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The social relations model for family data: A multilevel approach

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Abstract
Multilevel models are proposed to study relational or dyadic data from multiple persons in families or other groups. The variable under study is assumed to refer to a dyadic relation between individuals in the groups. The proposed models are elaborations of the Social Relations Model. The different roles of father, mother, and child are emphasized in these models. Multilevel models provide researchers with a method to estimate the variances and correlations of the Social Relations Model and to incorporate the effects of covariates and test specialized models, even with missing observations.

Families are complex groups. Although this is a platitude, all too often the statistical models used have failed to capture very much of that complexity. Data about relations between family members are especially difficult to analyze statistically in a way that does justice to the complexity of the data structure. One way of modeling family data is to consider the measurements to be a function of the family as a whole, the individual family members, and the dyads within the family (e.g., mother–father, mother–child). For example, the degree to which a mother reports feeling coldness toward her son may be a function of how much coldness exists in that family overall, as compared to other families. It may also be a function of the mother—she may be a very cold person. The mother–son coldness score may also be a function of the son—he may be a difficult child and, on average, people may not feel so warm toward him. Finally, this score may reflect a unique relational component such that, compared to the relations of the mother to the other family members, and compared to the relations of this son with other family members, the mother feels especially cold toward her son. The Social Relations Model (SRM; Kashy & Kenny, 1990; Kenny & La Voie, 1984) is a model that captures these multiple levels of analysis inherent in families.

This article is about relational data—that is, data referring to relations between individuals, where, moreover, the individuals are nested in groups. Because relations are between pairs or dyads of individuals, relational data is also called *dyadic data*. Although the methods can be applied to other kinds of groups (e.g., task-related groups with or without a formal role structure) we focus on relations in families. Thus, each observation refers to the relation of one family member directed to another family member. For instance, each family member may report how warm his or her relationship is with each of the other family members. Typically, the family consists of a father, a mother, and one or more children.

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Dyadic data are typically directed—that is, the measured relationship of person i to person j is different from the measured relationship of person j to person i. A data-collection method where family members report on their relationship or behavior towards, or their perception of, other family members leads to directed data. Person A reporting of her relation to person B may be different from person B reporting about his relation with person A. The respondent reports his or her perception that may differ from the perception of the other person in the relationship. This reflects the distinction between the person (the actor) who reports the perceptions and the other person who is the partner in the relationship. The two individuals involved in each data point are referred to as the *actor* and the *partner* of the relation. Thus, a mother (actor) might judge how warm her child (partner) is. Another way to indicate this is that the relation is *from* person i (the actor) *to* person j (the partner). There is an essential asymmetry between the roles of the two individuals in a dyadic relationship. Actor and partner are referred to as the *positions* of the individual with respect to a given directed relation, retaining *role* for the roles of father, mother, and child. Actor and partner effects are used as generic terms. In other contexts, different terms might be used. For example, if the data are intensities of observed communications, then *sender* and *receiver* would be more appropriate terms; if flows of resources, then *giver* and *receiver* would be used; and if perceptions are measured, then *perceiver* and *target* would be used.

The essentials of the SRM are deceptively simple. A dyadic measurement is assumed to be a function of the actor, partner, and a residual. The actor effect refers to how the actor generally behaves with or sees others. The partner effect refers to how people generally behave with or see the person. The residual or relationship effect is the interaction of actor and partner. The formal statistical model is presented later.

There are various difficulties in the estimation of the SRM parameters with traditional methods such as random effects analysis of variance (ANOVA) (Bond & Lashley, 1996; Kenny & La Voie, 1984) or structural equation modeling (Bollen, 1989; Kashy & Kenny, 1990; Kenny, 1979). These difficulties are elaborated later in this article and can be summarized as follows. First, there is not one method of estimation but several, one for which group members do not have fixed roles and one for which they do. Second, missing data is especially problematic for traditional methods. It can happen that the loss of just one observation results in the loss of an entire group. Third, covariates cannot easily be incorporated into the traditional approach. Fourth, specialized models cannot be easily estimated by traditional methods.

The innovative estimation strategy presented in this article solves all of these problems. First, and foremost, it is a single unified approach to estimation. Moreover, maximum likelihood estimation is used, and so estimates are statistically optimal and model comparisons can be made. Second, the data are allowed to be incomplete in the sense that data are missing for some of the dyads. The number of children may vary, and some families may have data from only one parent. The methods that we present do not require that every family have the same number of children and both parents. Third, covariates, such as age or gender, can be taken into consideration. Fourth, specialized models can be estimated. For instance, we can force variances to be equal to each other or to zero.

