$s_{i8}^{\text{net}}(x) = \sum_{j=1}^{n} x_{ij} \sum_{\substack{h=1\\h\neq i,j}}^{n} (b_0 - |x_{ih} - x_{jh}|) ,$$

where b_0 is a constant included to reduce the correlation between this effect and the density effect, defined by

$$b_0 = \frac{1}{(M-1)n(n-1)(n-2)} \sum_{m=1}^{M-1} \sum_{\substack{i,j=1\\h\neq i,j}}^n \sum_{\substack{h=1\\h\neq i,j}}^n |x_{ih}(t_m) - x_{jh}(t_m)| .$$

(In SIENA versions before 3.17w, this was divided by n-2, which for larger networks tended to lead to quite large estimates and standard errors. Therefore in version 3.17w, the division by n-2 – which had not always been there – was dropped.)

- 9. number of distances two effect, defined by the number of actors to whom i is indirectly tied (through at least one intermediary, i.e., at sociometric distance 2), $s_{i9}^{\text{net}}(x) = \#\{j \mid x_{ij} = 0, \max_h(x_{ih} x_{hj}) > 0\};\$ endowment effect only likelihood-based;
- 10. number of doubly achieved distances two effect, defined by the number of actors to whom iis not directly tied, and tied through twopaths via at least two intermediaries, $s_{i10}^{\text{net}}(x) = \#\{j \mid x_{ij} = 0, \sum_{h} (x_{ih} x_{hj}) \ge 2\};$ endowment effect only likelihood-based;

- 11. number of dense triads, defined as triads containing at least c ties, $s_{i11}^{\text{net}}(x) = \sum_{j,h} x_{ij} I\{x_{ij} + x_{ji} + x_{ih} + x_{hi} + x_{jh} + x_{hj}\} \ge c\},$ where the 'indicator function' $I\{A\}$ is 1 if the condition A is fulfilled and 0 otherwise, and where c is either 5 or 6; (this effect is superfluous and undefined for symmetric networks);
- 12. number of (unilateral) peripheral relations to dense triads, $s_{i12}^{\text{net}}(x) = \sum_{j,h,k} x_{ij}(1-x_{ji})(1-x_{hi})(1-x_{ki})I\{(x_{jh}+x_{hj}+x_{jk}+x_{kj}+x_{hk}+x_{kh}) \ge c\},$ where c is the same constant as in the dense triads effect; for symmetric networks, the 'unilateral' condition is dropped, and the definition is $s_{i12}^{\text{net}}(x) = \sum_{j,h,k} x_{ij}(1-x_{hi})(1-x_{ki})I\{(x_{jh}+x_{hj}+x_{jk}+x_{kj}+x_{hk}+x_{kh}) \ge c\};$
- 13. in-degree related popularity effect (earlier called popularity or popularity of alter effect), defined by the sum of the in-degrees of the others to whom i is tied, s^{net}_{i13}(x) = ∑_j x_{ij} x_{+j} = ∑_j x_{ij} ∑_h x_{hj}; until version 3.17p, this effect was multiplied by a factor 1/n;
- 14. in-degree related popularity (sqrt) effect (earlier called popularity of alter (sqrt measure) effect), defined by the sum of the square roots of the in-degrees of the others to whom i is tied, $s_{i14}^{\text{net}}(x) = \sum_j x_{ij} \sqrt{x_{+j}} = \sum_j x_{ij} \sqrt{\sum_h x_{hj}};$ this often works better in practice than the raw popularity effect; also it is often reasonable to
 assume that differences between high in-degrees are relatively less important than the same
 differences between low in-degrees;
- 15. out-degree related popularity effect (earlier called activity or activity of alter effect), defined by the sum of the out-degrees of the others to whom *i* is tied, $s_{i15}^{\text{net}}(x) = \sum_j x_{ij} x_{j+} = \sum_j x_{ij} \sum_h x_{jh}$; until version 3.17p, this effect was multiplied by a factor 1/n;
- 16. out-degree related popularity (sqrt) effect (earlier called activity of alter (sqrt measure) effect), defined by the sum of the square roots of the out-degrees of the others to whom *i* is tied, $s_{i16}^{\text{net}}(x) = \sum_j x_{ij} \sqrt{x_{j+}} = \sum_j x_{ij} \sqrt{\sum_h x_{jh}};$ this often works better in practice than the raw activity effect for the same reasons as mentioned above for the sqrt measure of the popularity effect;
- ⊙ for non-directed networks, the popularity and activity effects are taken together as "degree effects", since in-degrees and out-degrees are the same in this case;
- 17. *in-degree related activity effect*, defined as the cross-product of the actor's in- and out-degrees, $s_{i17}^{\text{net}}(x) = x_{i+} x_{+i};$ endowment effect only likelihood-based;
- 18. in-degree related activity (sqrt) effect, defined by $s_{i18}^{\text{net}}(x) = x_{i+1}\sqrt{x_{i+1}}$;
- 19. out-degree related activity effect, defined as the squared out-degree of the actor, $s_{i19}^{\text{net}}(x) = x_{i+}^2$; endowment effect only likelihood-based;
- 20. out-degree related activity (sqrt) effect (earlier called out-degree (1.5)), defined by $s_{i20}^{\text{net}}(x) = x_{i+}^{1.5} = x_{i+} \sqrt{x_{i+}}$ endowment effect only likelihood-based;

21. out-degree up to c, where c is some constant (internal effect parameter, see above), defined by $s_{i21}^{net}(x) = \max(x_{i+}, c);$

this is left out in later versions of SIENA;

- 22. square root out-degree, defined by $s_{i22}^{\text{net}}(x) = \sqrt{x_{i+}};$ this is left out in later versions of SIENA;
- 23. squared (out-degree c), where c is some constant, defined by $s_{i23}^{\text{net}}(x) = (x_{i+} c)^2$, where c is chosen to diminish the collinearity between this and the density effect; this is left out in later versions of SIENA;
- 24. sum of (1/(out-degree + c)), where c is some constant, defined by $s_{i24}^{\text{net}}(x) = 1/(x_{i+} + c);$ endowment effect only likelihood-based;
- 25. sum of (1/(out-degree + c)(out-degree + c + 1)), where c is some constant, defined by $s_{i25}^{\text{net}}(x) = 1/(x_{i+} + c)(x_{i+} + c + 1);$ endowment effect only likelihood-based.
- 26. out-out degree (1/c) assortativity, which represents the differential tendency for actors with high out-degrees to be tied to other actors who likewise have high out-degrees, $s_{i26}^{\text{net}}(x) = \sum_j x_{ij} x_{i+}^{1/c} x_{j+}^{1/c};$ c can be 1 or 2 (the latter value is the default);
- 27. out-in degree (1/c) assortativity, which represents the differential tendency for actors with high out-degrees to be tied to other actors who have high in-degrees, $s_{i27}^{\text{net}}(x) = \sum_j x_{ij} x_{i+}^{1/c} x_{+j}^{1/c};$ c can be 1 or 2 (the latter value is the default);
- 28. in-out degree (1/c) assortativity, which represents the differential tendency for actors with high in-degrees to be tied to other actors who have high out-degrees, $s_{i28}^{\text{net}}(x) = \sum_j x_{ij} x_{+i}^{1/c} x_{j+}^{1/c};$ c can be 1 or 2 (the latter value is the default);
- 29. in-in degree (1/c) assortativity, which represents the differential tendency for actors with high in-degrees to be tied to other actors who likewise have high in-degrees, $s_{i29}^{\text{net}}(x) = \sum_j x_{ij} x_{+i}^{1/c} x_{+j}^{1/c};$ c can be 1 or 2 (the latter value is the default);

The effects for a dyadic covariate w_{ij} are

- 30. covariate (centered) main effect, $s_{i30}^{\text{net}}(x) = \sum_{j} x_{ij} (w_{ij} - \bar{w})$ where \bar{w} is the mean value of w_{ij} ;
- 31. covariate (centered) × reciprocity, $s_{i31}^{\text{net}}(x) = \sum_{j} x_{ij} x_{ji} (w_{ij} - \bar{w}).$
- Three different ways can be modeled in which a triadic combination can be made between the dyadic covariate and the network. In the explanation, the dyadic covariate is regarded

as a weighted network (which will be reduced to a non-weighted network if w_{ij} only assumes the values 0 and 1). By way of exception, the dyadic covariate is not centered in these three effects (to make it better interpretable as a network). In the text and the pictures, an arrow with the letter W represents a tie according to the weighted network W.

32. WW => X closure of covariate, $s_{j22}^{\text{net}}(x) = \sum_{i \neq b} x_{ij} w_{ib} w_{hj};$

 $s_{i32}^{net}(x) = \sum_{j \neq h} x_{ij} w_{ih} w_{hj};$ this refers to the closure of W - W two-paths; each W - W twopath $i \stackrel{W}{\rightarrow} h \stackrel{W}{\rightarrow} j$ is weighted by the product $w_{ih}w_{hj}$ and the sum of these product weights measures the strength of the tendency toward closure of these W - W twopaths by a tie.



Since the dyadic covariates are represented by square arrays and not by edgelists, this will be a relatively time-consuming effect if the number of nodes is large.

- 33. $WX \Longrightarrow X$ closure of covariate, $s_{i33}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} w_{ih} x_{hj};$ this refers to the closure of mixed W - X two-paths; each W - Xtwo-path $i \xrightarrow{W} h \rightarrow j$ is weighted by w_{ih} and the sum of these weights measures the strength of the tendency toward closure of these mixed W - X twopaths by a tie;
- 34. XW => X closure of covariate,

 $s_{i34}^{\text{net}}(x) = \sum_{j \neq h} x_{ij} x_{ih} w_{hj};$ this refers to the closure of mixed X - W two-paths; each X - Wtwo-path $i \rightarrow h \xrightarrow{W} j$ is weighted by w_{hj} and the sum of these weights measures the strength of the tendency toward closure of these mixed X - W twopaths by a tie.



j

i

For actor-dependent covariates v_j (recall that these are centered internally by SIENA) as well as for dependent behavior variables (for notational simplicity here also denoted v_j ; these variables also are centered), the following effects are available:

- 35. covariate-alter or covariate-related popularity, defined by the sum of the covariate over all actors to whom *i* has a tie, $s_{i35}^{\text{net}}(x) = \sum_{j} x_{ij} v_j;$
- 36. covariate squared alter or squared covariate-related popularity, defined by the sum of the squared centered covariate over all actors to whom *i* has a tie, (not included if the variable has range less than 2) $s_{i36}^{net}(x) = \sum_{i} x_{ij} v_j;$
- 37. covariate-ego or covariate-related activity, defined by i's out-degree weighted by his covariate value, $s_{i37}^{\text{net}}(x) = v_i x_{i+};$

38. covariate-related similarity, defined by the sum of centered similarity scores \sin_{ij}^{v} between i and the other actors j to whom he is tied,

 $s_{i38}^{\text{net}}(x) = \sum_{j} x_{ij} (\sin_{ij}^{v} - \sin^{v}),$ where $\widehat{\sin^{v}}$ is the mean of all similarity scores, which are defined as $\sin_{ij}^{v} = \frac{\Delta - |v_i - v_j|}{\Delta}$ with $\Delta = \max_{ij} |v_i - v_j|$ being the observed range of the covariate v (this mean is given in the output file just before the "initial data description");

- 39. covariate-related similarity × reciprocity, defined by the sum of centered similarity scores for all reciprocal dyads in which *i* is situated, $s_{i39}^{\text{net}}(x) = \sum_{i} x_{ij} x_{ji} (\sin_{ij}^{v} - \widehat{\sin^{v}});$
- 40. same covariate, which can also be called covariate-related identity, defined by the number of ties of *i* to all other actors *j* who have exactly the same value on the covariate, $s_{i40}^{\text{net}}(x) = \sum_j x_{ij} I\{v_i = v_j\},$ where the indicator function $I\{v_i = v_j\}$ is 1 if the condition $\{v_i = v_j\}$ is satisfied, and 0 if it is not;
- 41. same covariate \times reciprocity, defined by the number of reciprocated ties between *i* and all other actors *j* who have exactly the same value on the covariate, $s_{i41}^{\text{net}}(x) = \sum_{j} x_{ij} x_{ji} I\{v_i = v_j\};$
- 42. covariate-ego × alter, defined by the product of *i*'s covariate and the sum of those of his alters, $s_{i42}^{\text{net}}(x) = v_i \sum_j x_{ij} v_j;$
- 43. covariate-ego × alter × reciprocity, defined by the product of *i*'s covariate and the sum of those of his reciprocated alters, $s_{i43}^{\text{net}}(x) = v_i \sum_j x_{ij} x_{ji} v_j;$
- 44. ego > alter for covariate, defined by the number of ties where *i*'s covariate is larger than alter's, while equality counts for half, $s_{i44}^{\text{net}}(x) = \sum_j x_{ij} \operatorname{dsign}(v_i - v_j),$ where $\operatorname{dsign}(d) = 0$ for d < 0, 0.5 for d = 0, and 1 for d > 0.
- 45. covariate of indirect ties, defined by the sum of the covariate over the actors to whom *i* is tied indirectly (at a geodesic distance of 2), $s_{i45}^{\text{net}}(x) = \sum_{i} (1 - x_{ij}) (\max_{h} x_{ih} x_{hj}) v_{j}.$
- 46. user-defined interaction effects as described in Section 6.6. The internal effect parameter is decomposed by SIENA into its two or three constituents, see in the mentioned section. The interaction is defined on a tie basis: if two interacting effects are defined by $s_{ia}^{\text{net}}(x) = \sum_j s_{ij}^a(x)$ and $s_{ib}^{\text{net}}(x) = \sum_j s_{ij}^b(x)$ (where a and b are calculated from the internal effect parameter c), then the interaction is defined by $s_{i46}^{\text{net}}(x) = \sum_j s_{ij}^a(x) s_{ij}^b(x)$.

Additional possible effects are documented in Steglich, Snijders and Pearson (2010) and for Model Type 2 in Snijders (2003).

15.1.2 Network endowment function

The network endowment function is the way of modeling effects which operate in different strengths for the creation and the dissolution of relations. The network endowment function is zero for creation of ties, and is given by

$$g^{\text{net}}(x) = \sum_{k} \gamma_k s_{ik}^{\text{net}}(x) \tag{3}$$

for dissolution of ties. In this formula, the γ_k are the parameters for the endowment function. The potential effects $s_{ik}^{\text{net}}(x)$ in this function, and their formulae, are the same as in the evaluation function; except that not all are available, as indicated in the preceding subsection. For further explication, consult Snijders (2001, 2005; here, the 'gratification function' is used rather than the endowment function), Snijders, Steglich, and Schweinberger (2007), and Steglich, Snijders and Pearson (2010).

15.1.3 Network rate function

The network rate function λ^{net} (lambda) is defined for Model Type 1 (which is the default Model Type) as a product

$$\lambda_i^{\rm net}(\rho, \alpha, x, m) = \lambda_{i1}^{\rm net} \lambda_{i2}^{\rm net} \lambda_{i3}^{\rm net}$$

of factors depending, respectively, on period m, actor covariates, and actor position (see Snijders, 2001, p. 383). The corresponding factors in the rate function are the following:

1. The dependence on the period can be represented by a simple factor

$$\lambda_{i1}^{\rm net} = \rho_m^{\rm net}$$

for m = 1, ..., M - 1. If there are only M = 2 observations, the basic rate parameter is called ρ^{net} .

2. The effect of actor covariates with values v_{hi} can be represented by the factor

$$\lambda_{i2}^{\text{net}} = \exp(\sum_{h} \alpha_h \, v_{hi}) \,.$$

3. The dependence on the position of the actor can be modeled as a function of the actor's outdegree, in-degree, and number of reciprocated relations, the 'reciprocated degrees'. Define these by

$$x_{i+} = \sum_{j} x_{ij}, \ x_{+i} = \sum_{j} x_{ji}, \ x_{i(r)} = \sum_{j} x_{ij} x_{ji}$$

(recalling that $x_{ii} = 0$ for all i).

The contribution of the out-degrees to $\lambda_{i3}^{\text{net}}$ is a factor

 $\exp(\alpha_h x_{i+}),$

if the associated parameter is denoted α_h for some h, and similarly for the contributions of the in-degrees and the reciprocated degrees.

Also an exponential dependence on reciprocals of out-degrees can be specified; this can be meaningful because the rate effect of the out-degree becoming a value 1 higher might become smaller and smaller as the out-degree increases. Denoting again the corresponding parameter by α_h (but always for different index numbers h), this effect multiplies the factor $\lambda_{i3}^{\text{net}}$ by

$$\exp(\alpha_h/x_{i+})$$
.

15.1.4 Network rate function for Model Type 2

For Model Type 2 (see Section 6.5), the network rate function is defined according to Snijders (2003) by

$$\rho_m \lambda_{i+}(s) = \rho_m \frac{\nu(s)\,\xi(s)}{1+\xi(s)}, \rho_m \lambda_{i-}(s) = \rho_m \frac{\nu(s-1)}{1+\xi(s-1)},$$

where $\rho_m \lambda_{i+}(s)$ and $\rho_m \lambda_{i-}(s)$ represent, respectively, the rate at which an actor of current outdegree s increases, or decreases, his out-degree by 1. The parameter ρ_m is a multiplicative effect of the observation period.

