# The co-evolution of one-mode and two-mode networks 

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## One-mode and two-mode networks

The combined consideration of one-mode and two-mode networks is very fruitful because it allows to consider the mutual dependencies between (one-mode) relational networks and (two-mode) activities and/or memberships and/or cognitions and/or internal structure and/or details of behavioral tendencies and/or ....

These slides are about the co-evolution of one-mode and two-mode networks according to the Stochastic Actor-oriented Model.

## One-mode - two-mode dependencies

Two-mode networks have less structure, so that there are fewer effects.
Within-dyad dependencies are undefined.
Actor-level dependencies are meaningful.
mixed activity
mixed popularity

$\Rightarrow$ activity


Closed triads are impossible in bipartite networks; but they are possible as mixed patterns.

One-with-two-mode triads.
One-mode tie $\Rightarrow$
two-mode agreement
'I go to places where my friends are'
association-based affiliation closure

Two-mode agreement $\Rightarrow$ one-mode tie
'Those who go to the same places become friends'
affiliation-based focal closure


The two different ways in which this mixed triadic closure can occur implies that, analogous to the distinction influence $\leftrightarrow$ selection in network-behavior co-evolution, in the co-evolution of a one-mode and a two-mode network there is the distinction between focal closure and affiliation closure, also called affiliation-based closure and association-based closure.
(One-mode: association;
two-mode: affiliation, focus).
E.g., Easley and Kleinberg (2010, Section 4.3); Lomi and Stadtfeld (2014).

## Example 1: Glasgow friends and pastimes

## Example:

West of Scotland 11-16 Study; West et al. (1996 and later).
One school year group from a Scottish secondary school starting at age 12-13 years, monitored over more than 2 years; total of 160 pupils, sociometric \& behavior questionnaires at three moments, at appr. 1 year intervals.

First network: friendship;
second network (two-mode): activities.
covariates:
gender, smoking of parents and siblings (binary), money available (range 0-40 pounds/week).
wave 1
girls: circles boys: squares
node size: pocket money

$$
\begin{array}{r}
\text { color: top }=\text { drinking } \\
\text { bottom }=\text { smoking } \\
\text { (orange }=\text { high) }
\end{array}
$$

wave 2
girls: circles
boys: squares
node size: pocket money
color: top = drinking bottom = smoking (orange = high)


Example 1 Descriptives

## Descriptives for friendship

Three waves $\sim$ two periods.
Average degrees 3.7; 3.5; 3.6.

Amount of stability in network ties measured by Jaccard coefficient

$$
J=\frac{N_{11}}{N_{01}+N_{10}+N_{11}}
$$

where $N_{h k}=$ number of tie variables
with value $h$ at one wave and value $k$ at the next.
$J=0.30 ; 0.35$ for the two periods.

## Descriptives for leisure activities

Three waves $\sim$ two periods.
Average degrees 4.7; 4.0; 3.9.

Amount of stability in activities also measured by Jaccard coefficient

$$
J=\frac{N_{11}}{N_{01}+N_{10}+N_{11}}
$$

where $N_{h k}=$ number of tie variables
with value $h$ at one wave and value $k$ at the next.
$J=0.51$ for both periods.

## Second mode: Leisure time activities

|  | daily | weekly | monthly | less |
| :--- | ---: | ---: | ---: | ---: |
| I listen to tapes or CDs | $\mathbf{3 8 8}$ | 23 | 5 | 16 |
| I look around in the shops | $\mathbf{6 5}$ | 290 | 48 | 30 |
| I read comics, mags or books | $\mathbf{1 8 6}$ | 121 | 65 | 60 |
| I go to sport matches | $\mathbf{3 0}$ | $\mathbf{1 1 3}$ | 90 | 200 |
| I take part in sports | $\mathbf{2 1 8}$ | 117 | 30 | 68 |
| I hang round in the streets | $\mathbf{2 1 6}$ | 64 | 26 | 125 |
| I play computer games | $\mathbf{1 5 7}$ | 109 | 45 | 122 |
| I spend time on hobby (e.g. art, instrument) | $\mathbf{1 1 4}$ | 113 | 36 | 170 |
| I go to something like B.B., Guides or Scouts | $\mathbf{3 6}$ | $\mathbf{8 1}$ | 1 | 314 |
| I go to cinema | $\mathbf{1 1}$ | $\mathbf{8 1}$ | 269 | 71 |
| I go to pop concerts, gigs | $\mathbf{7}$ | $\mathbf{6}$ | $\mathbf{9 2}$ | 326 |
| I go to church, mosque or temple | $\mathbf{2}$ | $\mathbf{5 2}$ | $\mathbf{1 0}$ | 368 |
| I look after a pet animal | $\mathbf{1 9 7}$ | 25 | 6 | 203 |
| I go to dance clubs or raves | $\mathbf{1 5}$ | $\mathbf{4 4}$ | $\mathbf{1 0 4}$ | 266 |
| I do nothing much (am bored) | $\mathbf{3 7}$ | 39 | 24 | 331 |

Number of students participating in each of a list of activities, summed over three waves, for Glasgow data. Bold-faced are categories counted as a tie.

