# Siena Advanced Users' Meeting 2016 

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## Overview

(1) Where to find information?
(2) New convergence criterion
(3) sienacpp
(9) Specification, effects
© Co-evolution
( Multilevel
(1) Missing data
(B) Effect sizes
(9) Hot issues

## 1. Where to look?

Siena is an evolving endeavour, which may be hard to follow.

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- Follow the Siena/Stocnet discussion list!
- The website
http://www.stats.ox.ac.uk/~snijders/siena/ notes important matters at the 'News' page:
list of incompatibilities and bugs; new developments; some interesting papers.
- Most recent versions can be downloaded from R-Forge and 'Downloads' page of website, and are announced at the Siena/Stocnet discussion list.
- Website 'News' page, and Appendix B in the manual, give description of changes in the new versions.


## Where to look? (2)

- Website 'Literature’ page has a section
'Presentations (teaching material)' including (e.g.) these slides.
- Recent (since late 2014) changes in manual:
$\Rightarrow$ elementary effects (treated below);
$\Rightarrow$ more about user-defined interaction effects;
$\Rightarrow$ changed section about convergence and how to use the algorithm options.
- Siena_algorithms.pdf now is at the Siena website (partial explanation of algorithms and code).
- The available effects of 'myeff' are given by effectsDocumentation(myeff).


## 2. New convergence criterion

The usual convergence criterion is tmax the absolute maximum of the $t$-ratios for convergence, considering simultaneously all parameters in the model.

It has appeared that for some models (e.g., with non-centered actor covariates) the usual criterion

$$
\operatorname{tmax} \leq 0.10
$$

is not sufficient.

Therefore, the overall maximum convergence ratio (included as tconv.max in sienaFit objects since some time) gets a new importance.

## 2a. Overall maximum convergence ratio

This is defined as the maximum $t$-ratio for convergence for any linear combination of the parameters,

$$
\text { tconv.max }=\max _{b}\left\{\frac{b^{\prime}\left(\bar{s}_{j}-s^{\mathrm{obs}}\right)}{\sqrt{b^{\prime} \Sigma b}}\right\} .
$$

This is equal to (use Cauchy-Schwarz inequality)

$$
\max _{c}\left\{\frac{c^{\prime} \Sigma^{-1 / 2}\left(\bar{s}_{j}-s^{\mathrm{obs}}\right)}{\sqrt{c^{\prime} c}}\right\}=\sqrt{\left(\bar{s}_{j}-s^{\mathrm{obs}}\right)^{\prime} \Sigma^{-1}\left(\bar{s}_{j}-s^{\mathrm{obs}}\right)} .
$$

The definition implies that
tconv.max $\geq$ tmax.

Studies comparing results of siena07() with the 'true estimate' (robust mean of many estimations) show:
(1) Distance from true estimate is much better indicated by tconv.max than by tmax.
(2) When tconv.max exceeds 0.30 , distances $d_{i}$ from the true value are too large.

New criterion

$$
\operatorname{tmax} \leq 0.10 \text { and tconv.max } \leq 0.25
$$

## Further options for siena07()

To improve the possibilities of siena07( ) to indeed produce estimates satisfying this new criterion, some new options were developed since version 1.1-285 (2015-05-20);
see ?sienaAlgorithmCreate and manual, Section 6.1.3; also see Siena_algorithms.pdf.

Since version 1.1-289 (2015-09-10), new defaults for MoM:
(1) doubleAveraging $=0$
(i.e., use double averaging right from subphase 1)
(2) diagonalize $=0.2$

## What is double averaging?

The regular Robbins-Monro update step is

$$
\hat{\theta}_{N+1}=\hat{\theta}_{N}-a_{N} \tilde{D}^{-1}\left(S_{N}-s\right)
$$

The algorithm with double averaging is

$$
\hat{\theta}_{N+1}=\bar{\theta}_{N}-N a_{N} \tilde{D}^{-1}\left(\bar{S}_{N}-s\right),
$$

where

$$
\bar{\theta}_{N}=\frac{1}{N} \sum_{n \leq N} \hat{\theta}_{n}, \quad \bar{S}_{N}=\frac{1}{N} \sum_{n \leq N} s_{n} .
$$

See Siena_algorithms.pdf.

Achieving this more stringent convergence criterion may require several repeated runs of siena07() linked by using the prevAns parameter.

The following page is an extension of siena07() like in the manual Section 6.1.3.

```
siena07ToConvergence <- function(alg, dat, eff, ans0=NULL,
                                    threshold, ...){
```

```
numr <- 0
```

numr <- 0
ans <- siena07(alg, data=dat, effects=eff, prevAns=ans0, ...) \#
repeat {
save(ans, file=paste("ans",numr,".RData",sep="")) \# to be safe
numr <- numr+1 \# count number of repeated runs
tm <- ans\$tconv.max \# convergence indicator
cat(numr, tm,"\n") \# report how far we are
if (tm < threshold) {break} \# success
if (tm > 10) {break} \# divergence without much hope
\# of good return
if (numr > 100) {break} \# now it has lasted too long
ans <- siena07(alg, data=dat, effects=eff, prevAns=ans, ...)
}
ans
}

```

Results for a moderately complicated data set \& model with a low threshold=0.1:
vdb.algo4 <- sienaAlgorithmCreate(seed=54321, nsub=4)
ans012 <- siena07ToConvergence(vdb.algo4, vdb.data012, vdb.eff012, threshold=0.1, useCluster=TRUE, nbrNodes=2)
(note the use of the dots ... parameter)
\begin{tabular}{llll}
10.1979708 & 90.1471743 & 170.1157999 & 250.1904938 \\
20.2461148 & 100.134066 & 180.10688 & 260.1478382 \\
30.1748373 & 110.1402179 & 190.1368976 & 270.1509909 \\
40.1445431 & 120.1298273 & 200.1147648 & 280.1306809 \\
50.1442089 & 130.1447915 & 210.1453033 & 290.157657 \\
60.133533 & 140.1548974 & 220.1220962 & 300.1727815 \\
70.1211533 & 150.1522713 & 230.1181492 & 310.09992554 \\
80.1203132 & 160.1177088 & 240.2007752 &
\end{tabular}

With 5 subphases it goes more quickly:
vdb.algo5 <- sienaAlgorithmCreate(seed=54321, nsub=5)
ans012 <- siena07ToConvergence(vdb.algo5, vdb.data012, vdb.eff012, threshold=0.1, useCluster=TRUE, nbrNodes=2)
10.1318489
20.1417612
30.0894502

\section*{And with 6:}
```