We use the Hierarchical Linear Model (HLM) or multilevel model to analyze family data. This model is becoming increasingly well-known and utilized in social science research (Bryk & Raudenbush, 1992; Goldstein, 1995; Snijders & Bosker, 1999). The multilevel model is a statistical model for the analysis of data with a hierarchical nested structure (e.g., individuals nested in groups). Because SRM data are, indeed, data with individuals nested in groups, it is clear that the multilevel model is applicable to this type of data. However, dyadic data have the specific complexity that each data point, corresponding to a directed relation from individual i to individual j in family k, refers to two individuals instead of only one. In hierarchical linear models, there is not a single "unit of analysis" (such as the "respondent" in traditional social science research), but several. The group or family, as well as the individual, as well as the dyadic relation between two individuals, all are in principle important as units of analysis. The effect of an individual as an actor is, in general, different from his or her effect as a partner. The dyadic relation between persons i and j is defined as the pair of relations between these individuals: the relation from i to j and the relation from j to i. Thus, dyadic measurements refer to at least three units of analysis: the group, the individual person (both actor and partner), and the dyad.

The multilevel model for dyadic data formulated below takes account of this complexity by *random effects* associated to each of the units of analysis, and by *fixed effects* of variables that depend on these units. The random effects allow a complicated within-family correlational structure that corresponds to the complex logical structure of the data. The fixed effects are similar to effects in traditional regression analysis and allow the inclusion of effects of covariates, depending on the family, the individual, the dyad, and the observation.

In SRM designs, the units at these levels are not neatly nested, because each relation belongs to two individuals, rather than one. This implies that actor and partner have crossed random effects within the groups. Therefore, this model does not have the usual form of the HLM as described in Bryk and Raudenbush (1992) and Goldstein (1995). However, special extensions of the HLM have been developed that do allow crossed random effects (cf. Goldstein, 1994, 1995, Chapter 8; Raudenbush, 1993; Snijders & Bosker, 1999, Chapter 11). We explain in this article how these extensions allow for the estimation of the SRM model by multilevel models.

Multilevel models have been used before with relational data. One example is that of Raudenbush, Brennan, and Barnett (1995). However, their approach is limited to dyadic over-time data on couples (i.e., different dyads necessarily contain different individuals). The models developed in this article are meant for designs where each group or family yields data on multiple relations; for instance, if the data are complete, this makes for a round-robin structure (everyone rates or interacts with everyone else). Further, we do not require repeated measures, although the proposed technique can be extended to repeated measures. An application of multilevel modeling to personal, or egocentric, networks (the set of relations of respondents who are themselves not relationally connected) is presented by Snijders, Spreen, and Zwaagstra (1994). A method for analyzing dichotomous dyadic data is proposed in Van Duijn and Lazega (1997) and Van Duijn and Snijders (1997).

Several software packages for estimation of multilevel models are available, each with its own advantages and disadvantages: for example, *MLn* (Rasbash & Woodhouse, 1995) and its Windows version *MLwiN* (Goldstein et al., 1998; information about these programs is available at the web site, http://www.ioe.ac.uk/multilevel/), *VARCL* (Longford, 1993), and *HLM* (Bryk, Raudenbush, & Congdon, 1996). Estimation of the models presented here relies on the estimation of random slope models with equality constraints on the random slope variances. This option is currently possible only in *MLn*. For the models presented in this article, formulations are given that allow them to be estimated using *MLn*. This program was used for the computations for the examples. *MLn* macros, which can be used to specify these models for the *MLn* and *MLwiN* programs, can be obtained from the first author's web site (http://stat.gamma.rug.nl/snijders/).

The Social Relations Model

First we review the Social Relations Model (SRM; Kashy & Kenny, 1990; Kenny, 1994; Kenny & La Voie, 1984) and indicate how it can be interpreted and estimated as a multilevel model. Consider a single group containing n individuals with no roles (e.g., mother or father) distinguishing the different group members. The basic data point is denoted Y_{ij}, indicating the relation from individual i to individual j. So, for instance, if the variable were smiling, then Y_{ij} would represent how often person i smiles at person j, and Y_{ji} would represent how much j smiles at i.