Function ξ (xi) is called the distributional tendency function and is represented according to Snijders (2003, formula (17)) by

$$\xi(s) = \exp\left(\alpha_1 - \alpha_2 \log(s+1) - \frac{\alpha_3}{s+1}\right)$$

where the names given in SIENA are

- α_1 : out-degrees effect;
- α_2 : logarithmic out-degree effect;
- α_3 : factorial out-degree effect.

The reasons for these names and interpretation of the effects can be found in Snijders (2003). To the exponent also effects of actor covariates can be added.

The so-called volatility function ν (nu) is defined as

$$\nu(s) = \left(1 + \alpha_4 \frac{1}{s+1}\right) \ .$$

Also to this exponent effects of actor covariates can be added.

15.2 Behavioral evolution

The model of the dynamics of a dependent actor variable consists of a model of actors' decisions (according to *evaluation* and *endowment functions*) and a model of the timing of these decisions (according to a *rate function*), just like the model for the network dynamics. The decisions now do not concern the creation or dissolution of network ties, but whether an actor increases or decreases his score on the dependent actor variable by one, or keeps it as it is.

15.2.1 Behavioral evaluation function

Effects for the behavioral evaluation function u^{beh} can be selected from the following. Here the dependent variable is transformed to have an overall average value of 0; in other words, z denotes the original input variable minus the overall mean, which is given in the output file under the heading *Reading dependent actor variables*.

1. behavioral shape effect,

 $s_{i1}^{\text{beh}}(x) = z_i \,,$

where z_i denotes the value of the dependent behavior variable of actor i;

2. average similarity effect, defined by the average of centered similarity scores \sin_{ij}^z between i and the other actors j to whom he is tied, $s_{i2}^{\text{beh}}(x) = x_{i+}^{-1} \sum_{i} x_{ij} (\sin_{ij}^z - \widehat{\sin}^z);$

 $s_{i2}^{\text{beh}}(x) = x_{i+}^{-1} \sum_{j} x_{ij} (\sin_{ij}^{z} - \widehat{\sin}^{z});$ (and 0 if $x_{i+} = 0$);

- 3. total similarity effect, defined by the sum of centered similarity scores \sin_{ij}^{z} between *i* and the other actors *j* to whom he is tied, $s_{i3}^{\text{beh}}(x) = \sum_{i} x_{ij} (\sin_{ij}^{z} - \widehat{\sin}^{z});$
- 4. indegree effect, $s_{i4}^{\text{beh}}(x) = z_i \sum_j x_{ji};$
- 5. outdegree effect, $s_{i5}^{\text{beh}}(x) = z_i \sum_j x_{ij};$
- 6. isolate effect, the differential attractiveness of the behavior for isolates, $s_{i6}^{\text{beh}}(x) = z_i I\{x_{+i} = 0\},$ where again $I\{A\}$ denotes the indicator function of the condition A;
- 7. average similarity × reciprocity effect, defined by the sum of centered similarity scores \sin_{ij}^{z} between *i* and the other actors *j* to whom he is reciprocally tied, $s_{i7}^{\text{beh}}(x) = x_{i(r)}^{-1} \sum_{j} x_{ij} x_{ji} (\sin_{ij}^{z} - \widehat{\sin}^{z});$ (and 0 if $x_{i(r)} = 0$);
- 8. total similarity × reciprocity effect, defined by the sum of centered similarity scores \sin_{ij}^{z} between *i* and the other actors *j* to whom he is reciprocally tied, $s_{i8}^{\text{beh}}(x) = \sum_{j} x_{ij} x_{ji} (\sin_{ij}^{z} - \widehat{\sin}^{z});$
- 9. average similarity × popularity alter effect, defined by the sum of centered similarity scores sim^z_{ij} between i and the other actors j to whom he is tied, multiplied by their indegrees, s^{beh}_{i9}(x) = x⁻¹_{i+} ∑_j x_{ij}x_{+j}(sim^z_{ij} − sim^z); (and 0 if x_{i+} = 0);
- 10. if the parameter for this effect is equal to 1: popularity alter effect, defined by the average in-degrees of the other actors j to whom i is tied, s^{beh}_{i10}(x) = z_ix⁻¹_{i+} ∑_j x_{ij}x_{+j}; (and 0 if x_{i+} = 0); if the parameter for this effect is not equal to 1: total similarity × popularity alter effect, defined by the sum of centered similarity scores sim^z_{ij} between i and the other actors j to whom he is tied, multiplied by their indegrees,

$$s_{i10}^{\text{beh}}(x) = \sum_{j} x_{ij} x_{+j} (\sin_{ij}^{z} - \sin^{z});$$

11. average similarity × reciprocity × popularity alter effect, defined by the sum of centered similarity scores \sin_{ij}^z between *i* and the other actors *j* to whom he is reciprocally tied, multiplied by their indegrees, $s_{i11}^{\text{beh}}(x) = x_{i(r)}^{-1} \sum_j x_{ij} x_{ji} x_{+j} (\sin_{ij}^z - \widehat{\sin}^z);$

and 0 if
$$x_{i(r)} = 0$$
;

12. total similarity \times reciprocity \times popularity alter effect, defined by the sum of centered similarity scores \sin_{ij}^{z} between *i* and the other actors *j* to whom he is reciprocally tied, multiplied by their indegrees, $s_{i12}^{\text{beh}}(x) = \sum_{j} x_{ij} x_{ji} x_{+j} (\sin_{ij}^{z} - \widehat{\sin}^{z});$

- 13. average alter effect, defined by the product of *i*'s behavior multiplied by the average behavior of his alters (a kind of ego-alter behavior covariance), $s_{i13}^{\text{beh}}(x) = z_i \left(\sum_j x_{ij} z_j\right) / \left(\sum_j x_{ij}\right)$ (and the mean behavior, i.e. 0, if the ratio is 0/0);
- 14. average reciprocated alter effect, defined by the product of *i*'s behavior multiplied by the average behavior of his reciprocated alters, $s_{i14}^{\text{beh}}(x) = z_i \left(\sum_j x_{ij} x_{ji} z_j\right) / \left(\sum_j x_{ij} x_{ji}\right)$ (and 0 if the ratio is 0/0);
- 15. effect of the behavior upon itself, which is like a quadratic shape effect, where the attractiveness of further steps up the behavior 'ladder' depends on where the actor is on the ladder: $s_{i15}^{\text{beh}}(x) = z_i^2$
- 16. dense triads effect, defined by the number of dense triads in which actor *i* is located, $s_{i16}^{\text{beh}}(x) = z_i \sum_{j,h} I\{x_{ij} + x_{ji} + x_{ih} + x_{hi} + x_{jh} + x_{hj}\} \ge c\},$ where *c* is either 5 or 6; *this is currently not correctly implemented in* SIENA 3;
- 17. peripheral effect, defined by the number of dense triads to which actor *i* stands in a unilateralperipheral relation, $s_{i17}^{\text{beh}}(x) = z_i \sum_{j,h,k} x_{ij}(1-x_{ji})(1-x_{hi})(1-x_{ki})I\{x_{ij} + x_{ji} + x_{ih} + x_{hi} + x_{jh} + x_{hj}) \ge c\}$, where *c* is the same constant as in the *dense triads* effect; for directed networks, the unilateral condition is dropped, and the effect is $s_{i17}^{\text{beh}}(x) = z_i \sum_{j,h,k} x_{ij}(1-x_{hi})(1-x_{ki})I\{x_{ij} + x_{ji} + x_{ih} + x_{hi} + x_{jh} + x_{hj}) \ge c\}$; this is currently not correctly implemented in SIENA 3;
- 18. reciprocated degree effect, $s_{i18}^{\text{beh}}(x) = z_i \sum_j x_{ij} x_{ji};$
- 19. average similarity × popularity ego effect, defined by the sum of centered similarity scores \sin_{ij}^z between *i* and the other actors *j* to whom he is tied, multiplied by ego's indegree, $s_{i19}^{\rm beh}(x) = x_{+i} x_{i+}^{-1} \sum_j x_{ij} (\sin_{ij}^z - \widehat{\sin}^z);$ (and 0 if $x_{i+} = 0$); because of collinearity, under the Method of Moments this cannot be estimated together with the average similarity × popularity alter effect.

For each actor-dependent covariate v_j (recall that these are centered internally by SIENA) as well as for each of the other dependent behavior variables (for notational simplicity here also denoted v_j), there is one main effect and one interaction effect, the latter of which is a choice among three, dependent on the internal parameter for this effect:

20. covariate effect,

 $s_{i20}^{\text{beh}}(x) = z_i v_i ;$

here too, the other dependent behavioral variables are centered so that they have overall mean 0;

21. depending on the parameter value (1, 2, or 3):

- value 1: interaction of actor variable with average similarity, $s_{i21}^{\text{beh}}(x) = (v_i/x_{i+}) \sum_j x_{ij} (\sin_{ij}^z - \widehat{\sin}^z);$ (and 0 if $x_{i+} = 0$);
- value 2: interaction of actor variable with total similarity, $s_{i21}^{\text{beh}}(x) = v_i \sum_j x_{ij} (\sin_{ij}^z - \widehat{\sin}^z);$
- value 3: interaction of actor variable with average alter, $s_{i21}^{\text{beh}}(x) = v_i z_i \left(\sum_j x_{ij} z_j\right) / \left(\sum_j x_{ij}\right)$ (and the mean behavior, i.e. 0, if the ratio is 0/0);
- 22. There are also *user-defined interaction effects* between actor variables, defined as the product of two or three grand mean centered variables. (If these include the dependent variable itself, special formulae are used for the change statistic.) How to specify these is described in Section 6.6.2.

Additional possible effects are documented in Steglich, Snijders and Pearson (2010).

15.2.2 Behavioral endowment function

Also the behavioral model knows the distinction between evaluation and endowment effects. The formulae of the effects that can be included in the behavioral endowment function e^{beh} are the same as those given for the behavioral evaluation function. However, they enter calculation of the endowment function only when the actor considers decreasing his behavioral score by one unit (downward steps), not when upward steps (or no change) are considered. For more details, consult Snijders, Steglich, and Schweinberger (2007) and Steglich, Snijders and Pearson (2010).

The statistics reported as *dec. beh.* (decrease in behavior) are the sums of the changes in actor-dependent values for only those actors who decreased in behavior. More precisely, it is

$$\sum_{m=1}^{M-1} \sum_{i=1}^{n} I\{z_i(t_{m+1}) < z_i(t_m)\} \left(s_{ik}^{\text{beh}}(x(t_{m+1})) - s_{ik}^{\text{beh}}(x(t_m))\right),\tag{4}$$

where M is the number of observations, $x(t_m)$ is the observed situation at observation m, and the indicator function $I\{A\}$ is 0 if event A is true and 0 if it is untrue.

15.2.3 Behavioral rate function

The behavioral rate function λ^{beh} consists of a constant term per period,

$$\lambda_i^{\text{beh}} = \rho_m^{\text{beh}}$$

for m = 1, ..., M - 1.

15.3 Exponential random graph model

The exponential random graph model, which is used if there is only one observation (i.e., for the non-longitudinal case), is defined by the probability function

$$P_{\theta}\{X=x\} = \exp\left(\theta' u(x) - \psi(\theta)\right), \qquad (5)$$

where u(x) is a vector of statistics. The following statistics are available. The selection of statistics is discussed extensively in Snijders, Pattison, Robins, and Handcock (2006), with attention especially to statistics 16–19. Note that SIENA will note whether the observed graph (x_{ij}) is symmetric or not, and choose accordingly between the statistics for undirected and directed graphs.

Also not that, as earlier, c is a constant that can be set by the user.

- 1. For undirected graphs, the number of edges $\sum_{i < j} x_{ij}$; for directed graphs, the number of arcs $\sum_{i \neq j} x_{ij}$.
- 2. The number of reciprocated relations $\sum_{i < j} x_{ij} x_{ji}$.
- 3. For undirected graphs, the number of transitive triads $\frac{1}{6} \sum_{i,j,h} x_{ij} x_{ih} x_{jh}$; for directed graphs, the number of transitive triplets $\sum_{i,j,h} x_{ij} x_{ih} x_{jh}$.
- 4. The number of three-cycles given for undirected graphs by $\frac{1}{6} \sum_{i,j,h} x_{ij} x_{jh} x_{hi}$ and for directed graphs by $\frac{1}{3} \sum_{i,j,h} x_{ij} x_{jh} x_{hi}$.
- 5. The number of out-twostars $\sum_{i} \sum_{h < k} x_{ih} x_{ik}$.
- 6. The number of in-twostars $\sum_{i} \sum_{h < k} x_{hi} x_{ki}$.
- 7. The number of two-paths (mixed twostars) given for undirected graphs by $\frac{1}{2} \sum_{i} \sum_{h \neq k} x_{hi} x_{ik}$ and for directed graphs by $\sum_{i} \sum_{h \neq k} x_{hi} x_{ik}$.
- 8. The number of *c*-instars, for *c* to be chosen by the user, $\sum_{i} {\binom{x_{i+}}{c}}$; starting from version 3.17t this was changed to $\sum_{i} {\binom{x_{i+}}{c}}/{(c!)}$ to let parameters be less small for larger *c*;
- 9. The number of *c*-outstars, for *c* to be chosen by the user, $\sum_{i} {\binom{x_{+i}}{c}}$; starting from version 3.17t this was changed to $\sum_{i} {\binom{x_{+i}}{c}}/{(c!)}$;
- 10. The sum of reciprocal out-degrees $\sum_{i} 1/(x_{i+} + c)$ for some constant c.
- 11. The sum of transformed out-degrees $\sum_{i} 1/[(x_{i+}+c)(x_{i+}+c+1)]$ for some constant c.
- 12. The alternating k-out-stars combination

$$c^{2} \sum_{i=1}^{n} \left\{ \left(1 - \frac{1}{c}\right)^{x_{i+1}} + \frac{x_{i+1}}{c} - 1 \right\} ,$$

for some value c.

13. The alternating k-in-stars combination

$$c^{2} \sum_{i=1}^{n} \left\{ \left(1 - \frac{1}{c}\right)^{x_{+i}} + \frac{x_{+i}}{c} - 1 \right\}$$

for some value c.

14. The number of pairs directly and indirectly connected,
i.e., tied pairs (i, j) for which there exists at least one h such that x_{ih} = x_{hj} = 1,
i.e., tied pairs for which there is at least one twopath from i to j;
for undirected graphs this is ∑_{i<j} x_{ij} max_{h≠i,j} {x_{ih} x_{hj}}, for directed graphs it is ∑_{i≠j} x_{ij} max_{h≠i,j} {x_{ih} x_{hj}}.

- 15. The number of indirectly connected pairs, i.e., pairs (i, j) for which there is at least one twopath from i to j;
 - for undirected graphs this is $\sum_{i < j} \max_{h \neq i,j} \{x_{ih} x_{hj}\}$, for directed graphs it is $\sum_{i \neq j} \max_{h \neq i,j} \{x_{ih} x_{hj}\}$.
- 16. The number of four-cycles given by $\sum_{i,j,h,k;\text{all distinct}} x_{ij} x_{jh} x_{hk} x_{ki}$.
- 17. The 3-parallel two-paths statistic, given by

$$\sum_{i,j,k_1,k_2,k_3;\text{all distinct}} \left\{ x_{ik_1} \, x_{k_1j} \, x_{ik_2} \, x_{k_2j} \, x_{ik_3} \, x_{k_3j} \right\} \; .$$

(Just for the sake of it: the 2-parallel two-paths are not included, because they are the same as the four-cycles.)

18. The alternating independent two-paths statistic, given for undirected graphs by

$$c \sum_{i < j} \left\{ 1 - \left(1 - \frac{1}{\lambda} \right)^{L_{2ij}} \right\}$$

which formula is replaced for directed graphs by

$$c \sum_{i,j} \left\{ 1 - \left(1 - \frac{1}{\lambda}\right)^{L_{2ij}} \right\}$$

- 19. The 2-triangles statistic, given by $\sum_{i,j,h,k} x_{ij} x_{jh} x_{hk} x_{ki} x_{ih}$.
- 20. The 3-triangles statistic, given by $\sum_{i,j,k_1,k_2,k_3;all \text{ distinct }} x_{ij} x_{ik_1} x_{jk_1} x_{ik_2} x_{jk_2} x_{ik_3} x_{jk_3}$.
- 21. The alternating k-triangles statistic, given for undirected graphs by

$$c\sum_{i < j} x_{ij} \left\{ 1 - \left(1 - \frac{1}{c}\right)^{L_{2ij}} \right\}$$

for some value c, where L_{2ij} is the number of two-paths from i to j, $L_{2ij} = \sum_{h} x_{ih} x_{hj}$; for directed graphs, the formula is

$$c\sum_{i,j} x_{ij} \left\{ 1 - \left(1 - \frac{1}{c}\right)^{L_{2ij}} \right\} ,$$

with the same definition of L_{2ij} .

- 22. For non-directed graphs, the number of coathangers given by $\sum_{i,j,h,k;all \text{ distinct }} x_{ij} x_{jh} x_{hi} x_{hk}$.
- 23. The number of three-paths given by $\sum_{i,j,h,k;all \text{ distinct }} x_{ij} x_{jh} x_{hk}$.
- 24. The number of cliques of order c, for c to be chosen by the user.
- 25. For each dyadic covariate w_{ij} , the sum $\sum_{i,j} x_{ij} w_{ij}$.
- 26. For each dyadic covariate w_{ij} , the associated reciprocity effect defined by $\sum_{i,j} x_{ij} x_{ji} w_{ij}$.
- 27. For each individual covariate v_i (changing or constant; recall that all covariates are centered), four effects are included.