vdb.algo6 <- sienaAlgorithmCreate(seed=54321, nsub=6)
ans012 <- siena07ToConvergence(vdb.algo6,
vdb.data012, vdb.eff012, threshold=0.1,
useCluster=TRUE, nbrNodes=2)

```
10.111879
20.08501629

\section*{Conjecture :}

If the initial value ('prevAns') is reasonably near the solution (say, tconv.max \(<0.4\) ), the successive ('prevAns') values of results of the Robbins-Monro procedure of siena07() are almost independent, with the distribution of tconv.max having an average value depending on the length of the last subphase.

Default length of subphase \(k\) is \(N_{\max }=(p+7) \times(2.52)^{k}\), with \(p=\) number of parameters.
This means for the length of the last phase:
\(k_{\max } N_{\text {max }}\)
4685
51727
64353
```

algo700 <- sienaAlgorithmCreate(seed=54321,
nsub=1, n2start=700)
vdb.eff012, threshold=0.01,

| 10.1566975 |  |
| :--- | :--- | :--- |
| 20.111309 | 110.1043422 |
| 30.1476088 | 120.1077261 |
| 40.1310093 | 130.1504494 |
| 50.1121832 | 140.1630273 |
| 60.1202841 | 150.1494122 |
| 70.1483798 | 160.09694043 |
| 80.08566426 | 170.09847209 |
| 90.117654 | 180.1935148 |
| 100.1345501 | 190.1862205 |

```
ans012.0 <- siena07(vdb.algo4, data=vdb.data012,
    effects=vdb.eff012, useCluster=TRUE, nbrNodes=2)
ans012r <- siena07ToConvergence(algo700, vdb.data012,
    useCluster=TRUE, nbrNodes=2, ans0=ans012.0)

3 smaller than 0.10; mean 0.13.
```

algo2000 <- sienaAlgorithmCreate(seed=54321,
nsub=1, n2start=2000)
ans012r <- siena07ToConvergence(algo2000, vdb.data012,
vdb.eff012, threshold=0.01,
useCluster=TRUE, nbrNodes=2, ans0=ans012.0)

| 10.1437418 | 110.3063748 |
| :---: | :---: |
| 20.1356098 | 120.07639885 |
| 30.1023418 | 130.127548 |
| 40.1790526 | 140.07809783 |
| 50.1562154 | 150.1082213 |
| 60.1009786 | 160.157643 |
| 70.1109657 | 170.08581463 |
| 80.09316735 | 180.07484955 |
| 90.08950204 | 190.1212121 |
| 100.1508801 | 200.1179153 |

```

6 smaller than 0.10; mean 0.13 (one outlier...)
```

algo4000 <- sienaAlgorithmCreate(seed=54321,
nsub=1, n2start=4000)
ans012r <- siena07ToConvergence(algo4000, vdb.data012,
vdb.eff012, threshold=0.01,
useCluster=TRUE, nbrNodes=2, ans0=ans012.0)

| 10.08891164 | 110.08027072 |
| :---: | :---: |
| 20.1013756 | 120.09768049 |
| 30.142784 | 130.1286098 |
| 40.1038731 | 140.0895482 |
| 50.1082759 | 150.08794522 |
| 60.1026858 | 160.1011986 |
| 70.1429859 | 170.09113767 |
| 80.07912083 | 180.09408743 |
| 90.06697233 | 190.07530766 |
| 100.1144773 | 200.1145916 |

```

10 smaller than 0.10; mean 0.10.
algo10000 <- sienaAlgorithmCreate(seed=54321, nsub=1, n2start=10000)
ans012r <- siena07ToConvergence(algo10000, vdb.data012, vdb.eff012, threshold=0.01, useCluster=TRUE, nbrNodes=2, ans0=ans012.0)
\begin{tabular}{ll}
10.06661866 & 110.1237608 \\
20.07738042 & 120.1458485 \\
30.110977 & 130.1231905 \\
40.0775174 & 140.09870046 \\
50.07146001 & 150.06263102 \\
60.099844 & 160.1059729 \\
70.1162311 & 170.07406779 \\
80.1294587 & 180.1081187 \\
90.07587712 & 190.09084813 \\
100.1355451 & 200.1250927
\end{tabular}

10 smaller than 0.10; mean 0.10 .

With larger and larger number of runs for estimation, for really large numbers of runs
the values of tconv.max do not get convincingly smaller.
What is limiting further decrease?

With larger and larger number of runs for estimation, for really large numbers of runs
the values of tconv.max do not get convincingly smaller.
What is limiting further decrease?

The length of phase 3.

Therefore now, a series of experiments with n2start \(=\mathrm{n} 3\).
Recall that for n2start \(=\mathrm{n} 3=700\), we had 3 smaller than 0.10; mean 0.13.
```

algo2000 <- sienaAlgorithmCreate(seed=54321,
nsub=1, n2start=2000, n3=2000)
ans012r <- siena07ToConvergence(algo2000, vdb.data012,
vdb.eff012, threshold=0.01,
useCluster=TRUE, nbrNodes=2, ans0=ans012.0)