We consider the data set of Table 7 in Warner, Kenny, and Stoto (1979). In their study, conversations were examined among eight different people. Each person was paired with each of the other seven people, resulting in 28 different conversations on three different days for 12 to 15 minutes. For each conversation, the percentage of time speaking was recorded. In our reanalysis, Y_{ij} is the percentage of time that i spoke to j, averaged over the 3 days; individual i is the actor for this variable, while j is the partner. The SRM describes the response Y_{ij} as the sum of an overall mean μ, an actor effect A_i, a partner effect B_j, and a dyadic or relationship effect E_{ij}:

$$Y_{ij} = \mu + A_i + B_j + E_{ij} \qquad (1)$$

Thus, an individual with a high value of A_i generally talks more in all conversations, an individual with a large value of B_j is generally talked to more in all conversations, and if E_{ij} is large then i talks more to j in his or her conversation than could be expected on the basis of the speaker's general effects A_i and B_j. The letters i and j are used to denote individuals in the actor and partner position, respectively, but refer to the same set of individuals. Thus, i and j take on values 1 through n, and A_1 and B_1 refer to the actor and partner effect of the same individual (i.e., the individual with number 1). These effects are assumed to have a zero mean and to be mutually uncorrelated across individuals, so that the variance of Y_{ij} is theoretically decomposed as the sum of an actor variance, a partner variance, and a dyadic variance. However, it is assumed that correlation exists, over the n individuals, between their actor effects A_i and their partner effects B_i. Individuals who talk more might elicit less talk from others (a negative correlation) or they might elicit more talk (a positive correlation).

In addition to the actor and partner correlation, another correlation might exist: the reciprocity, or mutuality, effect. This effect expresses that there is a special correspondence between the relation from i to j and the relation from j to i. Reciprocity effects are known to be fundamental in social network analysis (cf. Kenny & La Voie, 1984, p. 157, and Wasserman & Faust, 1994). For dyadic measurements that have the interpretation of friendship or affection, usually a positive correlation exists between these two reciprocal relations. For relational variables that express a directional inequality, like influence from i on j, or i teaching something to j, there is likely a negative correlation between the reciprocal relations. Therefore, it is necessary to assume that a correlation can exist between the two reciprocal dyadic effects, E_{ij} and E_{ji}. For the Warner et al. (1979) data, positive reciprocity would imply that in dyads where one person talks a great deal to one partner, that partner also talks a great deal in return.

Except for the correlations mentioned up to now, all correlations between random effects in Equation 1 are assumed to be zero. The general characteristics of individual i are reflected in the actor and partner effects of this individual, while there are dyad-specific deviations from these individual patterns.

To estimate the SRM as a multilevel model, our strategy is as follows. There are two levels: the group and the observation. We follow the procedure to estimate crossed random effects also used in *MLn* (as described in Rasbash & Woodhouse, 1995, and in Goldstein et al., 1998). Dummy variables are created for each individual actor and partner within the group, denoted a_1 to a_n for the actors and p_1 to p_n for the partners. Also, we define $n(n-1)$ dummy variables for each dyad, denoted as d_{12} (for the dyad composed of persons 1 and 2) through $d_{n-1,n}$. The dummy variables are defined as follows:

$a_i = 1$ if actor is individual i (zero otherwise),
$p_i = 1$ if partner is individual i (zero otherwise),

for i from 1 to n, and

$d_{ij} = 1$ if actor is individual i and partner individual j (zero otherwise),

for i and j being unequal and running from 1 to n. Note that in the Warner et al. (1979) example, $n = 8$ so $n(n-1) = 56$. The model implied by Equation 1 can be rewritten as

$$Y_{ij} = \mu + \Sigma_s A_s a_s + \Sigma_t B_t p_t + \Sigma_{s \neq t} d_{st} E_{st} \quad (2)$$

where the summations are over all individuals s (in their role as actors) and t (as partners) in the group. Models 1 and 2 are fully identical; the reason for working with Model 2 is only its amenability for implementation in multilevel software. In the terminology of multilevel analysis, the random effects A_i, B_j, and E_{ij} take on the role of random slopes at group level (level two) that multiply the dummy variables a, p, and d. In the multilevel terminology, level two is defined here as the group level. In the SRM model of the present section, there is just one group.

The group level is not superfluous here, however, because it is used by the *MLn* program to deal with the cross-classified random actor and partner effects (cf. the discussion in Goldstein et al., 1998, Appendix A, about modeling random effects of cross-classified category structures). The variances of all these random actor slopes are forced to be equal and that variance is the actor variance; likewise, the variances of all random slopes for partner are forced to be equal; and finally the variance of residual slopes are forced to be equal. Further, the covariances between A_i and B_j for different individuals i and j are constrained to be zero, and the n covariances between A_i and B_i, referring to the same individual i, are set equal. Finally, the covariance between E_{ij} and E_{ji} is the dyadic covariance or dyadic reciprocity. The formulation in Equation 2 allows the SRM to be estimated by *MLn* and *MLwiN* as a model with cross-classified random effects.

