MULTILEVEL LONGITUDINAL NETWORK ANALYSIS

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Social networks:

structures of relations between individuals,
represented by graphs and digraphs.

Traditional focus of network analysis:
single groups, *case studies* of networks.

Cross-sectional networks are snapshots,
the results of untraceable history.
Therefore, explaining them has limited importance.

Recently: more attention for *longitudinal* network analysis.
Evolving networks can show the rules of relation choice.

Repeated measurements on social networks:
at least 2 measurements (preferably more).

Statistical methods for such data have been developed (Snijders, 1996, 2001; Steglich, Snijders, & Pearson, 2010; etc.) in which *stochastic simulation models* are used as *statistical models* for data.

These can be applied using MCMC methods for statistical inference (stochastic approximation for determining MoM and ML estimates).

However, general theoretical questions are about groups in general, and should be examined empirically by investigating many groups.

This presentation gives a summary of Snijders & Baerveldt (2003): *meta analysis of several longitudinal network studies*.

Multilevel Social Network Analysis

For generalizable studies of group processes:
parallel studies of each group in a population of groups.

micro level: single network evolution study,
macro level: combination of these network studies.

Problems are caused by the fact that we are combining studies which by themselves are technically quite complicated and computer-intensive.

Example:

Classical theoretical issue in criminology:

relation between delinquent behavior and social ties.

Social control theory (Hirschi):

birds of a feather flock together,

delinquents select each other for friendship.

Differential association theory (Sutherland & Cressie):

delinquent behavior is learned from delinquent friends.

contagion or selection?

This is an example of a question
about social network evolution & behavior evolution
which can be studied only by investigating many groups.

The present example focuses on the selection question;
contagion question can also be considered.

Data and criminological background:
Social Behavioral Study by Chris Baerveldt (University of Utrecht).
Data for 17 schools are used.

These data are available from
`http://www.stats.ox.ac.uk/~snijders/siena/BaerveldtData.html` .

Multilevel network analysis

micro level:

actor-oriented network evolution model (Snijders, 2001).

macro level:

simple two-step multilevel approach distinguishing between *true* and *unreliable* parameter variation.

This is an approach also followed in *random effects meta-analysis*, developed by Cochran (1954) also see, e.g., Hedges & Olkin (1985).

First the micro-level network model will be explained,

then the combination of these in the meta-analysis.

Micro-level analysis

The network evolution in each school class is analyzed separately
using an actor-driven network evolution model
with a common model specification,
but potentially different parameter values.

Summary of the actor-driven approach to network evolution:

* between the network observations, time runs on continuously and networks change unobserved in many 'mini steps';
* actors in the network control their outgoing relations and 'try to obtain a favorable pattern of relationships';
* network changes can be explained only incompletely $\Rightarrow$ residual random component.

See Snijders (*Sociological Methodology, 2001*),
computer package *RSiena* in R, see
`http://www.stats.ox.ac.uk/~snijders/siena/`

Model components:

1. evaluation function

represents what actors regard
as a favorable pattern of relationships;

2. gratification/endowment function

extends the evaluation function
to represent differential effects for creating vs. breaking ties;

3. rate function

represents differences between actors
in the rates of change of their outgoing relations.

Denote network (digraph) by x, actors by i.

evaluation function:

$$f_i(\beta, x) = \sum_{k=1}^{L} \beta_k \, s_{ik}(x)$$

where β_k are statistical coefficients

indicating the weight of the network effects $s_{ik}(x)$.

Endowment function has similar form.

In a sequence of *mini-steps*, randomly designated actors
(depending on their rates of change)
change one of their outgoing relations
(create new tie or dissolve existing tie)
according to a random utility model
aimed at a myopic (non-strategic) maximization
of evaluation function + endowment function + random residual.

Model specification amounts to the determination
of the components of the evaluation, endowment,
and rate functions.

Parameter estimation amounts to estimation (*MoM !*)
of the weights of these components such as β_k .

Model specification :

Choose possible network effects for actor i, e.g.:
(others to whom actor i is tied are called here i's 'friends').