| 10.1154304 | 110.1346359 |
| :---: | :---: |
| 20.08335217 | 120.08379924 |
| 30.1090684 | 130.06819761 |
| 40.07606212 | 140.09366772 |
| 50.1121068 | 150.06833264 |
| 60.09108999 | 160.0710875 |
| 70.1344702 | 170.0623443 |
| 80.1006035 | 180.1140778 |
| 90.1010324 | 190.1103848 |
| 100.1177485 | 200.08337979 |

```

10 smaller than 0.10; mean 0.10.
```

algo4000 <- sienaAlgorithmCreate(seed=54321,
nsub=1, n2start=4000, n3=4000)
ans012r <- siena07ToConvergence(algo4000, vdb.data012,
vdb.eff012, threshold=0.01,
useCluster=TRUE, nbrNodes=2, ans0=ans012.0)

| 10.07859669 | 110.06140544 |  |
| :--- | :--- | :--- |
| 20.04700076 | 120.0761522 |  |
| 30.08937357 |  | 130.05954446 |
| 40.05405991 |  | 140.07722966 |
| 50.06668956 |  | 150.07653949 |
| 60.06828561 | 160.09723554 |  |
| 70.07328622 | 170.04959409 |  |
| 80.07626721 | 180.09825669 |  |
| 90.05873874 | 190.0545597 |  |
| 100.06239783 | 20 | 0.08574505 |

```
all smaller than 0.10; mean 0.07.
algo10000 <- sienaAlgorithmCreate(seed=54321, nsub=1, n2start=10000, n3=10000)
ans012r <- siena07ToConvergence(algo10000, vdb.data012, vdb.eff012, threshold=0.01, useCluster=TRUE, nbrNodes=2, ans0=ans012.0)
\begin{tabular}{ll}
10.04208639 & 110.04782997 \\
20.03915605 & 120.03918571 \\
30.04588603 & 130.05879899 \\
40.05221235 & 140.05735232 \\
50.04504675 & 150.05569969 \\
60.04463104 & 160.04880436 \\
70.07108124 & 170.04088348 \\
80.06551668 & 180.07608058 \\
90.07060039 & 190.06220432 \\
100.06158192 & 200.04426156
\end{tabular}
all smaller than 0.08; mean 0.05 .

\section*{Conclusion}

If a low value of tconv.max is not easily achieved, for getting better convergence:
\(\Rightarrow\) Use 5 or 6 subphases;
or
\(\Rightarrow\) Starting from a decent prevAns, use an algorithm with nsub=1, n2start='large', noting that 'large' \(>(p+7) \times(2.52)^{k}\)
with default \(k=4\); use a smaller firstg (e.g., 0.02);
\(\Rightarrow\) If tconv.max still too big, further increase n2start.

\section*{3. sienacpp()}

RSiena has two rooms:
(1) front office: user interface in R
(2) back office: simulations going on in C++

In siena07(), only the simulations are done in C++; the further calculations for the Robbins-Monro estimation algorithm are done in R .

Starting from version 1.1-290 (2016-01-31), RSienaTest contains sienacpp() which produces the same as siena07(), but with all calculations in C++.
(Some options are not yet included, e.g., multigroup data.)

Parallellization options may be different.
sienacpp() has a small efficiency advantage, which is relatively important only for small data sets / small amounts of total change.

\section*{4. Specification; effects}
(1) GWESP
(2) Structural equivalence: Jaccard distances
(3) Multivariate degree effects on behaviour
(1) Distance-two effects
© Elementary effects
© Influence effects
O Influence from incoming alters
(3) Miscellaneous

\section*{Effects (1): GWESP}

GWESP (geometrically weighted edgewise shared partners) (cf. ERGM) is intermediate between transTrip and transTies.
\[
\operatorname{GWESP}(i, \alpha)=\sum_{j} x_{i j} e^{\alpha}\left\{1-\left(1-e^{-\alpha}\right)^{\sum_{n} x_{i n} x_{n j}}\right\} .
\]
for \(\alpha \geq 0\) (effect parameter \(=100 \times \alpha\) ).
Default \(\alpha=\log (2)\), parameter \(=69\).

\section*{GWESP (contd.)}


Weight of tie \(i \rightarrow j\) for \(s=\sum_{h} x_{i h} x_{h j}\) two-paths.

The implementation of GWESP is an elementary effect:
For creation of a new tie, only its role as \(i \rightarrow j\) in the formula is counted, not its role as \(i \rightarrow h\).

GWESP sometimes yields better fit than transTrip or transTies.

The implementation of GWESP is an elementary effect:
For creation of a new tie, only its role as \(i \rightarrow j\) in the formula is counted, not its role as \(i \rightarrow h\).

GWESP sometimes yields better fit than transTrip or transTies.
The GWESP effect exists also for multivariate networks: gwespFFMix etc.

\section*{New effects (1): Structural equivalence}

A good way of expressing structural equivalence,
i.e., being connected to the same others, is the Jaccard similarity between rows, or columns:
\[
\begin{aligned}
J_{\text {out }}(i, j) & =\frac{\sum_{h} x_{i h} x_{j h}}{x_{i+}+x_{j+}-\sum_{h} x_{i h} x_{j h}} \\
J_{\text {in }}(i, j) & =\frac{\sum_{h} x_{h i} x_{h j}}{x_{+i}+x_{+j}-\sum_{h} x_{h i} x_{h j}}
\end{aligned}
\]

Based on these (by summing over the outgoing ties of \(i\) ), the effects Jout and Jin are defined.

For multivariate networks: JoutMix, JinMix.

\section*{Specification}

Basic specification for the usual type of networks:
- outdegree, reciprocity
- transitive closure: gwespFF or transTrip or ... or perhaps Jin and Jout could do just as well?
- interactions between this and reciprocity (Per Block): transRecTrip, gwespFF \(\times\) recip
(possible because gwespFF is an elementary effect!)
- inPop; outAct; inAct or outPop (or ...sqrt)
- (if available) representation of meeting opportunities

\section*{New effects (2): Multivariate degree effects}