The parameter estimates from *MLn* for the Warner et al. (1979) data are given in Table 1. When we use conventional SRM estimation methods (Bond & Lashley, 1996; Kenny, 1994) or the maximum likelihood method of Dempster, Rubin, and Tsutakawa (1981) or Wong (1982), we obtain exactly the same estimates as those in Table 1. It happens that the actor position contributes more than twice as much to the variance as does the partner position. Thus, who is talking is much more important for the talking frequency than who the listener is. The total variance is the sum of the three variance terms, and equals 91.8 + 40.9 + 78.4 = 211.1. The total variance due to individual (actor and partner) equals (91.8 + 40.9)/211.1 = 63% of the total variance of the variable. The correlation between the individual's actor effect, A_i,

Table 1. *Parameter estimates with standard errors (SE) for the Warner et al. data set*

Parameter	Interpretation	Estimate	*SE*
μ	Constant term	50.8	2.7
Variance (A_i)	Actor variance	92.0	53.9
Variance (B_j)	Partner variance	40.9	27.8
Covariance (A_i, B_i)	Actor–partner covariance	−40.4	32.0
Variance (E_{ij})	Dyadic variance	78.4	18.2
Covariance (E_{ij}, E_{ji})	Within-dyad covariance	−27.8	18.2

and the partner effect of the same individual, B_i, is $-40.4/\sqrt{(91.8 \times 40.9)} = -.66$, substantially negative. Thus, people who talk more are talked to less by others. This suggests an individual social-dominance effect.

There is an indication of dyadic reciprocity in the conversations. The dyadic correlation is $-27.8/78.4 = -.35$. This negative value suggests that within the dyads there is a process of asymmetry: One partner talks more and the other less than would be expected on the basis of only the partner and actor effects. Note that this negative within-dyad correlation is distinct from the negative correlation between actor and partner effects (in other data sets, it is possible that one of these correlations would be negative and the other positive).

If the reciprocity correlation (i.e., the correlation between E_{ij} and E_{ji}) is positive, then an alternative but equivalent expression of Model 1 is possible that is simpler because it does not rely on the host of dummy variables d. In this alternative formulation, there are three levels: Level one is the observation, level two the dyad, being the pair of reciprocal relations from i to j and from j to i, and level three the group. The reciprocity effect now is represented as a *dyad effect*, which is the random effect at level two.

This leads to a modification of the usual SRM equation:

$$Y_{ij} = \mu + A_i + B_j + R_{(ij)} + E_{ij} \qquad (3)$$

where $R_{(ij)}$ is the dyad effect, subject to the restriction $R_{(ij)} = R_{(ji)}$, *and* E_{ij} and E_{ji} now are assumed to be uncorrelated. In writing $R_{(ij)}$, the letters i and j are put between parentheses to indicate that this effect refers to the dyad (i,j) without distinguishing between the two involved individuals—that is, dyad (i,j) here is equivalent to dyad (j,i). Equation 3 can be re-expressed using the dummy variables a_i and p_j (but not d_{ij}) by

$$Y_{ij} = \mu + \Sigma_s A_s a_s + \Sigma_t B_t p_t + R_{(ij)} + E_{ij} \quad (4)$$

If the reciprocity correlation is positive, then the models implied by Equations 1, 2, 3, and 4 all are equivalent. In that case, the reciprocity correlation, which in Equations 1 and 2 is the correlation between E_{ij} and E_{ji}, is given in Equations 3 and 4 by the ratio of $\text{var}(R)$ to $(\text{var}(R) + \text{var}(E))$. Note that this demonstrates that there are two equivalent ways of conceptualizing dyadic reciprocity: The first is the residual correlation between the two reciprocal relationships, and the other is the variance at the dyad level. The dyad effect $R_{(ij)}$ refers to the shared perception of individuals i and j about their mutual relationship, whereas the residual E_{ij} in Equations 3 and 4 refers only to the individual perception of i about j and to measurement error. The way of specifying reciprocity in Equations 1 and 2 is more general because it allows negative reciprocity correlations. To take advantage of the simpler formulation in Equation 4 as compared to Equation 2, one can first estimate Model 4; only if Model 4 leads to an estimated variance of R equal to zero is it necessary to use formulation of Equation 2, which then leads to a zero or negative reciprocity correlation.

Extension of the SRM with covariates

The multilevel formulation of the SRM allows straightforwardly for the inclusion of covariates, for missing data on the dependent variable (provided that the data are missing by design or at random), and the estimation of specialized models (e.g., equal actor and partner variance). These three elements are not possible in the earlier estimation methods for the SRM.