Examples:

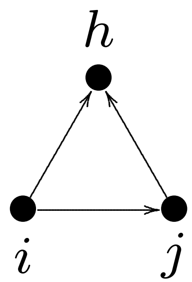
1. *out-degree effect*,
   $s_{i1}(x) = x_{i+} = \sum_j x_{ij}$
2. *reciprocity effect*, number of reciprocated relations
   $s_{i2}(x) = \sum_j x_{ij} \, x_{ji}$

3. *transitive triplets effect*,

number of transitive patterns in i's relations

$(i \to j,\ j \to h,\ i \to h)$

$s_{i3}(x) = \sum_{j,h} x_{ij}\, x_{jh}\, x_{ih}$



transitive triplet

4. *number of actors at distance two effect*,

number of actors j to whom i is indirectly related
(through at least one intermediary: $x_{ih} = x_{hj} = 1$)
but not directly ($x_{ij} = 0$),
= number of geodesic distances equal to 2,

$$s_{i4}(x) = \#\{j \mid x_{ij} = 0, \max_h(x_{ih}\, x_{hj}) > 0\}$$

For each actor covariate v_i , three kinds of evaluation function effect

5. *covariate-related popularity*,

   sum of covariate over all of i 's friends

   $s_{i5}(x) = \sum_j x_{ij}\, v_j$;

6. *covariate-related activity*,

   i's out-degree weighted by covariate

   $s_{i6}(x) = v_i\, x_{i+}$;

7. *covariate-related similarity*,

   sum of covariate similarity between i and his friends,

   $s_{i7}(x) = \sum_j x_{ij} \left(1 - \,|v_i - v_j|\,/r_V\right)$

   where r_V is the range of V.

evaluation function effect for dyadic covariate w_{ij} :

8. *covariate-related preference*,

   sum of covariate over all of i's friends,

   i.e., values of w_{ij} summed over all others to whom i is related,

   $s_{i8}(x) = \sum_j x_{ij} w_{ij}$.

   If this has a positive effect, then the value of a tie $i \rightarrow j$ becomes higher when w_{ij} becomes higher.

In this example the included effects are (*evaluation function*) :

1. out-degree
2. reciprocity
3. popularity (attractiveness of others with high in-degrees)
4. activity (attractiveness of others with high out-degrees)
5. transitivity (transitive triplets $i \to j,\ j \to h,\ i \to h$)
6. number of actors at distance 2
7. balance (preference for others who make the same choices)

For each of the the actor covariates (V)
gender, importance of school friends for the actor,
and level of delinquent behavior:

8. V-related popularity
9. V-related activity
10. V-related (dis)similarity

For the dyadic covariate (W) same ethnicity:

11. W-related preference .

Endowment function

represents differential effects for creating and breaking ties,

$$g_i(\gamma, x, j) = \sum_{h=1}^{H} \gamma_h \, r_{ijh}(x)$$

where γ_h are weights of the effects $r_{ijh}(x)$.

Included effects:

1. reciprocity
2. indirect relations
3. gender similarity
4. same ethnicity
5. similarity level of delinquent behavior .

Rate function can depend on:

1. in-degrees
2. out-degrees
3. actor's number of reciprocated relations
4. level of delinquent behavior .

Macro-level analysis

Each coordinate of parameter vector analysed separately.

Take one coordinate, call it θ .

N school classes $j = 1, ..., N$,

each with own true parameter value θ_j .

Assumption:

θ_j are random sample from a population with mean and variance

$$\mu_\theta = \mathsf{E}\,\theta_j \ , \quad \sigma^2_\theta = \mathsf{var}\,\theta_j \ .$$

Program:

1. test $H_0^{(0)}$: $\mu_\theta = \sigma_\theta^2 = 0$
   (all $\theta_j = 0$), effect θ absent altogether.
2. estimate μ_θ
3. test $H_0^{(1)}$: $\mu_\theta = 0$
4. test $H_0^{(2)}$: $\sigma_\theta^2 = 0$ (all $\theta_j = \mu_\theta$)
5. estimate σ_θ^2 .