Combined degrees affect behavior.
Number to whom \(i\) is tied in network \(X_{1}\) and network \(X_{2}\) : \(\mathrm{F}=\) 'Forward', \(\mathrm{B}=\) 'Backward', \(\mathrm{R}=\) 'Reciprocal'
(1) double outdegree effect (FFDeg), \(s_{i 1}^{\text {beh }}(x, z)=z_{i} \sum_{j} x_{1 i j} x_{2 i j}\);
(2) double indegree effect (BBDeg),
\(s_{i 2}^{\text {beh }}(x, z)=z_{i} \sum_{j} x_{1 j i} x_{2 j i} ;\)
(3) combined out-indegree effect (FBDeg), \(s_{i 3}^{\text {beh }}(x, z)=z_{i} \sum_{j} x_{1 i j} x_{2 j i} ;\)
(9) combined out-reciprocated degree effect (FRDeg), \(s_{i 4}^{\text {beh }}(x, z)=z_{i} \sum_{j} x_{1 i j} x_{2 i j} x_{2 j i}\);
(3) combined in-reciprocated degree effect (BRDeg), \(s_{i 5}^{\text {beh }}(x, z)=z_{i} \sum_{i} x_{1 j i} x_{2 i j} x_{2 j i}\).

\section*{New effects (3): Influence}

The triple avSim - totSim - avAlt now is a quartet with a \(2 \times 2\) structure:
\{ sim, alt \} \(\times\) \{ av, tot \}
totAlt was implemented for regular influence effects, influence from reciprocated alters, and influence from other covariates (non-dependent / exogenous).

New effects:
(1) totAlt (next to avAlt, totSim, avSim)
(2) totRecAlt (next to avRecAlt)
(3) totXAlt (next to avXAlt, the old AltsAvAlt)

\section*{Incoming influence effects}

The effects avAlt - totAlt - avXAlt - totXAlt now also have analogues for influence from incoming ties:
(4) avinAlt
(5) totInAlt
(6) avXInAlt
(3) totXInAlt
\(i\) is influenced by
incoming ties \(j_{1}-j_{3}\)


\section*{Extreme influence effects}
(8) maxAlt
(9) minAlt

\section*{New effects (4): Distance-two}

There now is the possibility to express influence at distance 2.
With the distinction average/total this leads to 4 possibilities: average vs. total at step 1 or step 2.
(10) avAltDist2
(1) totAltDist2
(1) avTAItDist2
(3) totAAltDist2
\(i\) is influenced by
the average/total of the alter averages/totals of \(j_{1}-j_{3}\)


\section*{New effects (4a)}
(14) The formula for avAltDist2 (average at both steps) uses
\[
\breve{z}_{j}^{(-i)}= \begin{cases}\frac{\sum_{h \neq i} x_{j h} z_{h}}{x_{j+}-x_{j i}} & \text { if } x_{j+}-x_{j i}>0 \\ 0 & \text { if } x_{j+}-x_{j i}=0 .\end{cases}
\]

The effect is
\[
s_{i 14}^{\mathrm{beh}}(x, z)=z_{i} \times \frac{\sum_{j} x_{i j} \check{z}_{j}^{(-i)}}{\sum_{j} x_{i j}}
\]
(and the mean behavior, i.e. 0 , if the ratio is \(0 / 0\) ).

\section*{New effects (4b)}
(ㄷ) totAltDist2 (total at both steps) is defined by
\[
s_{i 15}^{\mathrm{beh}}(x, z)=z_{i} \sum_{j} x_{i j} \sum_{h \neq i} x_{j h} z_{h}=z_{i} \sum_{j} x_{i j}\left(x_{j+}-x_{j i}\right) \check{z}_{j}^{(-i)} .
\]

\section*{New effects (4c)}
(10) avTAltDist2 (average of totals) is defined by
\[
\begin{aligned}
s_{i 16}^{\mathrm{beh}}(x, z) & =z_{i} \times \frac{\sum_{j} x_{i j}\left(x_{j+}-x_{j i}\right) \check{z}_{j}^{(-i)}}{\sum_{j} x_{i j}} \\
& =z_{i} \times \frac{\sum_{j} x_{i j} \sum_{h \neq i} x_{j h} z_{h}}{\sum_{j} x_{i j}}
\end{aligned}
\]
and the mean behavior, i.e. 0 , if the ratio is \(0 / 0\).
(1) totAAltDist2 (total of averages) is defined by
\[
s_{i 17}^{\mathrm{beh}}(x, z)=z_{i} \times\left(\sum_{j} x_{i j} \check{z}_{j}^{(-i)}\right) .
\]

\section*{New effects (5)}

The same for distance-2 averages and totals of covariates:
(B) avXAItDist2
(1) totXAltDist2
(2) avTXAItDist2
(2) totAXAltDist2

\section*{New effects (6): outgoing - incoming}

The same for distance-2 averages and totals where the second step is for incoming ties:
(23) avInAltDist2
(33) totInAltDist2
(23) avTInAltDist2
(3) totAInAltDist2
(20) avXInAltDist
(3) totXInAItDist2
(3) avTXInAltDist2
(2) totAXInAItDist2

\(i\) is influenced by the incoming alter averages of \(j_{1}-j_{3}\). Also ‘sim' versions (simEgolnDist2 etc.)

\section*{New effects (6a)}

The *InAltDist2 effects are also available for two-mode networks.


This means that it is now possible to model influence from those out-alters who have the same affiliations as the focal actor.

\section*{Structural equivalence again}

These distance-two outgoing-incoming effects
can be regarded as representing influence from actors who are structurally equivalent (w.r.t. outgoing ties).

An alternative would be to use Jaccard measures (cf. Jin, Jout) for defining influence effects.
This is still for future consideration.

\section*{Elementary effects}

SAOM effects have been framed in the triple
(1) evaluation
(2) maintenance/endowment
(3) creation
effects.
If the parameters for a creation and corresponding maintenance effect are the same, then it can be represented just as well by an evaluation effect.

These kinds of effects differ in how they contribute to the probability of a particular choice in the ministep.

The contributions to probabilities are based on
evaluation function \(f^{\mathrm{ev}}\)
maintenance function \(f^{m t}\)
creation function \(f^{\text {cr }}\).
Evaluation function plays a role for any step; creation function only for upward change; maintenance function only against downward change.

The definition is on the following page.