Covariates lead to a more elaborate fixed part of the multilevel model. The term *fixed part* refers to the fact that the equations for multilevel models are usually decomposed into a fixed and a random part. The fixed part comprises regression coefficients for explanatory variables, like a regression model. In the model implied by Equation 1, the fixed part consists only of the constant effect, μ. The random part comprises the total contributions of all random effects, indicated here by capital Roman letters, A_i, B_j, and E_{ij}.

The inclusion of covariates is illustrated here by the gender of the individuals. Individuals 1, 2, 5, 6 in the data set presented by Warner et al. (1979) are male, the other are female. The gender of the conversation partners can be coded in three variables: GA_i is the gender of actor i, GP_j is the gender of partner j (both coded as -1 for males and 1 for females), and GD_{ij} equals one if the conversation partners have a different gender, and zero otherwise. The SRM equation with these covariates is expressed as

$$Y_{ij} = \mu + \beta_{GA}GA_i + \beta_{GT}GT_j + \beta_{GD}GD_{ij} + \Sigma_s A_s a_s + \Sigma_t B_t a_t + d_{ij}E_{ij} \quad (5)$$

Of the seven terms on the right-hand side, the first four terms constitute the "fixed part" of the model and the last three the "random part." Parameter β_{GA} is the effect of the actor's gender, β_{GT} the effect of the partner's gender, and β_{GD} the effect of a gender difference between the conversation partners. These three parameters and the constant term μ are just like regression coefficients, and the constant term in a traditional analysis of covariance (ANCOVA) or regression analysis.

When the model specified by Equation 5 is estimated, the variance and covariance parameters change only slightly compared to Table 1. We turn our attention to the effect of gender. Tests of the individual fixed effects can be carried out on the basis of the t-ratio, the parameter estimate divided by standard error. If the number of observations is large, the significance of this statistic can be calculated with reference to the standard normal distribution. The estimated regression coefficients are -1.63 (standard error or $SE = 3.82$) for the actor's gender, 3.95 ($SE = 2.23$) for the partner's gender, and -0.45 ($SE = 1.97$) for the gender difference. When considering t-statistics, it can be concluded that none of these effects are statistically significant. This is not surprising for this small data set of 8 individuals and 28 dyads. The largest effect, which is also the effect closest to significance ($t(7) = 3.95/2.23 = 1.77$, $p = .12$), is that of the partner's gender. This indicates that there is a (not quite significant) tendency for individuals to talk more to women than to men.

It is possible to test for the three covariates jointly by comparing the deviance (Goldstein, 1995; Snijders & Bosker, 1999) for the model expressed by Equation 1 and the model expressed by Equation 5. The *deviance* (technically equal to minus twice the log-likelihood) can be computed for each model and represents the degree to which the model is consistent with the data. One model can be tested against another by subtracting the deviance, which is tested using a chi-squared distribution where the number of degrees of freedom is the difference in the number of parameters between the two models. (This test is called the "likelihood ratio test"). These deviances are 424.64 and 421.37, respectively, for this data set. The deviance difference between nested models is distributed as chi-squared under the null hypothesis that the additional parameters are zero. So the test that the three covariates have no effect is a difference of deviances and is a $\chi^2(3) = 3.27$, $p > .15$. The degrees of freedom equal 3 because the difference between the two models consists of the three regression parameters. Because the chi-squared test is not significant, the coefficients for the covariates do not appear to be jointly significantly different from zero.

To show the flexibility of the multilevel model, we estimated a model without covariates but with the actor and partner variances set equal to each other. Such a model cannot be estimated by traditional SRM methods. This led to a deviance of 425.84. Comparing this to the deviance of the model of Equation 1 led to $\chi^2(1) = 1.20$ ($p > .20$); thus we find no statistical support that the variances are different. Note that, although the estimates of the actor and partner variances in Table 1 are quite different, their standard errors are large, which is not surprising given the small amount of data.

Relations Within Multiple Groups or Families

It is generally not very useful to consider the SRM for single families or groups, unless the groups are rather large (say, 20 or more individuals), because small groups result in large standard errors and low statistical power. It is more useful, however, to consider data for relations within several families or groups. In that case there are not only effects related to actors, partners, and dyads, but also to families. Whereas in the multilevel formulation in Equation 2 of the SRM of the previous section, level two consisted of only one group, now each family constitutes a group at level two. It is necessary to index the observations by the family and the individuals within the family; the family is indicated by k, the actor by i, and the partner by j. Actors and partners are assumed to be nested in families. The variable under study is still a dyadic variable, designated as Y_{ijk}. It is not required that all families have the same size, and so the size of family k is denoted by n_k. Thus, the indices i as well as j refer to persons in a family, from 1 to n_k, where k is the family being considered.