Approach:

two-stage weighted least squares (Cochran, 1954).

In micro-level analysis, θ_j is estimated with statistical error:

$$\hat{\theta}_j = \theta_j + E_j$$

Standard error denoted by s_j .

Note:

Observation in group j is not θ_j but $\hat{\theta}_j$,
random variable with mean μ_θ
and variance $\sigma_\theta^2 + s_j^2$.

Assumption includes independence between E_j and s_j.

Estimation

Preliminary unbiased estimator for μ_θ:

$$\hat{\mu}_\theta^{\text{OLS}} = \frac{1}{N} \sum_j \hat{\theta}_j \ .$$

with

$$\text{s.e.}\left(\hat{\mu}_\theta^{\text{OLS}}\right) = \sqrt{\frac{1}{N}\left(\sigma_\theta^2 + \bar{s}^2\right)}$$

where

$$\bar{s}^2 = \frac{1}{N} \sum_j s_j^2 \ .$$

Unbiased estimator for σ_θ^2 is

$$\hat{\sigma}_\theta^2 = \frac{1}{N-1} \sum_j \left(\hat{\theta}_j - \hat{\mu}_\theta^{\text{OLS}}\right)^2 - \bar{s}^2$$

(observed variance minus unreliable variance).

Weighted least squares (WLS, 2SLS) estimator for μ_θ :

$$\hat{\mu}_\theta^{\mathsf{WLS}} = \frac{\sum_j \left(\hat{\theta}_j/(\hat{\sigma}_\theta^2 + s_j^2)\right)}{\sum_j \left(1/(\hat{\sigma}_\theta^2 + s_j^2)\right)}$$

with

$$\text{s.e.}\left(\hat{\mu}_\theta^{\mathsf{WLS}}\right) = \frac{1}{\sqrt{\sum_j 1/(\sigma_\theta^2 + s_j^2)}} \; .$$

Assumption:

θ_j and s_j^2 independent in level-2 population.

Testing

Assumption: $\widehat{\theta}_j$ normally distributed

with mean θ_j and variance s_j^2 .

For testing

$$H_0^{(0)} : \ \mu_\theta = \sigma_\theta^2 = 0$$

(i.e., all $\theta_j = 0$) , use

$$T^2 = \sum_j \left(\frac{\widehat{\theta}_j}{s_j} \right)^2$$

in chi-squared distribution with $d.f. = N$.

For testing

$$H_0^{(1)} : \; \mu_\theta = 0$$

use

$$t_{\mu_\theta} = \frac{\widehat{\mu}_\theta^{\text{WLS}}}{\text{s.e.}\left(\widehat{\mu}_\theta^{\text{WLS}}\right)}$$

in the standard normal distribution.

For testing

$$H_0^{(2)}: \ \sigma_\theta^2 = 0$$

use

$$Q = T^2 - \tilde{t}^2$$

where

$$\tilde{t} = \frac{\sum_j \hat{\theta}_j / s_j^2}{\sqrt{\sum_j 1/s_j^2}}$$

in chi-squared distribution with $d.f. = N - 1$.

These procedures are contained in function *siena08()*
in the *RSiena* package.
This is post-processor for *sienaFit* objects produced by *siena07()*,
and requires that these all have estimated the same model.

Example: Stepwise model selection

Forward selection.

In each step:
estimate the same micro-level model for each school separately
(with school-dependent parameters).

Aggregate estimates at the macro level
and exclude non-significant effects in the next step.

First make such steps for control effects;
then test delinquency effects.

Tested effects

(f indicates evaluation function,

g indicates endowment function,

l indicates rate function) :

1. f density and reciprocity
2. f network closure effects:
   transitive triplets, indirect relations, balance
3. f popularity and activity
4. g reciprocity and indirect relations
5. l degrees

6. covariates

   f gender: popularity, activity,

   f, g gender similarity,

   f importance of school friends:

   popularity, activity;

   f, g same ethnicity

7. and finally the effects of

   delinquent behavior

   f, g similarity

   f activity, popularity

   l rate of change.