The probability that, given a current network \(x\) and actor \(i\) making the ministep, the network changes to \(x^{ \pm i j}\), is
\[
\frac{\exp \left(u_{i}\left(x, x^{ \pm i j}\right)\right)}{1+\sum_{h \neq i} \exp \left(u_{i}\left(x, x^{ \pm i h}\right)\right)}
\]
where the objective function is
\[
\begin{aligned}
u_{i}\left(x, x^{*}\right)=f_{i}^{\mathrm{ev}}\left(x^{*}\right)-f_{i}^{\mathrm{ev}}(x) & +\Delta^{+}\left(x, x^{*}\right)\left(f_{i}^{\mathrm{cr}}\left(x^{*}\right)-f_{i}^{\mathrm{cr}}(x)\right) \\
& +\Delta^{-}\left(x, x^{*}\right)\left(f_{i}^{\mathrm{mt}}\left(x^{*}\right)-f_{i}^{\mathrm{mt}}(x)\right)
\end{aligned}
\]
and
\[
\begin{aligned}
& \Delta^{+}\left(x, x^{*}\right)= \begin{cases}1 & \text { if tie is created }\left(x^{*}=x^{+i j}\right) \\
0 & \text { if tie is dropped, or no change }\end{cases} \\
& \Delta^{-}\left(x, x^{*}\right)= \begin{cases}1 & \text { if tie is dropped }\left(x^{*}=x^{-i j}\right) \\
0 & \text { if tie is created, or no change. }\end{cases}
\end{aligned}
\]

However, not all probabilities of change can be based on changes in some (evaluation-type) function.

Example : transitive triplets
The transitive triplets effect is defined as
\[
s_{i}(x)=\sum_{j, k} x_{i j} x_{i k} x_{k j}
\]
with change statistic
(change when adding tie \(i \rightarrow j\) )
\[
\delta_{i j}(x)=\sum_{k} x_{i k}\left(x_{k j}+x_{j k}\right) .
\]


The first part refers to creating the tie \(i \rightarrow j=h\), the second part to creating the tie \(i \rightarrow j=\ell\).

But one could be interested in only transitive closure, as defined by closing of an open two-path \((i \rightarrow j=h)\), as distinct from creating ties to those with the same out-choices, which is a kind of structural equivalence ( \(i \rightarrow j=\ell\) ).

This cannot be represented
as a change in an evaluation function.

But one could be interested in only transitive closure, as defined by closing of an open two-path ( \(i \rightarrow j=h\) ), as distinct from creating ties to those with the same out-choices, which is a kind of structural equivalence ( \(i \rightarrow j=\ell\) ).

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This cannot be represented as a change in an evaluation function.

Therefore we need a different kind of effect:
elementary effect

\section*{Elementary effect}

An elementary effect is a term of the objective function \(u_{i}\left(x, x^{*}\right)\) used to define change probabilities for ministeps, referring to creation and/or maintenance of a tie \(i \rightarrow j\), without being necessarily a difference \(f_{i}\left(x^{ \pm i j}\right)-f_{i}(x)\) of some function \(f_{i}\) (or similar with multiplication by \(\Delta^{+}\)or \(\Delta^{-}\)).

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Evaluation function is only about the result; elementary effect can express the detailed process / step that leads to a given configuration.

Example : transTrip1 and transTrip2
transTrip1 (transitive closure)
\[
s_{i j}(x)=x_{i j} \sum_{k} x_{i k} x_{k j}
\]
transTrip2
(structural equivalence outgoing ties)
\[
s_{i j}(x)=x_{i j} \sum_{k} x_{i k} x_{j k}
\]


Elementary effects can lead to the same configuration and therefore have the same target statistic (such as transTrip1 and transTrip2).

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However, they can be be distinguished empirically by estimation by the Generalized Method of Moments (under development)
and by likelihood-based methods (Maximum Likelihood, Bayes).

The use of elementary effects can give a more fine-grained representation of the process of network change; but this will require more data;
like also distinction creation-maintenance requires more data.

\section*{Other example of elementary effects}
(30) \(\mathrm{XWX1}\) : like XWX , dependent variable is only one of the XWX ties: \(i \rightarrow j\).
(3) XWX2: dependent variable here is \(i \rightarrow k\).


XWX1 and XWX2 are elementary effects.

\section*{Still other elementary effects}
(32) cl.XWX1: like XWX1 but for dependent network.
(33) cl.XWX2: like XWX2 but for dependent network.
(33) sameXInPop, indegree popularity from same covariate number of incoming ties received by those to whom \(i\) is tied and sent by others who have the same covariate value as \(i\),
\[
S_{i 34}^{\mathrm{net}}(x)=\sum_{j} x_{i j} \sum_{h} x_{h j} l\left\{v_{i}=v_{h}\right\} .
\]
(3) altXOutAct, outd. activity weighted by alter's covariate squared sum of ties weighted by alter's covariate values, \(s_{i 35}^{\text {net }}(x)=\left(\sum_{j} x_{i j} v_{j}\right)^{2}\);
makes sense especially for non-centered covariates.
(36) transRecTrip2, another reciprocity \(\times\) transTrip interaction.

(3) reciPop: reciprocal degree popularity
(3) reciAct: reciprocal degree activity
(3) gwesp.. effects have endowment and creation effects.

They also are allowed to interact with other effects (interactionType = "dyadic").
(10) And various others
(e.g., interactions between networks and covariates).