Model with a group effect

A multiple group version of the SRM is given by

$$Y_{ijk} = \mu + F_k + A_{ik} + B_{ik} + E_{ijk} \quad (6)$$

where the additional term F_k is the random main effect of family k. The difference of this model from the SRM model of Equation 1 is the random effect for families, denoted by F_k and having a zero mean. The random family effect requires one additional statistical parameter, the family-level variance. The interpretation is that in some families, the variable Y tends to be consistently higher (positive F_k) in all relations, and in other families Y tends to be consistently lower (negative F_k) in all relations, compared to the average family.

If the reciprocity correlation, the correlation between E_{ijk} and E_{jik}, is positive, then again the reciprocity effect can be represented as a *dyad effect* in a three-level model, where level one is the observation, level two the dyad, and level three the family. This is the analogue of Equation 3 and here leads to a modification of Equation 6:

$$Y_{ijk} = \mu + F_k + A_{ik} + B_{ik} + R_{(ij)k} + E_{ijk} \quad (7)$$

where $R_{(ij)k}$ is the dyad effect, subject to the restriction $R_{(ij)k} = R_{(ji)k}$, and E_{ijk} and E_{jik} now are assumed to be uncorrelated. If the reciprocity correlation is nonnegative, then the Models 6 and 7 are equivalent and correspond in the same way that was discussed above for Models 3 and 4.

To use multilevel modeling for estimating the model implied by Equation 7, we again create the dummy variables a and p, but now they are double subscripted to refer to person and group. The summations go from 1 to n_{max}, which is defined as the maximum group size. Because reciprocity is now modeled in the three-level model of Equation 7 by the term R, there is no need for the dummy variables d. The full model is

$$Y_{ijk} = \mu + F_k + \Sigma_s A_{sk} a_{sk} + \Sigma_t B_{tk} p_{tk} + R_{(ij)k} + E_{ijk} \quad (8)$$

The level-one variables are a_{ik}, p_{ik}, and E_{ijk}, the only level-two variable is $R_{(ij)k}$, and the level-three variable is F_k. The parameters of the model and their interpretation are as follows:

μ: the constant
variance of A_{ik}: actor variance
variance of B_{ik}: partner variance
covariance of A_{ik} with B_{ik}: actor–partner covariance of same individual
variance of $R_{(ij)k}$: dyad variance (positive reciprocity)
variance of E_{ijk}: error variance

The actor–partner correlation can be computed from these parameters by an application of the well-known formula, $\mathrm{cov}(A_{ik}, B_{ik})/\sqrt{\{\mathrm{var}(A_{ik} \times \mathrm{var}(B_{ik})\}}$.

Group model with roles

When the SRM is applied to families, it is natural to account for the fact that the roles of fathers, mothers, and children may be different. The factor with values "mother," "father," and "child" are referred to by the term "role." The effect of role has two aspects: the average role effect, which might be called the "cultural effect" in the population from which the families were sampled; and the deviations from this average as they occur in the individual families, which might be called "family idiosyncracies."

Consider first the average role effect that is modeled in the fixed part of the model and expressed by regression coefficients of the father, mother, and child roles. We assume that families consist of mother, father, and children although some families may be missing one or more of these roles. For families with mother (M), father (F), and two or more children (C), there are seven types of relations or observations: MF, MC, FM, FC, CM, CF, and CC where the first term represents the actor and the second the partner. So CM refers to the child's perception of the mother, and so on. We can think of the means of these seven types of observations as being arrayed in a 3×3 design: relations *from* the father, mother, and child *to* the mother, father, and child. Two of the cells are missing by design (father to father and mother to mother). The CC cell refers to a child–child relation where actor and partner are two different children. One reasonable model of the data is to allow for effects due to actor role, effects due to partner role, and the interaction between the two. We use dummy variables to model these effects. The dummy variables are denoted *f*, *m*, and *c* and indicate the different roles in the family. These dummy variables have values 0 and 1 and are defined by

f_{ik}: 1 if individual i in family k is the father and 0 otherwise,
m_{ik}: 1 if individual i in family k is the mother and 0 otherwise, and
c_{ik}: 1 individual i in family k is a child and 0 otherwise.