## Results

The table of results is distributed over 4 pages:

- friendship: the basis
- friendship: effects of leisure activities
- leisure: the basis
- leisure: sex-related specializations.


## Friendship: basic

| Effect | par. | (s.e.) |
| :--- | :---: | :---: |
| rate period 1 | 12.383 | $(1.217)$ |
| rate period 2 | 9.870 | $(1.132)$ |
| Friendship: endogenous effects |  |  |
| outdegree (density) | $-3.633^{* * *}$ | $(0.258)$ |
| reciprocity | $3.337^{* * *}$ | $(0.311)$ |
| GWESPFF: creation $(\alpha=0.69)$ | $3.350^{* * *}$ | $(0.301)$ |
| GWESPFF: maintenance $(\alpha=0.69)$ | 0.273 | $(0.385)$ |
| indegree - popularity | $-0.079^{* * *}$ | $(0.020)$ |
| outdegree - activity | $0.121^{* * *}$ | $(0.036)$ |
| reciprocated degree - activity | $-0.303^{* * *}$ | $(0.071)$ |
| indegree - activity | 0.001 | $(0.056)$ |
| Covariate effects |  |  |
| girls alter | -0.124 | $(0.085)$ |
| girls ego | 0.032 | $(0.086)$ |
| same gender | $0.446^{* * *}$ | $(0.082)$ |

## Friendship: effects of leisure activities

| Effect | par. | (s.e.) |
| :--- | :---: | :---: |
| Friendship: effects of leisure |  |  |
| leisure outdegree popularity | -0.046 | $(0.037)$ |
| leisure outdegree activity | $-0.087^{*}$ | $(0.037)$ |
| affiliation-based closure | $0.213^{* *}$ | $(0.073)$ |

## Leisure: basic

| Effect | par. | (s.e.) |
| :--- | :---: | :--- |
| Activities | 4.386 | $(0.293)$ |
| rate period 1 | 4.254 | $(0.313)$ |
| rate period 2 | $-2.149 * * *$ | $(0.333)$ |
| Endogenous effects of activities |  |  |
| outdegree (density) | $0.0272^{* * *}$ | $(0.0073)$ |
| 4-cycles | $0.0269^{* *}$ | $(0.0084)$ |
| indegree - popularity | $0.389^{* * *}$ | $(0.086)$ |
| outdegree - activity | $-0.0128^{* * *}$ | $(0.0027)$ |
| out-in degree assortativity |  |  |
| Effects of friendship on activities |  |  |
| friendship indegree activity | 0.001 | $(0.039)$ |
| friendship outdegree activity | $-0.148^{*}$ | $(0.073)$ |
| association-based closure | $0.351^{* * *}$ | $(0.062)$ |

## Leisure: two-mode sex homophily

Homophily in two-mode networks is treated in
https://www.stats.ox.ac.uk/~snijders/siena/Twomode_s.pdf

| Effect | par. | (s.e.) |
| :--- | :---: | :--- |
| Effects of sex on activities |  |  |
| girls ego | $-0.870^{* *}$ | $(0.313)$ |
| 4-cycles among girls | 0.0027 | $(0.0065)$ |
| girls $\times$ outdegree - activity | $0.066^{*}$ | $(0.029)$ |
| indegree - popularity within girls | $0.0242^{*}$ | $(0.0098)$ |
| indegree - popularity within boys | 0.0091 | $(0.0103)$ |

Leisure homophily only for girls!
The leisure-only model did show leisure homophily also for boys.
This is 'explained away' here by association-based closure.

## Example 2: American high school

Other example, based on Fujimoto, Snijders, \& Valente (NWS, 2018).
US high school, $X=$ friendship, $Z=$ sport activities.

## Descriptives

Two waves $\sim$ one period.
$n=309$ students, $m=16$ sports,
$X=$ friendship, $Z=$ sport participation in past 12 months.
Average friendship degrees 6.6, 6.2; Jaccard similarity 0.25 .

Average sport out-degrees 1.2, 1.1; Jaccard similarity 0.44 .

Again, four pages of results.