\section*{5. Co-evolution}

Evolution of multiple networks is studied more and more.
Various new effects have been constructed for this purpose: see Section 12.1.2 of the manual.

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When a monadic or dyadic variable is regarded as a control variable,
it still may be advisable to use it as a dependent variable in the SAOM analysis, rather than as a covariate, because this will allow the 'control' variable much better to maintain its correspondence during the simulations with the focal dependent variables.

Results using a 'control network' as a covariate will differ quite appreciably from results obtained while using it as a co-evolving dependent network; and similarly for monadic variables.

Example: acquaintance or communication as a control network variable for advice to study the properties of the 'purified' advice relation, conditional on the condition of acquaintance.

\section*{6. Multilevel Analysis of Networks}

See MultiMetaSAOM_s.pdf, at website.
Emmanuel Lazega and Tom A.B. Snijders (eds).
Multilevel Network Analysis
for the Social Sciences.
Cham: Springer, 2016.


Special issue of Social Networks ‘Multilevel Social Networks’, edited by Alessandro Lomi, Garry Robins, and Mark Tranmer, vol. 44 (January 2016).

\section*{Analysis of Multilevel Networks}

Multilevel network (Wang, Robins, Pattison, Lazega, 2013):
Network with nodes of several types, distinguishing between types of ties according to types of nodes they connect.

Thus, if types of nodes are \(A, B, C\), distinguish between \(A-A, B-B, C-C\) ties, etc., (within-type) and between \(A-B, A-C\), etc., ties (between-type).

Some may be networks of interest, others may be fixed constraints, still others may be non-existent or non-considered.

This generalizes two-mode networks
and multivariate one mode - two mode combinations.

\section*{See paper}

Tom A.B. Snijders, Alessandro Lomi, and Vanina Torlò (2013).
A model for the multiplex dynamics of two-mode and one-mode networks, with an application to employment preference, friendship, and advice.
Social Networks, 35, 265-276;
Analysis of longitudinal multilevel networks in RSiena is possible by a trick (thanks to James Hollway).

Consider multilevel network with two node sets, \(A\) and \(B\).
There are two one-mode networks internal to \(A\) and \(B\), and two two-mode networks \(X_{1}\) from \(A\) to \(B ; X_{2}\) from \(B\) to \(A\).

Specification for RSiena possible by employing one joint node set \(A \cup B\) and two dependent networks:
\(A\)
\(B\)
\(A\)\(\left(\begin{array}{cc}A & A \\ \text { internal } A & 0 \\ 0 & \text { internal } B\end{array}\right) ~\left(\begin{array}{cc}0 & \text { two-mode } A \times B \\ \text { networks } A, B & \text { network } X_{2}\end{array}\right.\)

For example:
\(A\) a set of organizations, \(B\) a set of individuals, \(X_{2}\) is a fixed membership relation, \(X_{1}\) is not there;
networks \(A\) and \(B\) could be taken apart
in two distinct networks;
if there are only ties between individuals within organizations,
\(B\) will be a network of diagonal blocks
and structural zeros between different organizations;
if there are essential differences between individual ties within organizations or across organizations, \(B\) can be decomposed in two further distinct networks.

For the 'Analysis of Multilevel Networks' using RSiena, possibilities exist in principle, as indicated above;
a first example is Snijders, Lomi, Torlò (2013) mentioned above;
the research program has been continued by James Hollway in his DPhil thesis (Oxford - Zürich - Genève);
further relevant effects have to be elaborated; and the field is open!

\section*{7. Missing Data in RSiena}

The internal treatment of missing tie values in RSiena is simple:
- Impute missing tie variables in wave 1 by 0 .
- Impute missing tie variables in later waves by Last Observation Carried Forward.
- Exclude these imputed values from the calculation of the statistics used for estimation in the MoM.

This can be improved if you have more knowledge of the data and also if you are willing to take more effort.

\section*{Missing Data: improvements}
\(\Rightarrow\) Sometimes there is enough information to make some imputations, based on knowledge of the data, with a high degree of confidence. If possible, do this!
\(\Rightarrow\) There was an error in the treatment of missings in non-centered monadic covariates until and including version 1.1-284.

\section*{Missing Data (contd.)}
\(\Rightarrow\) New option imputationValues in coCovar, varCovar: these values will be used for imputation of missings for the simulations, but (like always happens for missings) are not taken into account for the statistics used for estimation.
Can be used if there are reasonable, not completely reliable values for imputation.

\section*{Missing Data (contd. further)}
\(\Rightarrow\) Papers about treatment of missing data in Social Networks by Hipp, Wang, Butts, Jose, Lakon (2015) and Wang, Butts, Hipp, Jose, Lakon (2016)