The first is the v_i -related popularity effect $\sum_i x_{+i} v_i$;
- 28. next is the v_i -related activity effect $\sum_i x_{i+} v_i$;
- for non-directed networks, these together are replaced by the single covariate effect $\sum_{i} x_{+i} v_i = \sum_{i} x_{i+} v_i$;
- 29. the v_i -related similarity effect, defined by the sum of centered similarity scores $\sum_{i,j} x_{ij} (\sin_{ij}^v - \widehat{\sin^v}),$ where $\widehat{\sin^v}$ is the mean of all similarity scores, which are defined as $\sin_{ij}^v = \frac{\Delta - |v_i - v_j|}{\Delta}$ with $\Delta = \max_{ij} |v_i - v_j|$ being the observed range of the covariate v;
- 30. and fourth the same v_i effect is defined by $\sum_{i,j} x_{ij} I\{v_i = v_j\}$, where the indicator function $I\{v_i = v_j\}$ is 1 if the condition $\{v_i = v_j\}$ is satisfied, and 0 if it is not; this can also be called the 'same V' effect;
- 31. user-defined interaction effects are possible as described in Section 6.6. The internal effect parameter is decomposed by SIENA into its two or three constituents, as described in the mentioned section. The change statistic for the interaction is the product of the change statistics of the two or three components.

16 Parameter interpretation

This section still is in development.

16.1 Longitudinal models

The main 'driving force' of the actor-oriented model is the evaluation function (in earlier publications called objective function, see Snijders, 2001, 2005) given in formula (2) (for the network) as

$$f^{\rm net}(x) = \sum_k \beta_k^{\rm net} s_{ik}^{\rm net}(x) \ .$$

The objective function can be regarded as the "attractiveness" of the network (or behavior, respectively) for a given actor. For getting a feeling of what are small and large values, is is helpful to note that the objective functions are used to compare how attractive various different tie changes are, and for this purpose random disturbances are added to the values of the objective function with standard deviations equal⁵ to 1.28.

An alternative interpretation is that when actor *i* is making a 'ministep', i.e., a single change in his outgoing ties (where no change also is an option), and x_a and x_b are two possible results of this ministep, then $f^{\text{net}}(x_b) - f^{\text{net}}(x_a)$ is the log odds ratio for choosing between these two alternatives – so that the ratio of the probability of x_b and x_a as next states is

$$\exp(f^{\rm net}(x_b) - f^{\rm net}(x_a)) \; .$$

Note that, when the current state is x, the possibilities for x_a and x_b are x itself (no change), or x with one extra outgoing tie from i, or x with one fewer outgoing tie from i. Explanations about log odds ratios can be found in texts about logistic regression and loglinear models.

The evaluation function is a weighted sum of 'effects' $s_{ik}^{net}(x)$. Their formulae can be found in Section 15.1.1. These formulae, however, are defined as a function of the whole network x, and in most cases the contribution of a single tie variable x_{ij} is just a simple component of this formula. The contribution to $s_{ik}^{net}(x)$ of adding the tie $i \to h$ minus the contribution of adding the tie $i \to j$ is the log odds ratio comparing the probabilities of i sending a new tie to h versus sending the tie to j, if all other effects $s_{ik}^{net}(x)$ yields the same values for these two hypothetical new configurations.

For example, suppose that actors j and h, actual or potential relation partners of actor i, have exactly the same network position and the same values on all variables included in the model, except that for some actor variable V for which only the popularity (alter) effect is included in the model, actor h is one unit higher than actor j: $v_h = v_j + 1$. It can be seen in Section 15.1.1 that the popularity (alter) effect is defined as

$$s_{ik}^{\text{net}}(x) = \sum_j x_{ij} v_j \; .$$

The contribution to this formula made by a single tie variable, i.e., the difference made by filling in $x_{ij} = 1$ or $x_{ij} = 0$ in this formula, is just v_j . Let us denote the weight of the V-alter effect by β_k . Then, the difference between extending a tie to h or to j that follows from the V-alter effect is $\beta_k \times (v_h - v_j) = \beta_k \times 1 = \beta_k$.

Thus, in this situation, β_k is the log odds ratio of the probability that h is chosen compared to the probability that j is chosen. E.g., if i currently has a tie neither to j nor to h, and supposing that $\beta_k = 0.3$, the probability for i to extend a new tie to h is $e^{0.3} = 1.35$ times as high as the probability for i to extend a new tie to j.

⁵More exactly, the value is $\sqrt{\pi^2/6}$, the standard deviation of the Gumbel distribution; see Snijders (2001).

16.1.1 Ego – alter selection tables

When some variable V occurs in several effects in the model, then its effects can best be understood by considering all these effects simultaneously. For example, if in a network dynamics model the ego, alter, and similarity effects of a variable V are specified, then the formulae for their contribution can be obtained from the components listed in Section 15.1.1 as

$$\beta_{\text{ego}} v_i x_{i+} + \beta_{\text{alter}} \sum_j x_{ij} v_j + \beta_{\text{sim}} \sum_j x_{ij} (\sin^v_{ij} - \widehat{\sin^v}) , \qquad (6)$$

where the similarity score is $\sin_{ij}^{v} = 1 - \frac{|v_i - v_j|}{\Delta_V}$, with $\Delta_V = \max_{ij} |v_i - v_j|$ being the observed range of the covariate v and where $\widehat{\sin^v}$ is the mean of all similarity scores. The superscript ^{net} is left out of the notation for the parameters in order not to clutter the notation.

Similarly to how it was done above, the contribution to (6) of the tie from i to j, represented by the single tie variable x_{ij} – i.e., the difference between the values of (6) for $x_{ij} = 1$ and $x_{ij} = 0$ – can be calculated from this formula. It should be noted that all variables are internally centered by SIENA, and that the mean values used for the centering are given near the beginning of the input file. This is made explicit in the following by the subtraction of the mean \bar{v} . The contribution of

$$\beta_{\text{ego}} \left(v_i - \bar{v} \right) + \beta_{\text{alter}} \left(v_j - \bar{v} \right) + \beta_{\text{sim}} \left(\operatorname{sim}_{ij}^v - \operatorname{sim}^v \right) = \beta_{\text{ego}} \left(v_i - \bar{v} \right) + \beta_{\text{alter}} \left(v_j - \bar{v} \right) + \beta_{\text{sim}} \left(1 - \frac{|v_i - v_j|}{\Delta_V} - \widehat{\operatorname{sim}^v} \right).$$
(7)

From this equation a table can be made that gives the outcome of (7) for some values of v_i and v_j .

This can be concretely carried using the data set s50 which is an excerpt of 50 girls in the data set used in Pearson and Michell (2000), Pearson and West (2003), Steglich et al. (2006) and Steglich et al. (2007). We refer to any of these papers for a further description of the data. The friendship network data over 3 waves are in the files s50-network1.dat, s50-network2.dat, and s50-network3.dat. We also use the attribute data for alcohol use, s50-alcohol.dat, as a dependent variable. It can be seen from the SIENA output file using these data that the alcohol use variable assumes values from 1 to 5, with overall mean equal to $\bar{v} = 3.113$, and mean of the similarity variable $\hat{\sin}^v = 0.6983$. Drug use is used as a changing actor variable, with range 1–4, average $\bar{v} = 1.5$ and average dyadic similarity $\hat{\sin}^v = 0.7533$.

Suppose that we fit a model of network-behavior co-evolution to this data set with for the network evolution the effects of outdegree, reciprocity, transitive ties, number of distances two, the ego, alter, and similarity effects of alcohol use, as well as the ego, alter, and similarity effects of drug use; and for the behavior (i.e., alcohol) dynamics the shape effect, the effect of alcohol on itself (quadratic shape effect), and the average similarity effect.

The results obtained are given in the following part of the output file.

	Network	Dynamics
--	---------	----------

1.	rate:	constant network rate (period 1)	8.2357	(1.6225)
2.	rate:	constant network rate (period 2)	5.6885	(0.8434)
З.	eval:	outdegree (density)	-2.1287	(0.1565)
4.	eval:	reciprocity	2.3205	(0.2132)
5.	eval:	transitive ties	0.2656	(0.2025)
6.	eval:	number of actors at distance 2	-0.9947	(0.2173)
7.	eval:	drink alter	0.0899	(0.1184)
8.	eval:	drink ego	-0.0100	(0.1087)
9.	eval:	drink similarity	0.8994	(0.5864)
10.	eval:	drug use alter	-0.1295	(0.1282)

11. 12.	eval: eval:	drug use ego drug use similarity	0.1362 0.6650	((0.1253) 0.3381)
Beha	avior Dy	vnamics			
13.	rate:	rate drink period 1	1.3376	(0.3708)
14.	rate:	rate drink period 2	1.8323	(0.4546)
15.	eval:	behavior drink shape	0.3618 (0.	1946)
16.	eval:	behavior drink average similarity	3.9689	(2.2053)
17.	eval:	behavior drink: effect from drink	-0.0600	(0.1181)

We interpret here the parameter estimates for the effects of drinking behavior and drug use without being concerned with the significance, or lack thereof. For the drinking behavior, formula (7) yields (rounded to two decimals)

$$-0.01 (v_i - \bar{v}) + 0.09 (v_j - \bar{v}) + 0.90 \left(1 - \frac{|v_i - v_j|}{\Delta_V} - 0.70\right) \,.$$

The results can be tabulated as follows.

1	2	3	4	5
0.10	-0.03	-0.17	-0.30	-0.44
-0.13	0.18	0.05	-0.09	-0.22
-0.37	-0.05	0.26	0.13	-0.01
-0.60	-0.29	0.03	0.34	0.21
-0.84	-0.52	-0.21	0.11	0.42
	$\begin{array}{c} 1 \\ 0.10 \\ -0.13 \\ -0.37 \\ -0.60 \\ -0.84 \end{array}$	$\begin{array}{c cccc} 1 & 2 \\ \hline 0.10 & -0.03 \\ -0.13 & 0.18 \\ -0.37 & -0.05 \\ -0.60 & -0.29 \\ -0.84 & -0.52 \end{array}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

This table shows the preference for similar alters: in all rows, the highest value is at the diagonal $(v_j = v_i)$. The ego and alter parameters are close to 0, therefore the similarity effect is dominant. However, note that the formula uses raw values for v_i and v_j but divides the values for the absolute difference $|v_i - v_j|$ by Δ_V which here is 5 - 1 = 4. Therefore the weight of 0.09 for the alter effect is not completely negligible compared to the weight of 0.90 for the similarity effect. The positive alter effect leads to a preference for ties to alters with a high v_j value which goes against the similarity effect for $v_i = 1$ but strengthens the similarity effect for $v_i = 5$. The table shows that the net resulting preference for similar others is strongest for actors (egos) high on drinking behavior, and weakest for actors in the middle and low range of drinking behavior.

For drug use, the formula yields

$$0.14 (v_i - \bar{v}) - 0.13 (v_j - \bar{v}) + 0.67 \left(1 - \frac{|v_i - v_j|}{\Delta_V} - 0.7533\right),$$

which leads to the following table.

$z_i \setminus z_i$	z_j 1	2	3	4
1	0.16	-0.19	-0.54	-0.89
2	0.08	0.17	-0.18	-0.53
3	-0.01	0.08	0.17	-0.18
4	-0.10	-0.00	0.09	0.18

In each row the highest value is at the diagonal, which shows that indeed everybody prefers to be friends with similar others also with respect to drug use. The negative alter effect supports this for low v_i values and counteracts it for high v_i values. This is seen in the table in the strong preference of low drug users ($v_i = 1$) for others who are low on drug use, and the very weak preference for high drug users ($v_i = 4$) for others also high on drug use.

An alternative specification uses the drink ego \times drink alter interaction together with the drink squared alter effect in the network dynamics model, and similarly for drug use; for the behavior dynamics, an alternative specification uses the average alter effect. This leads to the following table of results.

1.	rate:	constant network rate (period 1)	8.0978	(1.5118)
2.	rate:	constant network rate (period 2)	5.7781	(0.9474)
З.	eval:	outdegree (density)	-2.1333	(0.2196)
4.	eval:	reciprocity	2.3033	(0.2184)
5.	eval:	transitive ties	0.2430	(0.2059)
6.	eval:	number of actors at distance 2	-1.0011	(0.2275)
7.	eval:	drink alter	0.1041	(0.1348)
8.	eval:	drink squared alter	0.0141	(0.1329)
9.	eval:	drink ego	0.0078	(0.1157)
10.	eval:	drink ego x drink alter	0.1655	(0.1095)
11.	eval:	drug use alter	-0.2603	(0.2436)
12.	eval:	drug use squared alter	-0.0249	(0.1945)
13.	eval:	drug use ego	-0.0214	(0.1454)
14.	eval:	drug use ego x drug use alter	0.1976	(0.1146)
Beha	avior D	ynamics			
15.	rate:	rate drink period 1	1.3218	(0.3632)
16.	rate:	rate drink period 2	1.7884	(0.5053)
17.	eval:	behavior drink shape	0.3820 (0	.2421)
18.	eval:	behavior drink average alter	1.1414	(0.6737)
19.	eval:	behavior drink: effect from drink	-0.5428	(0.2839)

For this specification, the formulae in Section 15.1.1 imply that the components in the network objective function corresponding to the effects of variable V are

$$\beta_{\rm ego} \left(v_i - \bar{v} \right) x_{i+} + \beta_{\rm alter} \sum_j x_{ij} \left(v_j - \bar{v} \right) + \beta_{\rm sq. \ alter} \sum_j x_{ij} \left(v_j - \bar{v} \right)^2 + \beta_{\rm e \times a} \sum_j x_{ij} \left(v_i - \bar{v} \right) \left(v_j - \bar{v} \right) \,. \tag{8}$$

The contribution of the single tie variable x_{ij} to this formula is equal to

$$\beta_{\text{ego}}\left(v_{i}-\bar{v}\right) + \beta_{\text{alter}}\left(v_{j}-\bar{v}\right) + \beta_{\text{sq. alter}}\left(v_{j}-\bar{v}\right)^{2} + \beta_{\text{e}\times a}\left(v_{i}-\bar{v}\right)\left(v_{j}-\bar{v}\right).$$

$$\tag{9}$$

Filling in the estimates for the effects of drinking behavior yields

 $0.01 (v_i - \bar{v}) + 0.10 (v_j - \bar{v}) + 0.01 (v_j - \bar{v})^2 + 0.17 (v_i - \bar{v}) (v_j - \bar{v}) .$

and this gives the following table.

$v_i \setminus v_j$	1	2	3	4	5
1	0.54	0.27	0.01	-0.23	-0.45
2	0.20	0.09	0.00	-0.07	-0.13
3	-0.15	-0.09	-0.01	0.08	0.19
4	-0.49	-0.26	-0.02	0.24	0.51
5	-0.83	-0.44	-0.03	0.39	0.83

For drug use we obtain the formula

$$-0.02(v_i - \bar{v}) - 0.26(v_j - \bar{v}) - 0.02(v_j - \bar{v})^2 + 0.20(v_i - \bar{v})(v_j - \bar{v}).$$

and the following table.

$v_i \setminus$	v_j	1	2	3	4
1		0.18	-0.18	-0.58	-1.04
2		0.06	-0.10	-0.31	-0.57
3		-0.06	-0.02	-0.03	-0.10
4		-0.18	0.06	0.24	0.38

The fact that we are using three variables involving alter (alter, alter squared, interaction) instead of two (alter and similarity) leads to greater freedom in the curve that is fitted: the top (or, in the rare case of a reversed pattern, bottom) of the attractiveness of alters is not necessarily obtained at the diagonal, i.e., at ego's value. Straightforward calculus shows us that (9) is a quadratic function and obtains its extreme value (a maximum if $\beta_{sq. alter}$ is negative, a minimum if it is positive – the latter is, in general, less likely) for

$$v_j = \bar{v} - \frac{\beta_{\text{alter}} + \beta_{\text{e} \times \text{a}} \left(v_i - \bar{v} \right)}{2\beta_{\text{sq. alter}}} \,. \tag{10}$$

If the effect $\beta_{\text{sq. alter}}$ of the squared alter's value is negative and the interaction effect $\beta_{e\times a}$ is positive, then this location of the maximum increases with ego's own value, v_i . Of course the number given by (10) will usually not be an integer number, so the actual value of v_j for which attractiveness is maximized is the integer in the range of V closest to (10).

For drinking there is a weak positive effect of squared drinking alter; the effect of squared drug use alter is weak negative. For drinking we see that the most attractive value for egos with $v_i = 1$ or 2 is no drinking, $v_j = 1$, whereas for egos with $v_i \ge 3$ the most attractive alters are those who drink most, $v_j = 5$. We also see that egos with the highest drinking behavior are those who differentiate most strongly depending on the drinking behavior of their potential friends.

For drug use the situation is different. Actors with $v_i = 1$ or 2 prefer friends with drug use $v_j = 1$; for actors with $v_i = 3$ the difference is hardly discernible, but if we consider the differences even though they are tiny, then they are most attracted to others with $v_j = 2$; actors with the highest drug use ($v_i = 4$) differentiate most strongly, and are attracted most to others with also the highest drug use.