Data

17 schools (‘MAVO grade 3’), two waves, one year interval

990 pupils completing both waves, 34 − 129 pupils per school.

Delinquency measured by self-report questionnaire
about frequencies of 23 minor offences
(Cronbach’s alpha = .87 and .91) .

Petty crime of pupils in MAVO-3 and MAVO-4. Percentages of pupils who commited an offence at least once, averaged over waves

shoplifting	40
changing price tags in shops	32
dodging fares	53
buying stolen goods	26
theft of (small) goods from school	35
theft of money from home	23
theft of money from fellow pupil	4
theft of jacket/coat of another pupil	1
burglary/forbidden entry in a house or shop	9
theft of a bike	16
theft of a motor bike	5
theft of something else	12
graffiti	32
vandalism in public transport	13
vandalism on the street	19
arson	32
damaging a bike	25
damaging a car	18
vandalism at school	22
smashing/throwing in a window	20
miscellaneous vandalism	6
unarmed fighting (kicking or hitting)	46
threatening with knife/other weapon	10

Relation: emotional support received and/or given.

Frequencies (in percent) of emotional support relationships within the pupil's network (wave two).

Type of relationship	Number of ties per respondent				
	0	1	2	3	4
support given	30.4	17.7	15.4	13.1	23.4
support received	30.8	20.7	17.6	12.1	18.8

Results

First the control effects:

* network closure:
  indirect relations effect stronger
  than balance or transitive triplets.

* popularity and activity effects very unstable; left out.

* endowment effect of indirect connections strong,
  of reciprocity weak

* rate depends strongly on out-degrees

Results (continued)

* importance of friends at school not significant
* gender: mainly similarity effects
* same ethnicity: weak but statistically significant.

Results for model without effects of delinquent behavior

Effect	N	T^2	$\hat{\mu}_\theta^{\text{WLS}}$	(s.e.)	$\hat{\sigma}_\theta$	Q	(p)
rate function							
out-degrees effect on rate	14	218	2.51	(0.18)	0.0	19.5	.11
evaluation function							
density	15	496	-2.24	(0.16)	0.38	37.7	.001
reciprocity	17	284	2.31	(0.14)	0.0	14.2	.58
transitivity	16	109	1.19	(0.15)	0.0	41.7	$<$ .001
indirect connections	17	349	-0.66	(0.18)	0.61	50.6	$<$ .001
same ethnicity	17	31	-0.29	(0.14)	0.0	26.5	.048
gender popularity of alter	17	49	-0.61	(0.10)	0.0	9.6	.89
gender activity of ego	16	34	0.43	(0.16)	0.36	22.4	.10
gender similarity	17	104	0.91	(0.11)	0.0	33.2	.007
endowment function: effects on creating the tie							
indirect connections	12	136	-1.06	(0.50)	1.16	135.3	$<$ .001
endowment function: effects on breaking the tie							
same ethnicity	16	33	-0.62	(0.57)	1.24	28.3	.020
gender similarity	13	44	-0.11	(0.81)	2.41	43.9	$<$ .001

N = number of schools on which statistics for this effect are based;

T^2 = statistic for testing that total effect is nil;

$\hat{\mu}_\theta^{\mathrm{WLS}}$ = estimated average effect size, with standard error;

$\hat{\sigma}_\theta$ = estimated true between-schools standard deviation of the effect size;

Q = statistic for testing that true effect variance is nil,
with p-value of associated test.

To test the effects of delinquency,
a ‘baseline model’ was constructed
(including all effects for which
the tests of delinquency effects are controlled).
This was the model with the main effects
resulting from the stepwise procedure.

Significant but weak effects were excluded to obtain stable model.
(Note that this model must be run in batch for 17 schools.)

To this ‘baseline model’,
effects of delinquent behavior were added.

Delinquency effects

Overall similarity effect:

$T^2 = 10.66,\ d.f. = 17,\ p = .87$; not significant!