criticize missing data treatment by RSiena; but they disregard the fact that imputed values are not used for the statistics for estimation, only for simulations. Thus the effect of these imputations is only indirect.
\(\Rightarrow\) In Wang et al. (2016) it is proposed to do multiple imputations by ERGMs for treating missing data in SAOMS. This might be an improvement of the current defaults, but it disregards the longitudinal dependence!

\section*{Intermezzo:}

Multiple imputation - how does it work?
Multiple stochastic imputation was developed by Don Rubin.
For a given incomplete data set,
the missing data is imputed independently \(D\) times by drawing from the conditional distribution of the missing data given the observed data.

This leads to \(D\) complete data sets, that differ only with respect to the imputed values.

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This leads to \(D\) complete data sets, that differ only with respect to the imputed values.

For each complete data set the desired analysis is executed; standard errors of parameters are a combination of the within-data set standard errors, and the variability of estimates between the data sets.

\section*{How to combine the multiple imputations}

The parameter of interest is denoted \(\gamma\).
Suppose that the \(d^{\prime}\) th randomly imputed data set leads to estimates \(\hat{\gamma}_{d}\) and estimated variances \(W_{d}\) ('Within'),
\[
W_{d}=\operatorname{var}\left\{\hat{\gamma}_{d} \mid \text { data set } d\right\} .
\]

Note that \(W_{d}\) underestimates true uncertainty, because it treats imputed data as real data.

The combined estimate is the average
\[
\bar{\gamma}_{D}=\frac{1}{D} \sum_{d=1}^{D} \hat{\gamma}_{d} .
\]

\section*{Combine multiple imputations....}

Compute the average within-imputation variance
\[
\bar{W}_{D}=\frac{1}{D} \sum_{d=1}^{D} W_{d},
\]
and the between-imputation variance
\[
B_{D}=\frac{1}{D-1} \sum_{d=1}^{D}\left(\hat{\gamma}_{d}-\bar{\gamma}_{D}\right)^{2} .
\]

Estimated total variability for \(\bar{\gamma}_{D}\) is
\[
T_{D}=\widehat{\operatorname{var}}\left(\bar{\gamma}_{D}\right)=\bar{W}_{D}+\frac{D+1}{D} B_{D}, \text { s.e. }\left(\bar{\gamma}_{D}\right)=\sqrt{T_{D}} .
\]

\section*{Another kind of multiple imputation}

The ML option in RSiena will give a model-based simulation of the missings in the second wave, if the first wave has complete data.

This can be used for getting model-based longitudinal imputations:
(1) If the first wave has any missings, estimate an ERGM and impute the missings in the first wave using this.
(2) Estimate the SAOM parameters provisionally using the default treatment of missing data.
(3) For each wave \(m, m=1, \ldots, M-1\) :
given the completed data set for wave \(m\), produce a model-based random draw from the missings in wave \(m+1\) from an ML simulation.
This is not as time-consuming as full ML estimation, because only one simulation is required.
(4) Use this complete data set to obtain one estimate \(\hat{\gamma}_{d}\).
(5) Repeat this procedure \(D\) times and use Rubin's rules for combining the estimates and standard errors.

The main disadvantage is that the future values are not used for the imputations.

This assumes 'missingness at random': i.e., observed data are sufficient for randomly generating missing data.

\section*{Example}

Waves 2-3-4 of the van de Bunt students data.
Wave 0 is complete, so no ERGM imputation is needed!
Number of missing actors in waves 0-4 are \(0 ; 2 ; 3 ; 5 ; 6\), out of 32 .

Impute wave 1 - then 2 - then 3 - then 4.
\begin{tabular}{l|ccccc}
\hline & \multicolumn{3}{|c}{ default } & \multicolumn{3}{c}{ multiple imputation } \\
Effect & par. & (s.e.) & par. & (s.e.) & m.f. \\
\hline Rate 1 & 4.207 & \((0.640)\) & & & \\
Rate 2 & 5.063 & \((0.668)\) & & & \\
\hline outdegree & \(-1.728^{* * *}\) & \((0.317)\) & \(-1.804^{* * *}\) & \((0.343)\) & .16 \\
reciprocity & \(2.024^{* * *}\) & \((0.233)\) & \(2.100^{* * *}\) & \((0.260)\) & .18 \\
trans. trip. & \(0.324^{* * *}\) & \((0.048)\) & \(0.329^{* * *}\) & \((0.049)\) & .12 \\
indeg. - pop. & 0.002 & \((0.038)\) & 0.024 & \((0.039)\) & .16 \\
outdeg. - pop. & \(-0.132^{* * *}\) & \((0.027)\) & \(-0.155^{* * *}\) & \((0.031)\) & .11 \\
outdeg. - act. & 0.014 & \((0.014)\) & 0.013 & \((0.014)\) & .09 \\
sex alter & \(0.409^{*}\) & \((0.200)\) & 0.323 & \((0.204)\) & .08 \\
sex ego & \(-0.386^{\dagger}\) & \((0.208)\) & -0.282 & \((0.218)\) & .13 \\
same sex & \(0.379^{*}\) & \((0.189)\) & \(0.362^{*}\) & \((0.193)\) & .07 \\
program sim. & \(0.604^{* *}\) & \((0.205)\) & \(0.687^{* *}\) & \((0.213)\) & .09 \\
\hline
\end{tabular}
par. \(=\) estimate; s.e. \(=\) standard error; m.f. \(=\) missing fraction;
\({ }^{\dagger} p<0.1\); * \(p<0.05 ;{ }^{* *} p<0.01\); *** \(p<0.001\);
convergence \(t\) ratios all \(<0.06\); overall maximum convergence ratio 0.08 .

\section*{Note:}
in waves 3 and 4 the proportion of missing actors is 0.15 ; proportion missing information is of about this size.