The differences between the results with the similarity effects and the interaction effects are minor. The extra degrees of freedom of the latter model gives a slightly closer fit to the data. However, the differences between the two fits are not significant, as can be shown e.g. by scoretype tests.

16.1.2 Ego – alter influence tables

In quite a similar way as in the preceding section, from the output tables and the formulae for the effects we can construct tables indicating how attractive various different values of the behavior are, depending on the behavior of the actor's friends.

In the first model, the estimated coefficients in the behavior evaluation function are as follows.

15.	eval:	behavior	drink	shape		0.3618	(0.1946)
16.	eval:	behavior	drink	average	similarity	3.9689	(2.2053)
17.	eval:	behavior	drink	effect	from drink	-0.0600	(0.1181)

The dependent behavior variable now is indicated Z. (In the preceding section the letter V was used, but this referred to any actor variable predicting network dynamics, whether it was also a dependent variable or not.) The formulae in Section 15.2.1 show that the evaluation function for this model specification is

$$u^{\text{beh}} = \beta_{\text{trend}} \left(z_i - \bar{z} \right) + \beta_{\text{drink}} \left(z_i - \bar{z} \right)^2 + \beta_{\text{av. sim}} \frac{1}{x_{i+}} \sum_j x_{ij} (\sin_{ij}^z - \widehat{\sin}^z) .$$
(11)

In the second model, the table gave the following results.

17.	eval:	behavior	drink	shape		0.3820	(0.2421)
18.	eval:	behavior	drink	average	alter	1.1414	(0.6737)
19.	eval:	behavior	drink	: effect	from drink	-0.5428	(0.2839)

Here the evaluation function is

$$u^{\text{beh}} = \beta_{\text{trend}} \left(z_i - \bar{z} \right) + \beta_{\text{drink}} \left(z_i - \bar{z} \right)^2 + \beta_{\text{av. alter}} \left(z_i - \bar{z} \right) \left(\bar{z}_{(i)} - \bar{z} \right) \,, \tag{12}$$

where $\bar{z}_{(i)}$ is the average Z value of *i*'s friends⁶,

$$\bar{z}_{(i)} = \frac{1}{x_{i+}} \sum_{j} x_{ij} \, z_j$$

Equation (12) is simpler than equation (11), because (12) is a quadratic function of z_i , with coefficients depending on the Z values of *i*'s friends as a function of their average, whereas (11) depends on the entire distribution of the Z values of *i*'s friends.

Suppose that, in model (11), the similarity coefficient $\beta_{\text{av. sim}}$ is positive, and compare two focal actors, i_1 all of whose friends have $z_j = 3$ and i_2 who has four friends, two of whom with $z_j = 2$ and the other two with $z_j = 4$. Both actors are then drawn toward the preferred value of 3; but the difference between drinking behavior 3 on one hand and 2 and 4 on the other hand will be larger for i_1 than for i_2 . In model (12), on the other hand, since the average is the same, both actors would be drawn equally strongly toward the average value 3.

For model (11), consider actors in the extreme situation that all their friends have the same behavior z_{ij} . For the parameters given above, the behavior objective function then reads

$$u^{\text{beh}} = 0.36 \left(z_i - \bar{z} \right) - 0.06 \left(z_i - \bar{z} \right)^2 + 3.97 \left(\sin_{ij}^z - \widehat{\sin^z} \right) \,.$$

This can be tabulated as follows.

⁶If *i* has no friends, i.e., $x_{i+} = 0$, then $\bar{z}_{(i)}$ is defined to be equal to \bar{z} .

$\bar{z}_{(i)}$	\	z_i	1	2	3	4	5
1			-0.05	-0.82	-1.71	-2.72	-3.84
2			-1.38	0.50	-0.39	-1.39	-2.52
3			-2.70	-0.82	0.94	-0.07	-1.20
4			-4.02	-2.14	-0.39	1.25	0.13
5			-5.35	-3.47	-1.71	-0.07	1.45

For the other model, filling in the estimated parameters in (12) yields

$$u^{\text{beh}} = 0.38 (z_i - \bar{z}) - 0.54 (z_i - \bar{z})^2 + 1.14 (z_i - \bar{z})(\bar{z}_{(i)} - \bar{z}) .$$

For a given average Z values of *i*'s friends, this is a quadratic function of z_i . The following table indicates the behavior objective function for z_i (columns) as a function of the average drinking behavior of *i*'s friends (rows).

$\bar{z}_{(i)}$	\	z_i	1	2	3	4	5
1			1.87	1.59	0.22	-2.23	-5.76
2			-0.55	0.32	0.09	-1.22	-3.61
3			-2.96	-0.95	-0.04	-0.20	-1.46
4			-5.37	-2.22	-0.16	0.81	0.70
5			-7.78	-3.49	-0.29	1.82	2.85

We see that, even though the squared function does not necessarily draw the actors toward the average of their friends' behavior, for these parameters the highest values of the behavior objective function are obtained indeed when the focal actor (i) behaves just like the average of his friends. It should be noted that no between-ego comparisons are made, so comparisons are meaningful only within rows. The values far away from the maximum contrast in this case more strongly than in the case of the model with the average similarity effect, but these differences here are not significant.

Another way to look at the behavior objective function is to consider the location of its maximum. This function here can be written also as

$$u^{\text{beh}} = (0.38 + 1.14(\bar{z}_{(i)} - \bar{z}))(z_i - \bar{z}) - 0.54(z_i - \bar{z})^2.$$

This function is maximal for

$$z_i = \bar{z} + 0.35 + 1.05 (z_i - \bar{z})$$
.

16.2 Exponential random graph models

The definition of the exponential random graph model is given by the probability function

$$P_{\theta}\{X=x\} = \exp\left(\sum_{k} \left(\theta_{k} u_{k}(x)\right) - \psi(\theta)\right), \qquad (13)$$

given above as (5). The parameter values can be interpreted by considering the conditional probability for a tie to exist from *i* to *j*, conditional on the rest of the graph. Let us, for a given graph *x*, denote by $\tilde{x}^{(1)}(i,j)$ the graph with all ties as *x* but with an tie from *i* to *j*, i.e., *x* possibly changed so that $(\tilde{x}^{(1)}(i,j))_{ij} = 1$; by $\tilde{x}^{(0)}(i,j)$ the graph with all ties as *x* but without a tie from *i* to *j*, so that $(\tilde{x}^{(0)}(i,j))_{ij} = 0$; and by $\tilde{x}(i,j)$ the graph *x* but without the information about the existence of this particular tie. Then it follows from this probability function that the conditional probability of a tie to exist from i to j, given the rest of the graph, is given by

$$P_{\theta}\{X_{ij} = 1 \mid \tilde{x}(i,j)\} = \exp\left(\sum_{k} \left(\theta_k(u_k(\tilde{x}^{(1)}(i,j)) - u_k(\tilde{x}^{(0)}(i,j))\right)\right).$$
(14)

For any of the elements u_k of the vector u, the quantity

$$u_k(\tilde{x}^{(1)}(i,j)) - u_k(\tilde{x}^{(0)}(i,j))$$
(15)

is called the *change statistic* (Wasserman and Pattison, 1996). In words: this is the increase in the function u_k obtained by changing x_{ij} from 0 to 1. Formula (14) implies that parameter θ_k , which is the weight of u_k in the exponential function used in (5), is the log odds ratio for the conditional probability (14) of the tie from *i* to *j*, associated to a one unit increase in the change statistic (15). The functions u_k can be found in Section 15.3. Further explanation about log odds ratios can be found in texts about logistic regression and loglinear models.

E.g., for the reciprocity effect, where the statistic is the number of reciprocated ties

$$u_2(x) = \sum_{i < j} x_{ij} \, x_{ji} \; ,$$

the change statistic is the increase in the number of reciprocated ties when increasing x_{ij} from 0 to 1; this is equal to x_{ji} , the indicator of a tie from j to i. Hence the reciprocity parameter θ_2 is the log odds ratio for the probability of the tie from i to j due to the existence of the tie from j to i. The existence of the latter tie will multiply the odds for the tie from i to j to exist by the factor $\exp(\theta_2)$. E.g., if the value of this parameter is $\theta_2 = 2$, then this factor is $e^2 = 7.4$. This is quite a usual value for the reciprocity parameter, which indicates that tie reciprocation indeed has a very strong effect.

17 Running Siena outside of StOCNET

The SIENA program consists of a basic computation part, which is associated with the StOCNET windows shell. The computational part can be used both directly and from StOCNET. The StOC-NET windows shell is easy for data specification and model definition. However, especially for frequent users it can be more convenient to run SIENA outside of the StOCNET environment. How to do this, is explained in this section.

The computational part of SIENA is composed of seven executable programs. These programs are:

- 1. Siena01.exe for the basic data input, using an existing basic information file (see below);
- 2. Siena02.exe for data description;
- 3. Siena03.exe for command line changes in parameters; see Section 18;
- 4. Siena04.exe for redefining the names of the effects after changes have been made in the preset (non-estimated) internal effect parameters; and for copying the model definition from one project to another;
- 5. Siena05.exe for simulations with fixed parameter values;
- 6. Siena07.exe for parameter estimation;
- 7. Siena08.exe for multilevel analysis, which is a stand-alone program anyway, and not discussed here (see Section 14).

The mainly used programs are Siena01.exe for initiating the project, which only needs to be done once; and Siena07.exe for parameter estimation.

These programs can be used also independently of StOCNET. To run them, the project name *must* be given in a command line, e.g.

Siena01 bunt

if bunt is the name of the project, and there exists a bunt.in file. This bunt is called a *command line parameter*. There are the following four ways to specify a command with a command line parameter in Windows:

- 1. The command line can be given at the DOS prompt (in a Windows environment);
- 2. it can be given in the Windows "Run" command (for Windows 98 and higher);
- 3. a batch file with extension BAT can be created, e.g., with filename SIE1.BAT, containing the single line

Siena01 bunt

so that the program Siena01 will be executed when the batch file is called (it may be necessary to close the DOS window after the program is ready);

4. or a shortcut to the executable file can be made, where the project name is indicated in the "target" in the "properties" of the shortcut.

The third and fourth ways are the most straightforward in Windows. I (T.S.) prefer the use of the batch file. Note that a shortcut is made by opening Windows Explorer, giving a right mouse-click on the executable file, and then giving a left mouse-click on "Create Shortcut". When the shortcut

has been made, the project name of the SIENA project must be added to the shortcut as follows: give a right mouse-click on the shortcut, then give a left mouse-click on "properties", and in the "Target" field add a space and the project name after the path-plus-filename of the executable file. To run SIENA outside of StOCNET, the steps taken are the following.

- 1. Write the basic information file *pname*.IN which describes the data files and variable names, according to Section 21.1. This file must be in ASCII (text) format. It is also possible to run SIENA once through StOCNET, which will produce the basic information file (recognizable from the extension name .in), instead of writing the basic information file oneself.
- 2. Make shortcuts or batch files as indicated above for each of the programs Siena01 and Siena07; and, if desired, also for Siena02, Siena04, and Siena05.
- 3. Give the session name (indicated here as *pname*) as the command line parameter in the shortcuts or batch files.
- 4. Click on the shortcut or batch file for Siena01. (This should be done only once to create the project, because calling Siena01 for an existing project will overwrite the output file!)
- 5. Open the file *pname*.MO in a text editor, edit it to obtain the desired model specification (see section 21.2.1), and save it as a text (ASCII, TXT) file. (Here it is more convenient to use a light-weight text editor such as Notepad, Textpad or Wordpad, which do not have the inbuilt preferences for formatting text that MS-Word has.)

Note that siena03 also can be used to modify the *pname*.MO file, see Section 18.

6. Click on the shortcut or batch file for Siena07.

The last two steps – modifying the *pname*.MO file and running Siena07 – can be repeated as much as one likes.

If you are working on a computer with one processor and you would like to do other things while Siena07 is plodding along, by changing the command in the batch file to something like

start /belownormal /w Siena07 bunt

- 7. Get some basic descriptive statistics by running Siena02.
- 8. Simulate the model for fixed parameter values by running Siena05. The statistics to be simulated are indicated in the file *pname*.si, which can be modified to add to or delete from the list of simulated statistics, see Section 21.2.2.

17.1Siena04: implementing internal effect parameters

The program Siena04 is of minor importance. Some of the model-defining statistics in the SIENA model contain numbers, or preset (non-changing)⁷ internal effect parameters, which can be set at other values by the user. Examples are the value 3 in "out-degree up to 3" for the longitudinal version of SIENA, and the value 2 in the linear combination of k-out-stars in the non-longitudinal ("exponential random graph" or " p^* " model) version of SIENA. (These numbers should not be confused with the statistical parameters that are estimated by Siena07.) These values are represented

you give the Siena07 a lower priority so that you can continue to work normally on other processes. Other possibilities are:

⁷They are non-changing in the sense that they are not modified by the estimation algorithm, although they can be modified by the user.

in the *pname*.MO file by the last of the 6 values in the line for this effect. When these values are modified, the change must be put into effect by calling Siena04. This will update the names of the corresponding effects in the *pname*.MO file.

17.2 Siena04: copying model and option definitions

Another use of Siena04 is to call it with two parameters, e.g.,

Siena04 bunt bunthelp

if bunt is the name of the project, and there also exists a bunthelp.mo file. Then the model definition for the *bunt* project, i.e., the effects included in the model as well their current parameter values and the preset parameters, and all the options, are imported from the bunthelp.mo file. This is useful, e.g., if one wishes to change a large number of projects to get an identical definition.

18 Command line changes in parameters and options

Starting with SIENA version 3.2, the program Siena03 has been added which gives the possibility of direct changes in parameter values and options, outside of StOCNET (see Section 17) and without using an editor 'by hand' to change the *pname.mo* file which contains the model specification as described in Section 21.2.1. The program operates on the *pname.mo* file, and reads its input from the command line or from a file (which then has to be indicated in the command line – this is option **f** below).

There are commands for the following changes.

- p parameters: values and inclusion/exclusion;
- st starting values;
- est estimation method;
- mc model code for longitudinal modeling, or kind of steps for Exponential Random Graph modeling;
- muf multiplication factor;
- nsub2 number of subphases on phase 2 of the estimation algorithm;
- nrun3 number of runs in phase 3 of the estimation algorithm;
 - seed random number seed;
 - ig initial gain parameter;
 - f read commands from a file.

Commands of type f use the other types of commands, which then must be communicated through the file. The syntax for these commands is given below. When the program cannot interpret the commands given, this will be mentioned in the log file.

Note that another way of changing model specifications is by copying model and option specifications in their totality from one SIENA project to another by siena04.exe, see 17.2.

Parameters

The command (which can be given in a DOS window or in a batch file)

siena03 pname p x3 x4 x5

leads to the following changes in parameters. Here *pname* is the project name, the next p indicates a change to a parameter, and depending on the strings x3 - x5 the following actions are carried out.

- if x3 = r, this is about effect x4 in the rate function;
 for x5 = + this is included, for x5 = this is excluded;
 for x5 equal to some number the effect is set to this number;
- if x3 = v, this is about effect x4 in the evaluation function; for x5 = + this is included, for x5 = - this is excluded; for x5 = t this parameter is fixed to its current value and tested by a score test; for x5 = e it is estimated (not fixed) and not tested; for x5 equal to some number the effect is set to this number;

- if x3 = e, this is about effect x4 in the endowment function;
 - for x5 = + this is included, for x5 = this is excluded;
 - for x5 = t this parameter is fixed to its current value and tested by a score test;
 - for x5 = e it is estimated (not fixed) and not tested;

for x5 equal to some number the effect is set to this number;

• if x3 = p, this is about the internal parameter of effect x4 in the evaluation and endowment functions.

This is set equal to the value x5.

The numbers of the effects can be found in the list which is given for ERG models near the start of the output file, for longitudinal models near the start of the log file. The evaluation/endowment effects can be found also in the *pname*.eff file.

Some example are the following. All these examples presuppose that a SIENA project bunt exists, on which the commands operate. The command

```
siena03 bunt p v 2 0.2
```

changes the current value of the parameter of the 2'nd effect in the evaluation function to 0.2. The command

siena03 bunt p v 3 -

excludes the 3'd effect from the evaluation function. The commands

siena03 bunt p v 6 0.0 siena03 bunt p v 6 t

change the current value of the parameter of the 6'th effect in the evaluation function to 0.0 and request to test this parameter by a score test. The command

.

```
siena03 bunt p p 12 4
```

changes, for the SIENA project bunt, the value of the internal effect parameter of the 12'th effect to 4.

Starting values

The command

siena03 pname st x3

changes the model option for the initial ('starting') values for parameter estimation into 'current starting values' for x3 = 0, or 'standard starting values' for x3 = 1; see Section 7.

Estimation method The command

siena03 pname est x3

changes the estimation method to the value x3, with codes as given in page 96.

Model code The command

siena03 pname mc x3

changes the model code for longitudinal modeling, or kind of steps for Exponential Random Graph modeling, to the value x3.

Multiplication factor

The command

siena03 pname muf x3

changes the multiplication factor to x3. The multiplication factor is important to tune the MCMC algorithm of the ERG model described in Section 11 and for the maximum likelihood and Bayesian estimation of the longitudinal model.