A multiple family version of the SRM with main effects of roles can be expressed as

$$Y_{ijk} = \mu + \beta_{FA} f_{ik} + \beta_{FP} f_{jk} + \beta_{MA} m_{ik} + \beta_{MP} m_{jk} + F_k + A_{ik} + B_{jk} + R_{(ij)k} + E_{ijk} \quad (9)$$

where β_{FA} is the effect of the father as actor, β_{FP} the effect of the father as partner, β_{MA} the effect of the mother as actor, and β_{MP} the effect of the mother as partner. In this formulation the child role is used as the reference category, so that parameter μ is the population mean for child–child relations. Equation 9 contains five fixed parameters (the constant term and four regression coefficients of dummy variables) for seven cells (cells MM and FF are missing). This leaves two degrees of freedom for interaction. Interaction parameters could be included for the father-to-mother and the mother-to-father roles. This is represented by adding the terms $\beta_{F \rightarrow M} f_{ik} m_{jk}$ and $\beta_{M \rightarrow F} m_{ik} f_{jk}$ to Equation 9. They would be interpreted as the difference in mother-to-father and father-to-mother interactions from what is expected on the basis of the main effects for the father and mother role variables that appear in parent–child and child–parent relations; said more briefly, these interactions parameters indicate the extent to which behavior of parents in parent–parent relations differs from their behavior in parent–child relations.

Next, consider the difference between families with respect to the roles of the fathers, mothers, and children. These family-dependent deviations are modeled in the random part of the model and expressed by variances between fathers, variances between mothers, and variances between children. In the model implied by Equation 9, the variance between the fathers is equal to the variance between the mothers and also to the variance between the children, because the random effects A_{ik} are assumed all to have the same actor variance, and similarly the partner effects B_{jk} have a common variance. However, this need not be the case. For example, it is possible that the roles of fathers and mothers are culturally more restricted than the child role, so that the variance between children would be larger than the variance between parents.

This kind of extension of the model represented by Equation 9, where variances of the actor and partner effects depend on the role of the person, can be formulated by replacing the random actor and partner effects by more complicated expressions involving the dummy variables f, m, and c. Specifically, the actor effect A_{ik} can be replaced by

$$A^F_{ik}f_{ik} + A^M_{ik}m_{ik} + A^C_{ik}c_{ik}$$

where A^F_{ik} denotes the actor effect for fathers, A^M_{ik} the actor effect for mothers, and A^C_{ik} the effect for children. Because exactly one of the dummy variables f_{ik}, m_{ik}, and c_{ik} is one whereas the others are zero, this formula just denotes that the right one is chosen of the three potential effects, corresponding to the father, mother, or child. Similarly, the partner effect B_{jk} can be replaced by

$$B^F_{jk}f_{jk} + B^M_{jk}m_{jk} + B^C_{jk}c_{jk}$$

where B^F_{jk} denotes the partner effect for fathers, B^M_{jk} the partner effect for mothers, and B^C_{jk} the partner effect for children.

Thus, six distinct variance parameters can be estimated: actor variances and partner variances for each of the three roles. As before, it is reasonable to allow nonzero covariances between actor and partner effects of the same individuals. The covariance matrix of the original observations Y_{ijk} depends on these variance parameters and, in addition, on the variance of the family effect, F_k, being shared by all observed relations in the family, the variance of the dyadic effect, $R_{(ij)k}$, and the variance of the residual, E_{ijk}. It is possible to include other covariances—for example, a nonzero covariance between father and mother partner effects to indicate that fathers and mothers are more (for a positive covariance) or less (for a negative covariance) alike in their role as partners than is expected from the general family effect.

If there are theoretical or empirical reasons to do so, the variances of the dyad effect $R_{(ij)k}$ or of the residual E_{ijk} may also vary across the roles involved (cf. Goldstein, 1995, Section 3.1, or Snijders & Bosker, 1999, Chapter 8). Thus, for instance, there could be a dyadic effect with a larger variance in relationships between children than in relationships between parents.

This complex but very interesting model can be estimated by multilevel software. In addition to the possibility that family sizes may be different, this estimation method also allows incomplete data in the Y variable. For example, some dyads may have only one observation available, and some dyads may be missing completely. If missingness is at random (i.e., the fact of an observation being missing is not systematically related to the unobserved value), incompleteness of data is not a problem for the multilevel approach to estimation because the missing observations can simply be omitted from the data set.