Standard errors of the two approaches are similar; estimates sometimes (3 cases) differ by about half s.e., in other cases differ hardly.

Further studies are needed to see how this procedure performs.

\section*{8. Relative Importance of Effects}

Natalie Indlekofer has contributed the function sienaRI (), which assesses the relative importance of effects.
From version 1.1-270.
Natalie Indlekofer and Ulrik Brandes (2013).
Relative importance of effects
in stochastic actor-oriented models.
Network Science 1.3, 278-304.
sienaRI() also gives (not explicitly used in her paper) the raw/total importance of effects.
sienaRIDynamics() still has difficulties
(temporarily withdrawn).

Expected importance of a parameter is defined as the change in choice probabilities if this parameter would be changed to the value 0 .

Expected relative importance is the same, relative to all effects
(i.e., rescaled to have sum =1).
sienaRI() also produces entropies
(cf. Snijders, Maths. and Soc. Sci., 2004).

Indlekofer \& Brandes (2013), formulae (3, 4):
\(\pi_{i}\) is the vector of probabilities for actor \(i\) in next ministep, and \(\pi_{i}^{(-k)}\) is the same if effect \(k\) obtains a weight of 0 ;
\[
I_{k}(X, i)=\frac{\left\|\pi_{i}-\pi_{i}^{(-k)}\right\|_{1}}{\sum_{\ell=1}^{K}\left\|\pi_{i}-\pi_{i}^{(-\ell)}\right\|_{1}} ;
\]
expected relative importance then is
\[
\frac{1}{N} \sum_{i=1}^{N} I_{k}(X, i) .
\]

Expected (raw / total) importance can then be defined as
\[
\frac{1}{N} \sum_{i=1}^{N}\left\|\pi_{i}-\pi_{i}^{(-k)}\right\|_{1}
\]

\section*{Example: Results for Glasgow data}
\begin{tabular}{l|cc}
\hline Effect & par. & (s.e.) \\
\hline basic rate parameter friendship & 11.207 & \((1.025)\) \\
\hline outdegree (density) & \(-2.023^{* * *}\) & \((0.249)\) \\
reciprocity & \(2.563^{* * *}\) & \((0.190)\) \\
transitive recipr. triplets & \(-0.323^{* * *}\) & \((0.086)\) \\
GWESP I -> K -> J (69) & \(2.172^{* * *}\) & \((0.145)\) \\
indegree - popularity & -0.016 & \((0.031)\) \\
outdegree - popularity & \(-0.135^{\dagger}\) & \((0.076)\) \\
outdegree - activity & \(-0.146^{* * *}\) & \((0.026)\) \\
sex alter & -0.101 & \((0.118)\) \\
sex ego & 0.076 & \((0.150)\) \\
same sex & \(0.691^{* * *}\) & \((0.118)\) \\
\hline\({ }^{\dagger} p<0.1 ;{ }^{*} p<0.05 ; * * p<0.01 ;{ }^{* * *} p<0.001 ;\) \\
convergence \(t\) ratios all <0.07. \\
Overall maximum convergence ratio 0.15.
\end{tabular}

\section*{Example: Results for Glasgow data}
\begin{tabular}{l|cccc}
\hline & \multicolumn{2}{|c}{ Exp. rel. importance } & \multicolumn{2}{c}{ Exp. importance } \\
Effect & wave 1 & wave 2 & wave 1 & wave 2 \\
\hline outdegree (density) & 0.2075 & 0.2193 & 0.8656 & 0.9122 \\
reciprocity & 0.1857 & 0.1691 & 0.7154 & 0.6701 \\
transitive recipr. triplets & 0.0369 & 0.0381 & 0.1650 & 0.1696 \\
GWESP I -> K -> J (69) & 0.1889 & 0.1831 & 0.8079 & 0.7839 \\
indegree - popularity & 0.0145 & 0.0149 & 0.0543 & 0.0551 \\
outdegree - popularity & 0.0900 & 0.0922 & 0.3361 & 0.3500 \\
outdegree - activity & 0.1486 & 0.1541 & 0.6608 & 0.6791 \\
sex alter & 0.0113 & 0.0109 & 0.0373 & 0.0365 \\
sex ego & 0.0062 & 0.0063 & 0.0244 & 0.0248 \\
same sex & 0.1104 & 0.1121 & 0.3798 & 0.3860 \\
& & & & \\
Entropy & & & 0.3632 & 0.3941 \\
\hline
\end{tabular}


Plot of relative importance of effects for first 25 actors and averaged for all actors (pie-chart).

The graph was produced by
plot(RI, actors=1:25, addPieChart = TRUE, legendColumns=5)
where RI was the object produced by sienaRI();
plot. sienaRI() was slightly improved in version 1.1-288,
with a new argument actors,
and better proportions of the pie chart.
Note: you can get the code of such a function by
RSIenaTest:: :plot.sienaRI
(no parentheses!) and then, if you know enough \(R\), modify as desired.

\section*{9a. Developments in current models}

There still is much more to do and explore within the confines of what has already been developed and implemented.
(1) The topics mentioned above are open for application / elaboration.
(2) Evaluation / creation / maintenance / elementary effects
(3) Evaluation / creation / maintenance / effects for behaviour
(9) Variants of non-directed models.
(5) Comparability of effects across models, data sets
~ 'marginal' effects

\section*{Developments in current models (contd.)}
(6) Model selection
(7) Importance of GoF for validity of results
(8) Extended auxiliary functions for GoF
(9) avAlt \(\Leftrightarrow\) avSim \(\Leftrightarrow\) totAlt \(\Leftrightarrow\) totSim
(10) Diffusion of innovations - event history analysis
(11) Two-mode networks
(12) Multivariate (e.g., signed) networks
(13) Ordered networks

\section*{9b. Hot Issues}
- Analysis of Multilevel Networks (see above!)
- Comparison SAOM \(\leftrightarrow\) ERGM (Per Block et al)
- JSiena (Felix Schönenberger)
- Generalized Method of Moments (Viviana Amati)
- Continuous dependent actor variables (Nynke Niezink)
- Settings model (Tom Snijders)
- Marginal effects
- Stable standard errors (Nynke Niezink)
- CUP Books!
```