Number of subphases in phase 2

The command

siena03 pname nsub2 x3

changes the number of subphases in phase 2 of the Robbins Monro algorithm to x3. The default value is 4, and there usually is no reason to change this. Further explanation is found in the Section on model options.

Number of runs in phase 3 The command

siena03 pname nrun3 x3

changes the number of runs in phase 3 of the Robbins Monro algorithm to x3. Explanation of this parameter for the algorithm is found in the Section on model options.

Random number seed

The command

siena03 pname seed x3

changes the random number seed to x3.

When this value is 0, the computer determines its own random number seed (depending on the clock). Further explanation is found in the Section on model options.

Initial gain parameter

The command

siena03 pname ig x3

changes the initial gain parameter to x3. This determines the step sizes in the updates of the Robbins Monro algorithm. Further explanation

This determines the step sizes in the updates of the Robbins Monro algorithm. Further explanation is found in the Section on model options.

Read commands from file

The command

siena03 pname f x3

reads commands from the file x3. Each line in this file must contain one command of the type

x2 x3 x4 x5

- in other words, the program name siena03 and the project name indicated above by pname must not be repeated because they are already known from the siena03 f ... command.

19 Limitations and time use

The estimation algorithm, being based on iterative simulation, is time consuming. The time needed for the default way of running the program is approximately proportional to pn^aC where p is the number of parameters, n is the number of actors, the power a is some number between 1 and 2, and C is the number of tie (and behavior) variables that changed between time m and time m + 1, summed over m = 1 to M - 1. For data sets with 30 to 40 actors and something like 5 parameters, the estimation process takes a minute or so on a fast PC. The number of actors n should not give a problem up to, a few hundreds. For large data sets and models, the estimation process may take more minutes up to several hours.

Section 26 indicates the constants in the program that define limitations for the data sets used.

20 Changes compared to earlier versions

There are a few as yet undocumented options that will be further disclosed when papers on these methods will be available; e.g., Bayesian estimation procedures and models for signed digraphs. Version 3.2 has the following extensions and improvements compared to version 3.1.

- 1. Compatibility with Lazarus, which will hopefully permit compilation also for Unix and Mac platforms.
- 2. Implementation of behavioral endowment effects.
- 3. Introduction of the multi-group option.
- 4. A more consistent treatment of structurally determined values that are changing between waves.
- 5. Correction of various bugs.
- 6. From version 3.14 onward: correction of an error in the "same covariate" (earlier called "covariate identity") effect.
- 7. User-defined interactions with actor variables for dependent behavior variables, and interactions of transitivity with reciprocity and actor variables.
- 8. Various effects were added.
- 9. Change of some computations in the non-longitudinal (ERGM) case, allowing using the program for larger networks.
- 10. The new option in the non-longitudinal (ERGM) case of conditioning on the out-degrees.
- 11. When running Siena01 the progress of the program is monitored in windows, which may put the user at ease especially for larger data sets.
- 12. The program siena03 has been added, for command line changes in parameters and options.

Version 3 is a major overhaul of the program. It contains

1. an entirely different internal representation of graphs and digraphs, employing edge lists instead of adjacency matrices to store them, which gives considerable speed increases for large networks;

(input also is possible through edge lists in Pajek format);

- 2. a faster algorithm for ERGM estimation;
- 3. a different internal representation of components of the objective function ('effects'), which should make it easier to add new effects to the program;
- 4. ML estimation procedures for network evolution models as described in Snijders, Koskinen and Schweinberger (2010);
- 5. the possibility to use the program for a number of actors that is limited only by computing time and available memory, not by constraints in the software;
- 6. a further improvement to the estimation of standard errors;
- 7. a corrected and expanded way of modeling longitudinal data of symmetric (i.e., non-directed) networks but this still needs to be documented;
- 8. the possibility to use changing dyadic covariates;
- 9. the option of user-defined interactions;
- 10. the option to use Pajek format for reading network data files;
- 11. various other innovations.

The main innovations in version 2.2 are

- 1. more efficient procedure for estimating standard errors, based on unbiased derivatives estimators (Schweinberger and Snijders, 2007);
- 2. extension of model specification possibilities, especially for dependent behavior variables.

The main changes in version 2.1 compared to version 1.98 are

- 1. extension by allowing dependent actor variables, inclusion of effects related to group position and an update of the similarity effects, implementing methods in Steglich, Snijders and Pearson (2010), see Section 5.5;
- the addition of Neyman-Rao goodness-of-fit tests according to Schweinberger (2005), see Section 9;
- 3. the possibility to analyse dynamics of non-directed networks, according to Snijders (2007);
- 4. statistical Monte Carlo studies, see Section 25;
- 5. extension of the specifications of the exponential random graph (" p^* ") model in line with Snijders, Pattison, Robins, and Handcock (2006), and slight modifications of the algorithm for this case to increase efficiency;
- 6. missing data handling is extended to covariates and dependent actor variables;
- 7. addition of the program Siena08 for the multilevel analysis of multiple network evolution processes, implementing methods in Snijders and Baerveldt (2003);
- 8. analysing the dynamics of non-directed networks (not yet documented);
- 9. possibility to specify structural (i.e., non-random) zeros and structural ones in the adjacency matrices, see Section 5.1.1;

- a new format for the project definition file *pname*.IN and the replacement of the internal project files *pname*.mo1 through *pname*.mo4 by files *pname*.MO for model definition and *pname*.SI for simulation directives (old project files still can be read);
- 11. correction of various errors.

The main changes in version 1.98 compared to version 1.95 are

- 1. the advanced option modeltype is added, implementing methods in Snijders (2003);
- 2. maximum number of actors increased to 500.

The main changes in version 1.95 compared to version 1.90 are

- 1. for the exponential random graph model some extra simulation options were added, and inversion steps were added to the algorithm;
- 2. some effects (3-star and 4-star counts) added to the exponential random graph model;
- 3. for changing covariates, the global rather than the periodwise mean is subtracted;
- 4. the program Siena02 for data description was added.

The main changes in version 1.90 compared to version 1.70 are

- 1. possibility to use more than two observation moments;
- 2. inclusion of the exponential random graph (" p^* ") model, corresponding to one observation moment;
- 3. possibility to have changes of composition of the network (actors leaving and/or entering);
- 4. changing actor covariates;
- 5. arbitrary codes allowed for missing data (instead of the automatic use of 6 and 9 as codes for missing data, the user now has to supply these codes explicitly);
- 6. small improvements in the user interface.

Part III Programmer's manual

The programmer's manual will not be important for most users. It is intended for those who want to look behind the screens.

The SIENA program consists of a basic computation part programmed by Tom Snijders, Christian Steglich, Michael Schweinberger, and Mark Huisman, in Turbo Pascal and Delphi; and (starting with version 3.2) made compatible with Lazarus by Ruth Ripley. Lazarus is an open source set of class libraries for Free Pascal that emulate Delphi. Associated to SIENA is the StOCNET windows shell, programmed by Peter Boer and Rob de Negro in Delphi, with first Evelien Zeggelink, then Mark Huisman, and later Christian Steglich as the project leader.

21 SIENA files

Internally the following files are used. Recall that *pname* is the name of the project, which the user can choose at will. The extension file names cannot be changed.

21.1 Basic information file

The basic information file is called *pname*.IN, and contains the definition of the numbers of cases and variables, the names of the files in which data are initially stored and their codes (including missing data identification), and the names of the variables. The requirements for the input data files are given in section 5. The data sets at the SIENA website are provided with some basic information files which can serve as examples.

The basic information file is written by StOCNET when the data are defined, and can also be written by any text editor that can produce ASCII (TXT) files; note that it *must* have extension name .IN. Therefore it is also called the .in file. One way of writing the basic information file is to let StOCNET do it. The basic information files written by StOCNET can be found in the directory that is mentioned in the StOCNET options as the Sessions directory. Usually to employ the basic information files. How to do this will be evident for users who know how to work with directory structures and with file and path names.

It is read by Siena01.exe. This file contains up to eight sections, each starting with a line containing the section number (@1 through @8). These sections must have the following contents:

- 1. Section @1 contains basic information about type and amount of data. This section is required for all SIENA projects. It must contain nine rows, each starting with an integer number:
 - number of observations of the network (1 for exponential random graph modeling, 2 or more for modeling network evolution over time; denoted by M);
 - number of actors (denoted further by n);
 - number of dependent network variables (must be equal to one in the current version of SIENA). The network data are further specified in Section @2;
 - number of dependent actor variables. Possible dependent actor variables are further specified in Section @3;
 - number of files with constant actor covariates (further specified in Section @4);
 - number of files with changing actor variables (further specified in Section @5);

- number of constant dyadic covariates (further specified in Section @6);
- number of exogenous changing dyadic covariates (from version 2.4 of SIENA onward). Section @7 contains the specification of this type of covariate data;
- indicator of file with times of composition change (0 means no change of network composition; 1-4 mean composition change; 1 is the default treatment of composition change, 2-3 are alternatives using missing data, 4 effectively transforms the composition change information to structural zeros). See Section 5.7. If there is composition change, the file must be further specified in Section **@8**.
- 2. Section **@2** contains information about the network data, as follows:
 - for each of the *M* network observations, the following three lines:
 - a line with the name of the data file;
 - a line with the codes that are regarded as a present arc in the digraph;
 - a line with the codes that are regarded as missing data;
 - a line with the name of the network variable.

All codes should be in the range from 0 to 9.

- 3. Section @3 contains information about the dependent actor variables, in this format:
 - for each dependent actor variable, the following three lines:
 - a line with the name of the data file;
 - a line with the code that is regarded as missing data;
 - $-\,$ a line with the name of the variable.
- 4. Section **@4** contains information about constant actor covariates, as follows:
 - For each file containing such covariates, there must be the following lines:
 - a line with the name of the data file;
 - a line with the number of variables in this file;
 - for each variable:
 - * a line with the code for missing data;
 - * a line with the name of the variable.

Note that this format differs from the one used in Sections **@3** and **@5** through **@7** because here, data files can potentially contain more than one covariate, while in these other sections, only one variable is given per file.

- 5. Section **@5** contains information about changing actor covariates, in the same format as the dependent actor variables are given:
 - for each changing actor covariate, the following three lines:
 - a line with the name of the data file;
 - a line with the code that is regarded as missing data.
 - a line with the name of the covariate.
- 6. Section **@6** contains information about constant dyadic covariates, in the same format:
 - for each constant dyadic covariate, the following three lines:

- a line with the name of the data file;
- a line with the code that is regarded as missing data.
- a line with the name of the covariate.
- 7. Section **@7** refers to changing dyadic covariates, with for each changing dyadic covariate the following lines:
 - for each observation moment 1 to M 1 (therefore not for the last!), the following two lines:
 - a line with the name of the data file;
 - a line with the code that is regarded as missing data.

followed by

 $-\,$ a line with the name of the covariate.

E.g., if there are 2 changing dyadic covariates and M = 4 observation moments, this requires a total of $2 \times (M - 1) = 6$ data files.

- 8. Section **@8** contains information about network composition change, namely:
 - a line with the name of the file containing times of network composition change.

Whenever a certain type of data is not present, leave out the entire section in the file *pname*.IN corresponding to this type. For example, if you do not have any files containing changing actor covariates, leave out Section @5. If there are problems in reading the basic input file, try deleting superfluous blanks and/or empty rows. See to it that the basic input file is an ASCII text file, with numbers separated by blanks, lines separated by hard returns.

The variable names given in the input file will be used in the output files. If no names are provided and SIENA is run in the StOCNET environment, SIENA uses the default variable names generated by StOCNET in the StOCNET Data menu. This is not recommendable because it can lead to identical names for different variables. If no names are provided and SIENA is run outside of the StOCNET environment, SIENA uses its own default variable names, and this problem does not occur.

An example for the basic input file is the following file bunt.IN. This refers to data files that are included with the program, collected by Gerhard van de Bunt. This example, which contains a file with three covariates, is used in van de Bunt (1999) and in van de Bunt, van Duijn, and Snijders (1999).

```
@1[general information about SIENA project <bunt>:]
2 [number of waves]
32 [number of actors]
1 [number of dependent network variables]
0 [number of dependent actor variables]
1 [number of files with constant actor covariates]
0 [number of exogenous changing actor covariates]
0 [number of constant dyadic covariates]
0 [number of exogenous changing dyadic covariates]
0 [indicator for file with composition change directives]
@2[network files in temporal order; names follow:]
vrnd32t2.dat
1 2 3 [code for tie]
6 9 [code for missing]
vrnd32t4.dat
1 2 3 [code for tie]
```

```
6 9 [code for missing]
friendship

@4[files with constant actor covariates:]
vars.dat
3 [number of covariates in this file; names follow:]
99 [code for missing]
gender
99 [code for missing]
program
99 [code for missing]
smoke
```

The basic data input is carried out by executing Siena01.exe. This programs reads the basic information file. Some preliminary output is given in the files *pname.out* and *pname.log*.

21.2 Definition files

The program writes and reads for internal use the following two definition files:

- *pname*.MO defines model specification and options for model estimation,
- pname.Sl defines statistics and number of runs for simulation.

These definition files are read in a format where certain lines are skipped entirely and other lines are skipped after reading a certain number. These skipped parts are between square brackets [...]. Their purpose is to give information to the human reader about the meaning of the lines. Note, however, that SIENA does not check for the brackets, but skips information on the basis of line numbers and reading numerical information.

The three files *pname*.IN, *pname*.MO and *pname*.SI must be compatible (as they contain some overlapping information) for successfully running SIENA.

21.2.1 Model specification through the MO file

The model specification options were already discussed in Section 12. These can be changed outside the StOCNET shell, by changing the *pname*.MO file by a text editor. They also can be changed using the siena03.exe program. By operating directly on the *pname*.MO file is also possible to use advanced SIENA options which are not yet available through StOCNET; whether such options exist, will depend on the versions of SIENA and StOCNET.

When looking at the *pname*.MO file, you can see that this file by and large contains the same information as the screen opening up in StOCNET when clicking the Model specification button. More precisely, the *pname*.MO file consists of several sections, marked by @-symbols, as follows:

- Section **@1** contains general information about the project, such as data format, numbers and names of variables. The information given in this section must be compatible to the information provided in the file *pname*.IN.
- Sections starting @2.x contain the model specification for dependent network variables (indexed by x). There are two subsections:
 - Subsections @2.x.1 contain the specification of the network rate function, which looks like this:

```
@2.1.1 [rate function effects for dependent network variable <#1>:]
11 [number of these effects :]
[each effect is given in two rows]
[first row contains label of the effect,]
[second row contains flags for inclusion, fixing and testing,]
[the starting value and potential extra parameters for effect calculation.]
basic network rate parameter
0 0 0 5.851107 0
outdegrees effect on rate
0 0 0 0.000000 0
[...]
```

The section must contain first a row with the number of effects, further down for each of these effects two rows, one containing the effect name, the other containing a sequence of numbers. These have the following meaning:

- * a 0/1 entry denoting whether the effect is included (1) or excluded (0);
- * a 0/1 entry denoting whether the effect is fixed (1) or not fixed (0) during the estimation process;
- * a 0/1 entry indicating whether a fixed effect shall be included (1) or excluded (0) in goodness-of-fit calculations (see Section 9);
- * the starting value of the parameter for the estimation procedure;
- * an internal effect parameter (for modeling the constants c in the mathematical definition of the effects, see Section 15.1.1.
- Subsections @2.x.2 contain the specification of the network decision rule (including the evaluation endowment functions), and which has the following shape:

```
02.1.2 [objective function effects for dependent network variable <#1>:]
46 [number of such effects :]
[first row contains label of the effect,]
[next three rows: functions in which effect can be included:]
[row 1: evaluation function, row 2: endowment function, row 3: reinforcement function,]
[columns correspond to: (a) inclusion, (b) random effects, (c) fixing, (d) testing,]
[(e) starting value, (f) potential extra parameters for effect calculation.]
density (outdegree)
1000
        -1.143698 0
0000
         0.000000 0
0000
         0.000000 0
reciprocity
1000
         1.467572 0
0000
         0.000000 0
0 0 0 0
         0.000000 0
[...]
```

The section must contain first a row with the number of effects, further down for each of these effects four rows, one containing the effect name, the other containing sequences of numbers. These have the following meaning:

- * a 0/1 entry denoting whether the effect is included;
- * a 0/1 entry denoting whether a corresponding random effect effect is included;
- * a 0/1 entry denoting whether the effect is fixed;
- $\ast\,$ a 0/1 entry indicating whether a fixed effect is included in goodness-of-fit calculations;
- * the starting value of the parameter for the estimation procedure;
- * an internal effect parameter.

Of the three rows that follow this pattern, the first row corresponds to the evaluation function, the second row corresponds to the endowment function, and the third row is reserved for future model extensions that allow the modeling of adaptive learning behavior.

In the current version of SIENA, but one dependent network variable can be analyzed at a time, so there will only be sections @2.1.y.

- Sections starting **@3.x** contain the model specification for dependent action variables (again indexed by x). as for the network variables, there are two subsections:
 - Subsections @3.x.1 contain the specification of the behavioral rate function,
 - Subsections @3.x.2 contain the specification of the behavioral decision rule (including the evaluation endowment functions).

These subsections have the same format as the corresponding subsections for the network evolution model.