## Results: friendship (1)

| Effect | par. | (s.e.) |
| :--- | :---: | :---: |
| outdegree | $-3.519^{* * *}$ | $(0.413)$ |
| reciprocity | $2.775^{* * *}$ | $(0.171)$ |
| transitive triplets | $0.398^{* * *}$ | $(0.032)$ |
| transitive reciprocated triplets | $-0.293^{* * *}$ | $(0.071)$ |
| 3-cycles | 0.101 | $(0.064)$ |
| transitive ties | $0.425^{* * *}$ | $(0.073)$ |
| indegree - popularity | $0.022^{* * *}$ | $(0.005)$ |
| outdegree - popularity | $-0.065^{* * *}$ | $(0.009)$ |
| outdegree - activity | 0.011 | $(0.023)$ |
| outdegree - activity $(\sqrt{ })$ | 0.154 | $(0.187)$ |
| reciprocal degree - activity | $-0.079^{* * *}$ | $(0.015)$ |
| outdegree positive | -0.776 | $(0.763)$ |
| gender (F) alter | -0.035 | $(0.041)$ |
| gender (F) ego | $0.093^{*}$ | $(0.042)$ |
| same gender | $0.363^{* * *}$ | $(0.047)$ |
| same gender $\times$ reciprocity | $-0.442^{* *}$ | $(0.136)$ |

## Results: friendship (2)

| Effect | par. | (s.e.) |
| :--- | ---: | :---: |
| hispanic alter | 0.013 | $(0.065)$ |
| hispanic ego | -0.045 | $(0.063)$ |
| same hispanic | $0.144^{*}$ | $(0.064)$ |
| grade alter | -0.021 | $(0.022)$ |
| grade ego | -0.026 | $(0.023)$ |
| grade similarity | $0.317^{* * *}$ | $(0.088)$ |
| same class | $0.564^{* * *}$ | $(0.091)$ |
| same class $\times$ reciprocity | -0.210 | $(0.154)$ |
| same class $\times$ same gender | -0.041 | $(0.107)$ |

## Results: sports

| Effect | par. | (s.e.) |
| :--- | :---: | :---: |
| outdegree | $-2.369^{* * *}$ | $(0.613)$ |
| 4-cycles | 0.041 | $(0.030)$ |
| indegree - popularity | $0.020^{* *}$ | $(0.007)$ |
| outdegree - activity | -0.029 | $(0.102)$ |
| outdegree positive | $-2.116^{* * *}$ | $(0.592)$ |
| gender ego (F) | 0.023 | $(0.184)$ |
| two-mode gender similarity | $1.750^{* * *}$ | $(0.416)$ |
| 4-cycles same gender | $-0.085^{*}$ | $(0.039)$ |
| hispanic ego | $-0.599^{* *}$ | $(0.222)$ |
| grade ego | $0.299^{* *}$ | $(0.115)$ |

Strong evidence for homophily!

## Results: cross-networks

| Effect | par. | (s.e.) |
| :--- | ---: | :---: |
| Sports $\Rightarrow$ Friendship |  |  |
| outdegree $(\sqrt{ })$ sports activity | $-0.106^{* *}$ | $(0.038)$ |
| affiliation-based closure | $0.159^{* *}$ | $(0.057)$ |

Friendship $\Rightarrow$ Sports

| friendship outdegree $(\sqrt{ })$ activity (eval.) | 0.171 | $(0.468)$ |
| :--- | :---: | :---: |
| friendship outdegree $(\sqrt{ })$ activity (maint.) | -1.386 | $(1.063)$ |
| association-based closure (evaluation) | $0.442^{*}$ | $(0.187)$ |
| association-based closure (maintenance) | 0.646 | $(0.452)$ |

Those mentioning more sports mention fewer friends;
shared sport activities lead to friendship;
friendship leads to shared sport activities
(not different for creating or maintaining activities).

## Discussion

$\Rightarrow$ See Snijders, Lomi \& Torlò in Social Networks, 2013 Fujimoto, Snijders \& Valente (Network Science, 2018), Lomi \& Stadtfeld (KZfSS, 2014).
$\Rightarrow$ It's a multilevel issue (but not nested): ties, dyads, actors, triads, subgroups, ...
$\Rightarrow$ Testing cross-network dependencies in dynamics of multiple networks gives interesting new possibilities for hypothesis testing.
$\Rightarrow$ Elaborated along the lines of actor-based modeling.
$\Rightarrow$ Compared to modeling dynamics of single networks, this approach attenuates the Markov assumption by extending the state space to a multiple network.
$\Rightarrow$ New perspectives possible by combining one-mode and two-mode networks.
$\Rightarrow$ The method is available in RSiena.
This works for a small number (e.g., 2-6) of networks, and a limited number of actors (up to a few hundred).
$\Rightarrow$ If there are implication relations between the networks, e.g., two networks might be mutually exclusive, or one might be a sub-network of the other, then this constraint is observed, noted in the print01Report, and respected in the simulations.
This gives possibilities for networks with valued ties by using different dichotomies.