Similarity effects in evaluation *and* endowment functions:

evaluation function $T^2 = 41.28,\ d.f. = 17,\ p = .001$

endowment function (breaking tie) $T^2 = 49.60,\ d.f. = 17,\ p < .001$

Similarity effect on tie creation: $\widehat{\mu}_\theta^{\mathrm{WLS}} = .49$

similarity effect on tie dissolution: $\widehat{\mu}_\theta^{\mathrm{WLS}} = 1.00 - .49 = .51$

Conclusion:

there is an effect of similarity of delinquent behavior,

but it is discovered only

if the effects on creating and breaking ties are differentiated.

With greater similarity in delinquent behavior,

ties are more readily created and more readily dissolved.

Delinquency-related activity and popularity:

activity $T^2 = 13.5,\ d.f. = 17,\ p = .71$,

popularity $T^2 = 13.1,\ d.f. = 17,\ p = .73$,

not significant.

Delinquency-related rate of change:

$T^2 = 19.1,\ d.f. = 14,\ p = .16$, not significant.

Results for model with effects of delinquent behavior

Effect	N	T^2	$\hat{\mu}_\theta^{\mathrm{WLS}}$	(s.e.)	$\hat{\sigma}_\theta$	Q	(p)
rate function							
out-degrees effect on rate	12	113	2.05	(0.20)	0.0	9.3	.59
evaluation function							
density	15	804	-2.07	(0.07)	0.0	35.5	.001
reciprocity	14	318	2.39	(0.40)	1.33	12.0	.53
transitivity	15	66	0.97	(0.15)	0.0	26.0	.026
indirect connections	15	484	-0.63	(0.09)	0.28	58.7	$<$ .001
gender popularity of alter	16	71	-0.64	(0.09)	0.0	18.0	.26
gender activity of ego	16	41	0.27	(0.10)	0.0	33.5	.004
gender similarity	16	111	0.67	(0.07)	0.0	25.2	.047
simil. delinquent behavior	17	38	-0.49	(0.12)	0.0	15.6	.48
endowment function: effects on creating the tie							
indirect connections	13	48	-1.24	(0.31)	0.0	31.5	.002
endowment function: effects on breaking the tie							
simil. delinquent behavior	15	47	-1.00	(0.36)	1.12	20.9	.10

Discussion

'Simple' multilevel network analysis is feasible
when network models are run in batch,
and macro-level analysis distinguishes
true and unreliable variance of micro-level effects.

For a larger number of networks (here $N = 17$),
macro-level explanatory variables can also be used.

References:
Snijders (*Sociological Methodology, 2001*),
Snijders & Baerveldt (*J. Math. Soc., 2003*)
Siena manual, `http://www.stats.ox.ac.uk/~snijders/siena/`

In a likelihood framework, it is possible in principle
to fully integrate the micro-level and macro-level models.

Work on ML estimation of random coefficient multilevel
dynamic network models is under way
(with Johan Koskinen, function *sienaBayes()*)
but ML is much more time-consuming than MoM.

The assumption of independence between θ_j and s_j
is not always reasonable.

Therefore, *siena08* also includes Fisher's combination of tests,
which does not need this assumption.

In cases where large estimated parameter values go along with
large standard errors, the two-stage method may overlook effects
that are discovered by Fisher's combination procedure.

Two one-sided version of Fisher's combination procedure
are included in *siena08*.

1. $H_0^{(R)}: \ \theta_j \leq 0$ for all j;
   $H_1^{(R)}: \ \theta_j > 0$ for at least one j.

Significance is interpreted here,

that there is evidence that in *some* (at least one) group,

parameter θ_j is positive.

2. $H_0^{(L)}: \ \theta_j \geq 0$ for all j;
   $H_1^{(L)}: \ \theta_j < 0$ for at least one j.

Significance is interpreted here,

that there is evidence that in *some* (at least one) group,

parameter θ_j is negative.

It is very well possible that both one-sided combination tests

are significant: some positive and some negative effects.

Further materials

In the RSiena manual

available from http://www.stats.ox.ac.uk/~snijders/siena/downloads

see Section 11.2.

See script *RscriptMultipleGroups.R*

also available from http://www.stats.ox.ac.uk/~snijders/siena/