- The final Section @4 contains specifications of various estimation options. Most of these are accessible in StOCNET, e.g., through the model options mentioned in Section 12. The consecutive options are the following. Some of these may not be accessible from StOCNET, and be possible only when working with SIENA outside of StOCNET.
 - 1. Estimation method:
 - code 0 for unconditional estimation;
 - code 1 or code 21 for estimation conditional on observed changes in the network;
 - codes 21+k for estimation conditional on observed changes on the dependent action variable k;
 - code 2 for maximum likelihood estimation;
 - code 3 for Bayesian estimation.

For exponential random graphs, another available option is to include incidental vertex parameters:

- code 10 for unconditional estimation with incidental vertex parameters;
- code 11 for conditional estimation with incidental vertex parameters.
- 2. For estimation method 2 and 3, an indicator of whether initial estimates should be computed:
 - code 0: no;
 - code 1: yes: unconditional estimates are computed and used as initial estimates.
- 3. For Bayesian estimation: Metropolis Hastings (M-H) algorithm for the fixed effects in the evaluation and endowment functions:
 - code 1: random walk M-H;
 - code 2: independence sampler;
 - code 3: first-order autoregressive M-H.
- 4. For Bayesian estimation: scale factor of the proposal distribution:
 - code 0: scale factor is calibrated during burn-in;
 - $\operatorname{code} c > 0$ (positive, real number): c is used as scale factor.
- 5. For models with actor-dependent random coefficients: Metropolis Hastings (M-H) algorithm for the random effects:

- code 1: random walk M-H;
- code 2: independence sampler;
- code 3: first-order autoregressive M-H.
- 6. For models with actor-dependent random coefficients: scale factor of the proposal distribution:
 - code 0: scale factor is calibrated during burn-in;
 - $\operatorname{code} c > 0$ (positive, real number): c is used as scale factor.
- 7. A code for the type of model.

For longitudinal data this is the Model Code described in the section on model options. For exponential random graph models, this code defines the steps used in the oneobservation case for simulating a random (di)graph (see also the description above):

- code 1: Gibbs steps for single tie variables ;
- code 2: Gibbs steps for dyads;
- code 3: Gibbs steps for triplets;
- code 4: Metropolis Hastings steps for single tie variables, version A; this is the default for directed networks;
- code 5: Metropolis Hastings steps for single tie variables, version B;
- code 6: Metropolis Hastings steps for single tie variables, version A, for non-directed graphs;
- code 7: Metropolis Hastings steps for single tie variables for antisymmetric graphs ('tournaments');
- code 8: Metropolis Hastings steps keeping the in-degrees and out-degrees fixed.
- code 9: Metropolis Hastings steps keeping the out-degrees fixed, version A;
- code 10: Metropolis Hastings steps keeping the out-degrees fixed, version B.

To each of these values, the number 10 may be added (so the values become 11-20). In that case a *continuous* chain is used: i.e., the last generated graph is used as the initial value in the MCMC sequence for simulating the next graph. Otherwise (i.e., for the values 1-10), the initial value is an independently generated random graph. For practical purposes, one should *always* use a continuous chain – i.e., only use codes 11-20. Since version 3.181, the special option 26, available only for non-directed graphs, conditions additionally on the connectedness of the network, provided that the observed graph is connected.

- 8. The number of subphases in phase 2 of the estimation algorithm (advice: 4). If the number given here is higher than 10, this signifies the request for additional estimation runs as long as the maximum *t*-ratio for convergence is higher than 0.15. The maximum additional number of repeated estimation runs then is the rounded value of number of subphases divided by 10; the real number of subphases is the remainder in this division.
- 9. The number of phase 3 iterations for the estimation algorithm (advice: 1000).
- 10. A number r proportional to the number of steps used for generating one graph in the one-observation case. The number of steps is $r n^2/2d(1-d)$ where n is the number of actors and d is the observed density of the graph; if the observed density is less than .05 or more than .95, the value d = .05 is used.
- 11. The initial value of the gain parameter in the estimation algorithm (advice: 0.2 for longitudinal MoM, smaller for ERGM).
- 12. An indicator for the initial value used in the estimation algorithm:

- code 0: current value as specified in the preceding sections,
- code 1: standardized starting value, meaning that a good starting value is chosen for the density effect and in the one-observation (exponential random graph) case also for the reciprocity effect. For still other configurations, also starting values for the rate parameters and the shape effect for dependent action variables are internally determined. All other effects then have a 0.0 starting value.
- 13. An indicator allowing for tests of temporal (between-period) homogeneity of parameters:
 - code 0: no test,
 - code m > 0: test for period m
- 14. An indicator allowing for tests of actor homogeneity of parameters:
 - code 0: no test,
 - code 1: test of the density (out-degree) effect,
 - code 2: test of the reciprocity effect.
- 15. A space-separated list giving the row numbers of the actors for such an actor homogeneity test.
- 16. A value determining the use of random numbers during the estimation process:
 - code 0: randomize seed,
 - $\operatorname{code} c > 0$ (positive integer): c is used as random seed.
- 17. An indicator for the method of estimating derivatives of expected values with respect to parameters:
 - code 0: Finite Differences (the older method of Snijders 2001);
 - code 1: Score Function method I, see Schweinberger and Snijders (2007); (not available for all models);

If you change anything in the *pname*.MO file, you must run Siena04.exe to let SIENA check (and if necessary: censor or re-define) the model specification.

21.2.2 Specification of simulations through the SI file

For running simulations, SIENA needs to be told which statistics it should simulate. This is done by manipulating the *pname*.SI file by an ASCII text editor before running the simulations. This file consists of two sections, again marked by @-symbols, as follows:

• Section @1 contains the list of possible statistics that can be simulated. It looks as follows:

```
@1 [statistics that can be generated]
60 [number of these statistics :]
[each statistic is given in two rows]
[first row contains label of the statistic,]
[second row contains flag for inclusion]
Amount of change
1
Number of ties
1
Number of reciprocated ties
0
[...]
```

As can be seen, for each effect, there are two rows:

- a row containing the statistic's name;
- a row contining an indicator whether the statistic shall be simulated.
- Section **@**2 contains options for simulation. Currently, there is but one option:
 - the default number of simulations for straight simulation (advice: 1000).

The file *pname*.SI by and large contains the same information as the screen opening up in StOCNET when clicking the Statistics specification button (which is possible as soon as Simulation is selected as the Run model).

If the number of simulations is specified as 1, then one complete data set is stored and written both as adjacency matrices and in Pajek format. For larger number of simulations, only aggregate information (means, standard deviations and covariance matrices of statistics) is reported.

21.3 Data files

After the initial project definition the original data files are not used any more, but the project data files are used. These are the following.

- pname.d01 Network data file time 1.
- pname.d02, etc. Network data file time 2, etc.
- pname.m01, etc. Network missings file time 1, etc.
- pname.s01, etc. Fixed part of network structure period 1, etc.
- *pname.dav* Data file constant actor-dependent variables (centered).
- *pname*.miv Missings file actor-dependent variables.
- *pname.dac* Data file with changing actor-dependent variables.
- *pname.mac* Missings file changing actor-dependent variables.
- pname.z01, etc. Data files non-changing dyadic covariates.
- pname.c01, etc. Data files changing dyadic covariates.
- pname.n01, etc. Missings files dyadic covariates.
- *pname*.dex Data file times of composition change.

The user does not need to care about these data files (but should not delete them either).

21.4 Output files

The main output for the user goes to *pname.out*. In addition, some secondary output files are generated.

•	pname.log	An extra output file, which in the first place is a log file of what the program did, and also contains some further secondary information.
•	<i>pname</i> .cck	This file contains a more detailed report of the estimation algorithm. It is overwritten with each new estimation procedure.
•	<i>pname</i> .bof	An excerpt of the estimation results for quick reference.
•	<i>pname</i> .bos	An even smaller excerpt of the estimation results: only the parameter estimates and standard errors.

Some other output files may be written; this is then announced in the *pname.out* file.

21.5 Other information files

SIENA makes a few files with some additional information. One is for the user to facilitate the construction of interaction effects. Others are designed for being used by StOCNET as a list of options. These files are the following.

- *pname*.eff The numbered list of effects for help in constructing interaction effects.
- *pname*.cdc List of model code options for the evolution model.
- *pname*.cdd List of options for the estimation of derivatives, required for the estimation of standard errors.
- *pname*.cde List of estimation options.

The user does not need to look at these information files (but should not delete them either).

22 Units and executable files

The basic computational parts of SIENA are contained in the following units. First, there are six basic units.

- 1. S_DAT contains the basic data structures together with some basics of the model definition; it uses unit EIGHT for some lower-level data storage, and also DIGRAPH for definition of network data and model ingredients.
- 2. S_CONSTANTS contains definitions of maximal values of numbers of actors, numbers of variables, etc.
- 3. S_BASE contains the basic model definition and the basic simulation procedures.
- 4. S_SIM is the unit for carrying out straight simulations.
- 5. S_EST contains the procedure for estimation, using S_TEST for goodness-of-fit tests.
- 6. S_DESC contains procedures for data description.

Then there are some special-purpose units, of which the most important ones are:

- S_Start contains the procedure ReadWriteData to start a project by reading the *pname*.IN file and the initial data files, and writing the internally used files. It uses only S_DAT. Procedure ReadWriteData from S_START must be followed always by procedure BeforeFirst-ModelDefinition from S_BASE.
- 8. S_ML contains procedures for maximum likelihood estimation.
- 9. S_RE contains procedures for random effect models (as yet undocumented).
- 10. S_signed contains procedures for models for signed digraphs (as yet undocumented).
- 11. S-ChangeModel contains procedures for command-line driven changes in the .mo file.

Further there are five units containing specific kinds of utilities. Their names do not start with S₋ because they do not use the other units (except perhaps S₋CONSTANTS).

- 12. EIGHT is a unit for storing the data. Its name was chosen for historical reasons (in SIENA version 1 a byte was used to store eight booleans). This unit also connects the procedures in DIGRAPH to those in S_BASE; this is necessary because DIGRAPH is defined independent of the data.
- 13. DIGRAPH is a unit for defining network data types and effect data types.
- 14. CHAINS defines data types for use in data augmentation procedures employed in maximum likelihood estimation.
- 15. SLIB is a library of various computational and input/output utilities.
- 16. RANGEN is a library for generation of random variables. It uses the URNS suite for random numbers generation.

22.1 Executable files

The basic data input is carried out by executing Siena01.exe. This program executes ReadWriteData and BeforeFirstModelDefinition, thereby reading the basic information file.

Data description is carried out by Siena02.exe which executes Describe.

Changes in the .mo file, defined in the command line, can be carried out by siena03.exe.

In the StOCNET operation the model specification is carried out by StOCNET changing the .MO file and then running Siena04.exe.

Siena04.exe is used for checking admissibility of a model specification, and for keeping consistency between the different session files. It reads and updates the .MO file, and also writes the corresponding .SI file. If you change *pname*.MO by an text editor outside of StOCNET, it is advisable to run Siena04.exe before proceeding.

Siena04.exe can also be used to copy a model definition from one .mo file to another.

The simulation is carried out by Siena05.exe which executes Simulate.

The estimation is carried out by executing Siena07.exe which executes Estimate.

The multilevel combination of estimation runs for several data sets according to the same basic model specification is done by Siena08.exe (see Section 14). This program is independent of the other programs, and reads the output files produced by Siena07.exe.

22.2 Running SIENA in console mode

For various purposes, e.g., high-performance computing, it can be useful to have a version of SIENA which runs in console mode without a Windows graphical user interface. This is possible by compiling SIENA under Lazarus and, in the Project / Compiler options / Other screen of Lazarus, add -dbatch to the Custom options. This yields versions of the SIENA executable files which run in console mode, and have no opportunity for user interaction while the program is running. During the program operation, some information about the progress of the computations is written to the screen (i.e., the DOS window).

23 Starting to look at the source code

If you wish to start with understanding the structure of the source code, it may be helpful to take the following tour of some essential ingredients.

- 1. Unit DIGRAPH defines network data types:
 - (a) TValuedDigraph, storing a valued directed graph, used as the basic network data structure;
 - (b) **TDigraph**, storing a valued directed graph, used for storing indicators of missing data and structurally determined data.

Both data types store valued digraphs as doubly linked arc lists, allowing to search arcs both from the sender node and from the receiver node, and using less memory space than the adjacency matrices used in versions 1 and 2 of SIENA.

In addition, unit DIGRAPH defines types that allow to define the model components:

- (a) **TEffect**, defining a component in the objective function for the longitudinal case, and a component in the log-odds for the non-longitudinal (ERGM) case;
- (b) TEffects, defining an array of TEffect and many operations on such arrays.

- Unit EIGHT contains the fundamental data structures, using the types defined in DIGRAPH.
 (i) Functions starting with y represent dependent variables.
 - (a) yn is the adjacency matrix of the dependent network variable (current value): yn(i,j) is the tie variable from i to j;
 - (b) ya is the vector of dependent individual variables (current value): ya(i,h) is the h'th individual variable for actor i;
 - (c) ynm gives the adjacency matrices of the dependent network variable (all values): ynm(i,j,m) is the tie variable from i to j at observation moment m.
 - (d) mis is the indicator matrix for missing values:
 mis(i,j,m) = 0 if ynm(i,j,m) is an observed value and 1 if it is missing.

Note that yn and ya change dynamically during the simulation process; the variables containing the data from which ynm and mis are calculated, are read in the input phase and then are left unaltered.

For the sake of flexibility, the variables in the list above are implemented as functions, not as arrays.

- (ii) Variables starting with a single z represent individual variables.
- (a) Array z contains the constant actor variables; the number of such variables is nz.
- (b) Array zc contains the changing actor variables; their number is nzc.
- (c) The number of dependent actor variables is nza; these variables are the first nza among the nzc changing actor variables.

(iii) Variables starting with zz represent dyadic variables.

- (a) Array zz contains the constant dyadic covariates; their number is nzz.
- (b) Array zzc contains the changing dyadic covariates; their number is nzzc.

Further, unit EIGHT contains procedures linking the methods in DIGRAPH to the data structures available in EIGHT.

- 3. Unit S_DAT contains the defining ingredients for the network models. The intended demarcation between S_DAT and S_BASE, is that the former contains the model definition and the latter the operation of the model dynamics. This is not realized completely because of various conflicting constraints.
- 4. Unit S_BASE contains the simulation procedures. The main procedures in this unit are the following.
 - (a) Function SimStats generates the required statistics and is called by procedure Simulate in S_SIM and (indirectly, through FRAN in unit S_ML) by Estimate in S_EST. The procedure SimStats first simulates the network and behavior by the procedure Runepoch and then calculates statistics by the procedures NetworkStatistics and Action-Statistics; the values of these statistics are output arguments of SimStats.
 - (b) Procedures NetworkStatistics and ActionStatistics calculate the statistics from the generated network (or adjacency matrix) and behavior variables, and is called by SimStats.
 - (c) Procedure Runepoch generates a stochastic network and action variables for given parameter values and a given initial situation by simulating the dynamic model for one period between two observations. This procedure is called by procedure SimStats. If there are M observations ($M \ge 2$), Runepoch is called M 1 times.

- (d) Procedure Runstep makes one stochastic step according to the actor-oriented evolution model, i.e., it either changes one entry (i, j) of the adjacency matrix to be changed, or it changes the value of one action variable for one. The time variable time is also increased by an amount depending stochastically on the rate functions. This procedure is called by procedure Runepoch.
- (e) Procedure ChangeTie is called at the end of procedure RunStep if a change in the network is made, and carries out the required change of the network and the associated updates of various statistics.
- (f) Procedure ChangeBehavior is called at the end of procedure RunStep if a change in behavior is made, and carries out the required change of the dependent action variable and the associated updates of various statistics.
- (g) Function NetworkLambda, which is the rate function for the network changes made by each actor, and is used in procedure Runstep. For Model Type 2, it uses functions ξ and ν.
- (h) Function ActionLambda, which is the rate function for the behavior changes made by each actor, and is used in procedure Runstep.
- (i) Procedure ChoiceProbabilities, which defines the probabilities with which a given actor i chooses to change the tie variable to actor j, for each of j = 1, ..., n.
- (j) Function contr_fa, which defines the contribution $s_{ih}^Z(x)$ of each given effect h to the evaluation function for the behavior, and is used in procedure Runstep.
- (k) Function contr_ga, which defines the contribution of each given effect to the endowment function for the behavior, and is used in procedure Runstep.
- 5. Unit S_ML contains procedures for maximum likelihood estimation, including the 'chain' data structure for the augmented data. The main function computed here is Scores, the score function for the augmented data. This unit includes the function ERAN ('*Eurotion Bandom*') which is the basic function in the

unit includes the function FRAN ('*Function Random*'), which is the basic function in the equation solved by procedure POLRUP in unit S_EST . Depending on the type of estimation requested, FRAN calls function SimStats in unit S_BASE , or function Scores in unit S_ML .

- 6. In unit S_EST, the Robbins-Monro algorithm is contained in the procedure POLRUP (for Polyak-Ruppert, see Snijders, 2001). When parameters are to be tested by Neyman-Rao tests, S_EST calls the procedure TestStatistic in the unit S_TEST.
- 7. Unit S_SIM is used for simulations, and calls the function SimStats in S_BASE.
- 8. The unit S_ERGM_EST and the file S_ERGM.PAS which is an include file used in S_BASE contain procedures used only for the ERGM (non-longitudinal / 1 observation) case.

23.1 Sketch of the simulation algorithm

The simulation algorithm, used in the Method of Moments estimation as well as for straight simulation, is explained here using the notation of Snijders (2005) and Snijders, Steglich and Schweinberger (2007). It is formulated here only for the case of two observation moments t_1 and t_2 , and no dependent behavior variables. The distinction between evaluation function and endowment function is obscured here; both are jointly referred to as objective function. The following notation is used; in typewriter font are the symbols in the source code.

n	number of actors)	n
X	dependent network (digraph)	matyn
x_{ij}	indicator variable of the from i to j in digraph X	yn(i,j)
$x(i \rightsquigarrow j)$	digraph x in which x_{ij} is replaced by $1 - x_{ij}$,	
	and the other elements of x remain unchanged	
λ_i	rate function for network change	networklambda
$K \ge 1$	number of terms in the objective function	
β_k	parameters in the objective function	alpa_f[k], alpa_g[k], theta[k]
$s_{ik}(x)$	components of the objective function	
$\mathcal{E}(\lambda)$	exponential distribution with parameter λ	

The schematic outline of the algorithm is as follows. In typewriter font the procedures and variables are mentioned that are most important for this step in the algorithm; the outline is followed by an indication of where these procedures and variables are defined. The algorithm as a whole is implemented in the procedure SimStats in unit S_Base.

Replacing a variable by the sign + means summation over this index.

"generate \sim " means to generate a random variable with the indicated distribution. "choose random \sim " means to generate a discrete random variable with probabilities *proportional* to the indicated values.

1.	Initial conditions:	network $X = x(t_1);$ time $t = 0.$	Initialise_y, matyn time
2.	Repeat		RunEpoch, RunStep
		$\begin{bmatrix} 3. & \text{generate } \Delta t \sim \mathcal{E}(\lambda_+); \end{bmatrix}$	rs_tau
		4. choose random $i \sim \lambda_i$;	NewRanp, i
		5. for all $k = 1,, K$ and $j = 1,, n$,	UtilityComponents
		calculate $s_{ik}(x(i \rightsquigarrow j));$	or
		see (16) , (17) in Section 24.1;	Contrib_n,Contrib_fn
		6. for all $j = 1, \ldots, n$, calculate	ChoiceProbabilities
		$f_i(x(i \rightsquigarrow j)) = \sum_{k=1}^K \beta_k s_{ik}(x(i \rightsquigarrow j));$	
		7. choose random $j \sim \exp(f_i(x(i \rightsquigarrow j)));$	NewRanp, j
		8. Set $t := t + \Delta t$;	
			ChangeTie
	until		
		10. a. if estimation method is conditional:	
		$\sum_{i,j} x_{ij} - x_{ij}(t_1) $	Distance
		$= \sum_{i,j} x_{ij}(t_2) - x_{ij}(t_1) ;$	ObservedNetworkDistance
		b. if est. method is unconditional:	
		$t \ge 1;$	time
11.	Replace	tie variables which are structurally fixed	
		at the end but not the beginning of the period	ForceNewStructurals
10		by the structurally fixed end values	
12.	Calculate	statistics $u(X)$.	NetworkStatistics

The procedures and variables in this outline are defined in the following data types, procedures, and units. The list is in the order of their occurrence in the outline.

Procedure / variable	Defined in type or procedure	in unit
Initialise_y matyn	ModelSpecification	S_Base Eight
time	ModelSpecification	S_Base
RunEpoch	ModelSpecification	$S_{-}Base$
RunStep	ModelSpecification	S_Base
rs_tau	RunStep	S_Base
NewRanp		RanGen
i	RunStep	S_Base
UtilityComponents	TEffects	Digraph
Contrib₋n	TEffects	Digraph
Contrib_fn	TEffects	Digraph
ChoiceProbabilities	ModelSpecification	S_Base
NewRanp		RanGen
j	RunStep	S_Base
ChangeTie	ModelSpecification	S_Base
Distance	ModelSpecification	S_Base
ObservedNetworkDistance	DataSpecification	S_Dat
time	ModelSpecification	S_Base
NetworkStatistics	ModelSpecification	S_Base

23.2 Remarks about the likelihood algorithm

The implementation of the likelihood algorithm (for maximum likelihood and Bayesian estimation) is not documented yet, except for the general algorithm descriptions in Snijders, Koskinen and Schweinberger (2010), (Koskinen, 2005), Koskinen and Snijders (2007), and Schweinberger and Snijders (2007). This section currently is no more than a loose collection of remarks, that are being written as the need arises.

If the boolean variable makechain (defined in S_ML) is true and the maximum likelihood option is chosen, then the sequence of changes from each observation to the next is written to file *pname.cha*. This is coded in the following format.

- A line with the letter A. Then the parameter values used for the simulation.
- For each period:
 - A line with the letter P and the number of the period.
 - For each change a line;

if it is a network change, the format is $N \ i \ j \ v \ 0$

where i and j are the actors for whom the tie is changed, and v is the number of the network variable (currently always equal to 1). This line is interpreted as a change of x_{ij} to $1 - x_{ij}$.

If it is a behavior change, the format is

Bi0vd

where i is the actor who makes the change, v is the number of the behavior variable, and d is the amount of change (which can be -1 or +1). This line is interpreted as a change of z_{vi} to $z_{vi} + d$.

• A line with the letter L, followed by a line with the log-likelihood of this sequence of changes.

24 Parameters and effects

In the source code there are two kinds of parameters: alpha (α ; in the source code: alpa_l for the rate function, alpa_f for the evaluation function, alpa_g for the endowment function) and theta (θ). The alpha parameters are used in the stochastic model, and each alpha parameter corresponds to one effect, independently of whether this effect is included in the current model specification. Their values are stored in the *pname*.MO file, which also indicates (by 0-1 codes) whether these variables are included in the model, and whether they are fixed at their current value in the estimation process. The theta parameters are the statistical parameters that correspond to the effects in the current model specification. Thus, the theta parameters are those elements of the alpha parameters that are currently active.

Procedures SetAlpha and SetTheta in unit S_BASE define the correspondence between the alpha and theta parameters. This correspondence sets the order of the theta parameters as follows:

1. First the parameters for network dynamics:

- (a) Rate parameters;
- (b) evaluation function parameters;
- (c) endowment function parameters;
- 2. Then the parameters for behavior dynamics (if there is a dependent behavior variable):
 - (a) Rate parameters;
 - (b) evaluation function parameters;
 - (c) endowment function parameters.

If there is more than one dependent behavior variable, then within these three categories the parameters are ordered according to the dependent variable whose dynamics they influence.

The distinction between the theta and alpha parameters in principle also allows linear (or other) restrictions between the alphas. In the present version, the possibility of such restrictions is not implemented, but this possibility may be elaborated in a later version.

The effects for the evolution model, distinguishing between effects for the network dynamics and effects for the behavior dynamics, are defined in several procedures, which of course must correspond. To each effect corresponds one statistic used for estimating the parameter for this effect. This is spelled out in Sections 24.1 and 24.2.

Originally unit S_DAT contained the data definition and S_BASE the model definition. This distinction has been a bit blurred in version 3.0 by the construction work on the program, but the plan is to reorganize the units again to make the distinction between the units more helpful.

In unit S_DAT, the numbers of the various types of effects are defined:

- 1. NetworkEffects_f, the number of effects in the evaluation function for the network dynamics;
- 2. NetworkEffects_g, the number of effects in the endowment function for the network dynamics;
- 3. NetworkEffects_I, the number of effects in the rate function for the network dynamics;
- 4. ActionEffects_f, the number of effects in the evaluation function for the behavior dynamics;
- 5. ActionEffects_g, the number of effects in the endowment function for the behavior dynamics;
- 6. ActionEffects_I, the number of effects in the rate function for the behavior dynamics;

- 7. NetworkFunctions, the number of statistics that can be calculated and which can be used if there are no dependent behavior variables; this number is equal to NetworkEffects_f + NetworkEffects_g + NetworkEffects_l.
- 8. ActionFunctions, the number of statistics that can be used additionally to the statistics mentioned before, in case that there are dependent behavior variables.

In units S_DAT and S_BASE , the definitions of the effects and statistics are given. These use the basic data and effect structures defined in unit DIGRAPH (which has a name not starting with S_b because it is itself independent of the other SIENA units), which is described below.

- A. For the rate function:
 - 1. Procedure DefineModel_Inames defines the names of the effects and the index numbers of the corresponding statistics;
 - 2. function NetworkLambda defines the network change rate function;
 - procedure Transform_I calculates some variables for more efficient calculations in NetworkLambda;
 - 4. for Model Type 2, the rate function for network change also depends on the functions ξ and ν ;
 - 5. function ActionLambda defines the behavior change rate function.
- B. For the evaluation and endowment functions of the network dynamics:
 - 1. Unit S_BASE has an include file S_EFFECTS which contains the basic definitions of the available effects, i.e., components of the evaluation and endowment functions. The effects are defined in the procedure DefineNetworkEffects. The effects in the evaluation and endowment functions are defined conjointly (see below).
- C. For the evaluation and endowment functions of the behavior dynamics:
 - 1. Procedure DefineModel_fnames and DefineModel_gnames define the names of the effects in the evaluation and endowment functions, respectively;
 - 2. function Contr_fa and Contr_ga give the contribution of each effect to the evaluation and endowment functions, respectively.
- D. For the statistics:
 - 1. Procedure DefineFunctionnames defines the names of the statistics; for the network statistics, this uses the names defined in procedure DefineNetworkEffects in file S_EFFECTS;
 - $2.\ {\rm procedures}\ {\sf NetworkStatistics}\ {\rm and}\ {\sf ActionStatistics}\ {\rm calculate}\ {\rm the}\ {\rm statistics}.$

24.1 Effect definition

Unit DIGRAPH is used not only to define data types but also for defining effects:

- 1. type **TEffect** defines an effect, which is a term in the evaluation function, and possibly a corresponding term in the endowment function;
- 2. type **TEffects** defines an array of effects.

The effects are defined by means of the arrays
- 1. ContributionWeight, giving weights for the contribution of this effect to the evaluation function;
- 2. ConfigWeight, giving weights for the definition of the corresponding statistic;

and the functions

- 3. flijc, defining a contribution of tie $x_{ij} = 1$ to the evaluation function; this function has arguments (dg, i, j, m, t, par), where dg is the valued digraph, (i, j) indicates the tie variable, m defines the period (observation number), t is shorthand for the current value of tie variable x_{ij} of digraph dg (avoiding its unnecessary calculation), and par is a fixed parameter incorporated in the effect;
- 4. flij, defining a contribution of tie $x_{ij} = 1$ to the statistic, with arguments (dg, i, j, m, par); often flij is equal to flijc, but sometimes flij can be computed in a more simple way; there can also be differences between these two functions in the way they deal with missing data;
- 5. fli, defining an extra contribution of actor i to the statistic, with arguments (dg, i, m, par).

The parameter par is a parameter that can be used to modify the definition of the effect (e.g., par could be the order k of a k-star), and which is given in the MO file as the last element of each line corresponding to an effect.

In many cases ContributionWeight = ConfigWeight, $f_{1ij}(dg, i, j, m, par) = f_{1ijc}(dg, i, j, m, 1, par)$, and $f_{1i} = 0$, but flexibility and possibilities for handling missing data are gained by this way of specifying; which contains some redundancies for data sets without any missings.

The contribution of the effect to the *evaluation function* is defined as follows: if the effect has weight α (a statistical parameter, to be distinguished from the preset parameter indicated by *par*), then increasing tie variable x_{ij} from 0 to 1 will increase the evaluation function by α times

$$c_{1} + c_{2} x_{ji} + c_{3} x_{i+} + c_{4} x_{+i} + c_{5} x_{j+} + c_{6} x_{+j} + c_{7} T P_{ij} + c_{8} O S_{ij} + c_{9} I S_{ij} + c_{10} T P_{ji} + f 1 i j c (dg, i, j, m, 0, par) ,$$
(16)

where $c_h = \text{ContributionWeight}[h]$; decreasing tie variable x_{ij} from 1 to 0 will decrease the evaluation function by α times

$$c_{0} + c_{1} + c_{2} x_{ji} + c_{3} x_{i+} + c_{4} x_{+i} + c_{5} x_{j+} + c_{6} x_{+j} + c_{7} TP_{ij} + c_{8} OS_{ij} + c_{9} IS_{ij} + c_{10} TP_{ji} + f_{1ijc}(dg, i, j, m, 1, par).$$
(17)

Here TP, OS, and IS are the numbers of two paths, outstars, and instars, respectively, defined by

$$TP_{ij} = \sum_{h} x_{ih} x_{hj}$$
$$OS_{ij} = \sum_{h} x_{hi} x_{hj}$$
$$IS_{ij} = \sum_{h} x_{ih} x_{jh} ;$$

dg refers to the current network x, and m to the current period. Equations (16), (17) are (except for the sign in the second case) equal to $s_{ik}(x(i \rightsquigarrow j)) - s_{ik}(x)$ in Snijders (2001, 2005) and Snijders, Steglich and Schweinberger (2007), where k is the index of the effect.

Equations (16), (17) are calculated in either of two ways: in procedure UtilityComponents, and in procedures Contrib_n and Contrib_fn, all in unit Digraph. (The procedures in Digraph are linked

to unit S_Base by means of procedures in unit Eight.) Procedure UtilityComponents calculates (16) and (17) and makes them available directly. Procedures Contrib_n and Contrib_fn compute the change in objective function, which is

$$\sum_{k} \alpha_k \{ s_{ik} (x(i \rightsquigarrow j)) - s_{ik}(x) \} .$$
(18)

This is calculated for changing tie variables from 0 to 1, using (16) and $\alpha_k = alpa_f[k]$, in procedures Contrib_n and Contrib_n_alpha in unit Digraph; for changing tie variables from 1 to 0 it is calculated, using (17) and $\alpha_k = alpa_f[k] + alpa_g[k]$, in procedures Contrib_fn and Contrib_fn_alpha in unit Digraph. How these procedures are placed in the simulation algorithm is indicated in Section 23.1.

The *statistic* used for estimating the weight α of the evaluation effect is given by

$$\sum_{m} \sum_{i \neq j} f_{1i}(dg^{m+1}, m, par) + \sum_{i \neq j} x_{ij}^{m+1} \left(c_1 + c_2 x_{ji}^{m+1} + c_3 x_{i+}^{m+1} + c_4 x_{+i}^{m+1} + c_5 x_{j+}^{m+1} + c_6 x_{+j}^{m+1} + c_7 T P_{ij}^{m+1} + c_8 O S_{ij}^{m+1} + c_9 I S_{ij}^{m+1} + c_{10} T P_{ji}^{m+1} + f_{1ij}(dg^{m+1}, i, j, m, par) \right), \quad (19)$$

where $c_h = \text{ConfigWeight}[h]$ and the superscript $^{m+1}$ refers to the observation moment. This statistic is calculated by procedure CalcFunctions_f in unit DIGRAPH.

The endowment function is defined only if $f_{1i} = 0$; then IncludeEndowment is true, else it is false. The contribution of the effect to the endowment function (i.e., an extra component of the loss incurred when changing tie variable x_{ij} from 1 to 0) is given by the corresponding statistical parameter multiplied by (17). The *statistic* used for estimating the weight α of the endowment effect is given by

$$\sum_{i \neq j} (1 - x_{ij}^{m+1}) x_{ij}^m \left(c_1 + c_2 x_{ji}^m + c_3 x_{i+}^m + c_4 x_{+i}^m + c_5 x_{j+}^m + c_6 x_{+j}^m \right. \\ \left. + c_7 T P_{ij}^m + c_8 O S_{ij}^m + c_9 I S_{ij}^m + c_{10} T P_{ji}^m + f 1ij(dg^m, i, j, m, par) \right), \qquad (20)$$

where $c_h = \text{ConfigWeight}[h]$ and the superscripts m and m+1 refer to the observation moments. Note that the factor $(1 - x_{ij}^{m+1})x_{ij}^m$ means that the summation extends only over (i, j) for which there was a tie at observation m which had disappeared at moment m + 1, while the subscripts mbetween the parentheses imply that the "quantity lost" is calculated by reference to moment m. This statistic is calculated by procedure CalcFunctions_g in unit DIGRAPH.

24.2 Changing or adding definitions of effects

Objective function effects for the network dynamics are defined by the procedure AddEffect* defined in unit DIGRAPH, and called in procedure DefineNetworkEffects in include file S_Effects which is part of unit S_DAT. Procedure AddEffect* defines the name, the arrays ContributionWeight and ConfigWeight, and the functions flijc, flij, and fli, all described in Section 24.1. Note that various versions AddEffect1, AddEffect2, etc., are available for procedure AddEffect*, where omitted arguments are 0 or nil. Usually when a new effect is defined, also new functions will have to be defined that are then used in the roles of flijc, flij, and/or fli. Many examples can be found in file S_Effects.

If new effects are added to the rate function for the network dynamics, these additions must be made in a coherent way to each of the following procedures.

- 1. Procedure DefineFunctionNames in unit S_DAT, which contains the names of all the statistics calculated from each simulation;
- 2. Procedure DefineModel_Inames in unit S_DAT, which contains the names of all effects in the network change rate function;
- 3. Function NetworkLambda in unit S_BASE, the network change rate function itself;
- 4. Procedure NetworkStatistics in unit S_BASE, for the statistics used to estimate the parameters by the Method of Moments.

(For the maximum likelihood estimation procedure, non-constant rate functions are not yet implemented.)

If new effects are added to the model for the behavior dynamics, these additions must be made coherently to each of the following procedures.

- For each kind of effect:
 - 1. Procedure DefineFunctionNames in unit S_DAT , which contains the names of all the statistics potentially calculated from each simulation;
 - Function MaxFunctions in unit S_DAT, which contains the number of these statistics (more precisely, the functions called by MaxFunctions);
 - 3. Procedure ActionStatistics in unit S_BASE, which calculates these statistics.
- For effects in the rate function for behavior change, the analogous procedures have to be changed as those for the rate function for network change: function ActionEffects_I in unit S_DAT, procedure DefineModel_Inames in unit S_DAT, function ActionLambda in unit S_BASE, and procedure ActionStatistics in unit S_BASE.
- For effects in the evaluation and endowment functions for behavior change, the following procedures have to be changed: function ActionEffects_f in unit S_DAT, procedure DefineModel_fnames in unit S_DAT, and functions Contr_fa and Contr_ga in unit S_BASE; the latter functions must be coordinated with procedure CalcComponents_fa in unit EIGHT.

The functions AddNoTies_yn, SubtractTies_yn, Contr_fa, Contr_ga, NetworkLambda, and Action-Lambda are evaluated very frequently by the algorithm. Therefore these have been written so that relatively few calculations are needed to evaluate them. Such calculations for a large part are replaced by updating and storing the basic numerical information needed to compute them. These updates are contained in the procedure ChangeTie in unit S_BASE, and the initialisation is contained in the procedure Initialise_Running_Statistics.

25 Statistical Monte Carlo Studies

According to Sir Ronald A. Fisher, there are three main statistical problems, model specification, model estimation, and problems of distribution. The last one concerns the distribution of statistics, such as the distribution of parameter estimates around the true (data-generating) parameter value or the distribution of test statistics, and can be studied by SIENA in various ways. One way is to use Siena05 and Siena07 repeatedly in batch files. It can be useful to know that if Siena05 is called with only one run, then one data set is simulated and also stored in the internal SIENA format under the project name *sisim*. Further, Siena07 gives brief estimation reports in the files *pname*.bof and *pname*.bos, which can be used more easily as summaries of repeated runs than the normal output file.

This can be used, for example, as follows. It is assumed that the reader knows how to run SIENA outside of StOCNET, see Section 17.

- Make a directory which includes the files Siena01.exe, Siena05.exe, Siena07.exe and all the data files of a basic project that will be emulated in the simulations.
- 2. Initialise a Siena project; for example, with project name sprj. Do this by constructing the file sprj.in and running a batch file with the contents

siena01.exe sprj
copy sprj.* sisim.*

- 3. The second line in the batch file has also copied all files sprj.* to sisim.*. This means that now, two identical projects are available. Of these, sprj.* will be used to generate simulated data, and sisim.* to analyse these.
- 4. Specify the file sprj.mo to have the desired effects and parameter values. Copy this file to sisima.mo.
- 5. In the file sprj.si, change the last line so that the number of simulation runs specified is 1.
- 6. Make a batch file which contains many repetitions of the lines

start /w siena05 sprj
copy sisima.mo sisim.mo
start /w siena07 sisim

For example, 1,000 repetitions, for 1,000 simulations.

The first line makes one simulation run with the specifications included in sprj.mo, and writes the data files in internal SIENA format to files sisim.d*. The second line specifies the correct model specification for the analysis. (If you wish to estimate parameters with a different specification, the file sisima.mo should be defined accordingly.) The third line estimates the model.

- 7. The directory should at this moment (before running the batch file) not contain a file called sisim.bos or sisim.bof. Run the batch file.
- 8. After the batch file is finished, the file sisim.bos now contains a summary of all the estimates and standard errors produced. It gives first a heading, then for each simulation/estimation

run a line with initial information, the estimates, and the standard errors. The initial information is the estimation method; if the maximum absolute t-value for convergence is larger than 0.1, then a letter t, else the sign *; if the maximum standard error is larger than 10.0, then a letter s, else the sign *; and the maximum absolute t-value for convergence. The file sisim.bof contains longer summaries of each estimation run, for reference purposes.

Note that the threshold of 0.1 for the *t*-ratios for convergence is very conservative, and somewhat larger values usually are acceptable.

To understand the contents of the file sisim.bos, it is good to compare some of the first lines following the heading with the corresponding results summarized in sisim.bof.

Another way to do simulation studies using SIENA, if one has access to a Delphi or Lazarus compiler, is as follows. Open the unit Siena_7 and go to the procedure TEstForm.FormActivate. The first statement in the procedure is simulate := false. Set the global variable simulate to true. SIENA will then simulate data sets according to the probability model specified in the *pname*.MO file.

Then, manipulate the global constant sequences declared in unit Siena_7 by setting it to some positive integer value k (the default is 1). The constant sequences gives the number of runs (sequences).

The result is that running SIENA will generate k data sets according to the probability model specified *pname*.MO file. From each data set, the parameters are estimated and test statistics are evaluated.

Some Matlab source files are (by default) generated by SIENA. The source code, when interpreted by Matlab, produces histograms of some statistics, in particular histograms of the parameter estimates and the test statistics.

It should be noted that SIENA generates networks with desired properties, but (by default) no covariates. If covariates are desired, suitable code must be added at the beginning of the procedure SimulateData in the unit S_EST. Please note that both internal and external storage (see Section 21.3) of generated covariates is required. Internal storage is difficult unless one knows SIENA -it is advisable to contact the authors in such cases.

26 Constants

The program contains the following constants. Trying to use a basic information file that implies a data set going beyond these constants leads to an error message in the output file and stops the further operation of SIENA.

name	meaning	in unit
pmax	maximum number p of included effects	S_Constants
ccmax	maximum number of possible statistics	S_Constants
nzmax	maximum number nz of individual variables	EIGHT
nzzmax	maximum number nzz of dyadic covariates	EIGHT

Reasonable values for these constants are the following: pmax = 70;

ccmax = 500; the maximum number of statistics depends on the number of available effects, the number of dependent behavior variables, and the number of observations M, and is given by MaxFunctions in unit S_Dat; this should not be more than ccmax;

nzmax = 30;

nzzmax = 20.

The number M of observations may not be higher than 99. Since the number of observations is dealt with by a dynamic array, this is not reflected by some constant. The only reason for the upper bound of 99 is that the index number of the observation is used in the internal data file extension names and may not have more than two digits. But 99 seems quite a high upper bound for practical data sets.

27 References

- Albert, A., and J.A. Anderson. 1984. On the existence of the maximum likelihood estimates in logistic regression models. *Biometrika*, **71**, 1 10.
- Boer, P., Huisman, M., Snijders, T.A.B., Steglich, C.E.G., Wichers, L.H.Y., and E.P.H. Zeggelink. 2006. StOCNET: An open software system for the advanced statistical analysis of social networks. Version 1.7. Groningen: ICS / SciencePlus. http://stat.gamma.rug.nl/stocnet/.
- de Federico de la Rúa, A. 2004. L'Analyse Longitudinal de Réseaux sociaux totaux avec SIENA
 Méthode, discussion et application. BMS, Bulletin de Méthodologie Sociologique, 84, October 2004, 5–39.
- de Federico de la Rúa, A. 2005. El análisis dinámico de redes sociales con SIENA. Método, Discusión y Aplicación. *Empiria*, **10**, 151–181.
- Fisher, R.A. 1932. Statistical Methods for Research Workers, 4th edn. Edinburgh: Oliver & Boyd.
- Frank, O. 1991. Statistical analysis of change in networks. *Statistica Neerlandica*, 45, 283–293.
- Frank, O., and D. Strauss. 1986. Markov graphs. Journal of the American Statistical Association, 81, 832 – 842.
- Gelman, A., and X.-L. Meng (1998) Simulating Normalizing Constants: From Importance Sampling to Bridge Sampling to Path Sampling. *Statistical Science*, 13, 163–185.
- Geyer, C.J., and E.A. Thompson. 1992. Constrained Monte Carlo maximum likelihood for dependent data. *Journal of the Royal Statistical Society*, ser. B, **54**, 657 699.
- Handcock, Mark S. 2002. "Statistical Models for Social Networks: Inference and Degeneracy." Pp. 229 – 240 in Dynamic Social Network Modeling and Analysis: Workshop Summary and Papers, edited by Ronald Breiger, Kathleen Carley, and Philippa E. Pattison. National Research Council of the National Academies. Washington, DC: The National Academies Press.
- Handcock, Mark S., and Hunter, David R. 2006. Inference in curved exponential family models for networks. Journal of Computational and Graphical Statistics, 15, 565–583.
- Hauck Jr. W.W., and Donner, A. 1977. Wald's test as applied to hypotheses in logit analysis. Journal of the American Statistical Association, 72, 851-853.
- Hedges, L.V., and Olkin, I. 1985. Statistical Methods for Meta-analysis. New York: Academic Press.
- Huisman, M.E., and T.A.B. Snijders. 2003. Statistical analysis of longitudinal network data with changing composition. *Sociological Methods & Research*, **32**, 253 287.
- Huisman, M., and C. Steglich (2008). Treatment of non-response in longitudinal network data. *Social Networks*, in press, doi:10.1016/j.socnet.2008.04.004.
- Jariego, I.M., and de Federico de la Rúa, A. 2006. El análisis dinámico de redes con Siena. Pp. 77-93 in J.L. Molina, A. Quiroga, J. Martí, I.M. Jariego, and A. de Federico (eds.), *Talleres de autoformación con programas informáticos de análisis de redes sociales*. Bellaterra: Universitát Autonoma de Barcelona.
- Koskinen, J. 2004. Essays on Bayesian Inference for Social Networks. PhD Dissertation. Department of Statistics, Stockholm University.
- Koskinen, J.H., and T.A.B. Snijders. 2007. Bayesian inference for dynamic network data. Journal of Statistical Planning and Inference. Journal of Statistical Planning and Inference 13: 3930–3938.
- Leenders, R.Th.A.J. 1995. Models for network dynamics: a Markovian framework. *Journal of Mathematical Sociology* 20: 1 21.

- Lepkowski, J.M. 1989. Treatment of wave nonresponse in panel surveys. In: Kasprzyk, D., Duncan, G., Kalton, G., Singh, M.P. (Eds.), *Panel Surveys*. Wiley, New York, pp. 348–374.
- Pearson, M.A., and L. Michell. 2000. Smoke Rings: Social network analysis of friendship groups, smoking and drug-taking. Drugs: education, prevention and policy, 7, 21–37.
- Pearson, Michael, Steglich, Christian, and Snijders, Tom. 2006. Homophily and assimilation among sport-active adolescent substance users. *Connections* 27(1), 47–63.
- Pearson, M., and P. West. 2003. Drifting Smoke Rings: Social Network Analysis and Markov Processes in a Longitudinal Study of Friendship Groups and Risk-Taking. *Connections*, 25(2), 59–76.
- Rao, C.R. 1947. Large sample tests of statistical hypothesis concerning several parameters with applications to problems of estimation. *Proceedings of the Cambridge Philosophical Society*, 44, 50-57.
- Ripley, B. 1981. Spatial Statistics, New York: Wiley.
- Robbins, H., and Monro, S. 1951. A stochastic approximation method. Annals of Mathematical Statistics, 22, 400–407.
- Robins, G., Snijders, T.A.B., Wang, P., Handcock, M., and Pattison, P. 2007. Recent developments in Exponential Random Graph (p*) Models for Social Networks. Social Networks. 29 (2007), 192–215.
- Schweinberger, M. 2005. Statistical Modeling of Network Dynamics Given Panel Data: Goodness-of-fit Tests. Submitted for publication.
- Schweinberger, M., and Snijders, T.A.B. 2006. Markov models for digraph panel data: Monte Carlobased derivative estimation. *Computational Statistics and Data Analysis* 51: 4465 – 4483.
- Schweinberger, M. and T.A.B. Snijders (2007a). Random effects models for digraph panel data. Working paper.
- Schweinberger, M. and T. A. B. Snijders (2007b). Bayesian inference for longitudinal data on social networks and other outcome variables. Working paper.
- Snijders, T.A.B. 1999. The transition probabilities of the reciprocity model. Journal of Mathematical Sociology 23: 241 – 253.
- Snijders, T.A.B. 2001. The statistical evaluation of social network dynamics. Pp. 361-395 in Sociological Methodology 2001, edited by M.E. Sobel and M.P. Becker. Boston and London: Basil Blackwell.
- Snijders, T.A.B. 2002. Markov Chain Monte Carlo Estimation of Exponential Random Graph Models. Journal of Social Structure, Vol. 3 (2002), No. 2.
 - Available from http://www.cmu.edu/joss/content/articles/volume3/Snijders.pdf.
- Snijders, T.A.B. 2003. Accounting for degree distributions in empirical analysis of network dynamics. Pp. 146-161 in: R. Breiger, K. Carley, and P. Pattison (eds.), *Dynamic Social Network Modeling* and Analysis: Workshop Summary and Papers. National Research Council, National Academy of Sciences USA. Washington, DC: The National Academies Press.
- Snijders, T.A.B. 2004. Explained Variation in Dynamic Network Models. Mathématiques, Informatique et Sciences Humaines / Mathematics and Social Sciences, 168(4).
- Snijders, T.A.B. 2005. Models for Longitudinal Network Data. Chapter 11 in P. Carrington, J. Scott, and S. Wasserman (Eds.), *Models and methods in social network analysis*. New York: Cambridge University Press.
- Snijders, T.A.B., 2006. Statistical Methods for Network Dynamics. In: S.R. Luchini et al. (eds.), Proceedings of the XLIII Scientific Meeting, Italian Statistical Society, pp. 281–296. Padova: CLEUP.
- Snijders, T.A.B. 2007. Analysing dynamics of non-directed social networks. In preparation. Transparencies available at internet.
- Snijders, Tom A.B, and Baerveldt, Chris, 2003. A Multilevel Network Study of the Effects of Delinquent Behavior on Friendship Evolution. Journal of Mathematical Sociology 27: 123–151.
- Snijders, T.A.B. and Bosker, R.J. 1999. Multilevel Analysis: An introduction to basic and advanced multilevel modeling. London: Sage.

- Snijders, T.A.B., Koskinen, J.H., and Schweinberger, M. (2010) Maximum Likelihood Estimation for Social Network Dynamics. Annals of Applied Statistics, to be published.
- Snijders, T.A.B., P.E. Pattison, G.L. Robins, and M.S. Handcock. 2006. New specifications for exponential random graph models. *Sociological Methodology*, 99–153.
- Snijders, Tom A.B., Steglich, Christian E.G., and Schweinberger, Michael. 2007. Modeling the coevolution of networks and behavior. In *Longitudinal models in the behavioral and related sciences*, edited by Kees van Montfort, Han Oud and Albert Satorra, pp. 41–71. Mahwah, NJ: Lawrence Erlbaum.
- Snijders, T.A.B., van de Bunt, G.G., and Steglich, C.E.G. (2010). Introduction to actor-based models for network dynamics. *Social Networks*, 32, 44–60.
- Snijders, T.A.B., and M.A.J. Van Duijn. 1997. Simulation for statistical inference in dynamic network models. Pp. 493 – 512 in *Simulating Social Phenomena*, edited by R. Conte, R. Hegselmann, and P. Terna. Berlin: Springer.
- Snijders, T.A.B., and van Duijn, M.A.J. 2002. Conditional maximum likelihood estimation under various specifications of exponential random graph models. Pp. 117–134 in Jan Hagberg (ed.), Contributions to Social Network Analysis, Information Theory, and Other Topics in Statistics; A Festschrift in honour of Ove Frank. University of Stockholm: Department of Statistics.
- Steglich, Ch., Snijders, T.A.B., and Pearson, M. 2010. Dynamic Networks and Behavior: Separating Selection from Influence. To be published, Sociological Methodology. Available from the SIENA website.
- Steglich, Ch.E.G., Snijders, T.A.B., and West, P. 2006. Applying SIENA: An Illustrative Analysis of the Coevolution of Adolescents' Friendship Networks, Taste in Music, and Alcohol Consumption. *Methodology*, 2: 48–56.
- Van de Bunt, G.G. 1999. Friends by choice. An actor-oriented statistical network model for friendship networks through time. Amsterdam: Thesis Publishers.
- Van de Bunt, G.G., M.A.J. van Duijn, and T.A.B. Snijders. 1999. Friendship networks through time: An actor-oriented statistical network model. *Computational and Mathematical Organization Theory*, 5, 167 – 192.
- van Duijn, M.A.J., E.P.H. Zeggelink, M. Huisman, F.N. Stokman, and F.W. Wasseur. 2003. Evolution of Sociology Freshmen into a Friendship Network. *Journal of Mathematical Sociology* 27, 153–191.
- Wasserman, S. 1979. A stochastic model for directed graphs with transition rates determined by reciprocity. Pp. 392 412 in *Sociological Methodology 1980*, edited by K.F. Schuessler. San Francisco: Jossey-Bass.
- Wasserman, S., and P. Pattison. 1996. Logit models and logistic regression for social networks: I. An introduction to Markov graphs and p^* . *Psychometrika*, **61**, 401 425.