The estimation of the model implied by Equation 9 requires only to add fixed effects to Model 7. The extension to role-dependent variances of the actor and partner effects implies that, instead of giving random slopes to the dummy variables a_{ik} and p_{ik}, random slopes should be given to product variables $f_{ik}a_{ik}$, $m_{ik}a_{ik}$, and $c_{ik}a_{ik}$ for actor and $f_{ik}p_{ik}$, $m_{ik}p_{ik}$, and $c_{ik}p_{ik}$ for partner. The complete equation is quite complex:

$$\begin{aligned} Y_{ijk} = {} & \mu + \beta_{FA} f_{ik} + \beta_{FP} f_{jk} + \beta_{MA} m_{ik} \\ & + \beta_{MP} m_{jk} + F_k + \Sigma_s a_{sk} (A^F{}_{sk} f_{sk} \\ & + A^M{}_{sk} m_{sk} + A^C{}_{sk} c_{sk}) \\ & + \Sigma_t p_{tk} (B^F{}_{tk} f_{tk} + B^M{}_{tk} m_{tk} \\ & + B^C{}_{tk} c_k) + R_{(ij)k} + E_{ijk}. \end{aligned}$$

Example: Recalled parental rearing styles

As an example of analysis of relational data with reciprocated relations, results are presented from a study by Gerlsma (1993; see also Gerlsma, Snijders, Van Duijn, & Emmelkamp, 1997) of parental rearing styles. As a part of that study, retrospective data were collected concerning how parents raised their children in each of 60 families. The parents and two children were asked to answer a questionnaire with four subscales about memories of the style in which the parent reared the child. The test used was the EMBU (*Egna Minnen Beträffande Uppfostran*, Perris et al., 1980; Dutch form by Arrindell, Emmelkamp, Brilman, Monsma, 1983). In the present article, results are presented about a scale labeled as Affection, consisting of nine items of the Emotional Warmth subscale of the EMBU (Gerlsma et al., 1997, p. 273). Each parent reported about each of the two children, and vice versa. For the previous example, there were seven types of relations, but for this design there are only four: mothers and fathers rating their childrearing style (MC and FC), and children rating their father's and mother's childrearing style (CF and CM). This means that of the 3 × 3 design mentioned above, only four cells are used. The measure presented here is the emotional warmth from the parent to the child, reported separately by the parent and the child. The person who reports the warmth is the actor, whereas the partner is the other person in the relationship. The restriction to two children implies that a complete data set would have eight relations per family. Because of incomplete answers, the total number of reported relations was 358, considerably less than the 480 that would have been obtained without any missing data. We can distinguish the following random effects:

family effect,
actor effect of father, mother, and child,
partner effect of father, mother, and child, and
dyadid reciprocity effect of father–child and mother–child relations.

The size of each of these effects is indicated by a variance parameter, reflecting the net variability among, respectively, families, actors, partners, and dyads. The family effect, compared to total variability, reflects general agreement within families (e.g., if the family variance is large, then members of some families tend to report the relations within their families as warm but members of other families report the relations within their families as less warm). The reciprocity effect is the variance at the dyad level (the variance of $R_{(ij)k}$ in Equation 7). Positive reciprocity in this example means that if a parent believes that she or he was warm to a particular child, the child also tends to see this parent as having been warm, controlling for the general actor and partner effects of the involved individuals. Reciprocity in this case reflects parent–child agreement about parental warmth.

Because we are not distinguishing between the two children's roles within the family, the fixed effects of the two children and the variances of their random effects were restricted to be equal. With the incompleteness of the design, this leaves four degrees of freedom for the fixed effects, corresponding to the MC, FC, CM, and CF cells. They were coded by a constant term and the effects of a Father-actor dummy (coded 1 when the father was the actor and zero otherwise), a Mother-actor dummy (coded 1 when the mother was the actor and zero otherwise), and a Father-mother Partner Difference dummy (coded 1 when the father was the partner, −1 when the mother was the partner, and zero otherwise), indicating the extent to which the children reported differently about the father than about the mother. Given the coding used, the intercept equals the average response for the pairs with the child in the actor role.

Two different models were estimated. In Model 1, there were no reciprocity effects whereas in Model 2 such effects were estimated. If there were no reciprocity effects, then the perceived warmth as reported by one person about another depends only on the involved individuals and the family, but not on the particular pair of persons. The results are presented in Table 2. With four degrees of freedom for the fixed effects, the fixed part of the models in Table 2, having four parameters, is said to be *saturated,* which means that the mean values for the four cells, MC, FC, CM, and CF, are fitted exactly by this model.

In Model 2, the dyad variance was allowed to differ between mothers and fathers, in the actor as well as in the partner role. This means that mother–child dyads were allowed to be more variable, or less variable, than father–child dyads.

Examining the fixed effects in Table 2, it can be concluded that, on average, children report less warmth than do the parents, but mothers and fathers do not differ much from each other as actors. As partners, fathers were seen as less warm than were mothers.

Several of the random effects are estimated as having a variance of zero. This is not uncommon for random coefficient models and is interpeted as follows. The observed variability on the component under consideration is less than what would be expected by chance, if the true variance component were indeed very small or even equal to zero. The zero variance estimate may be interpreted as an indication that the variance of this component is, in any case, not significantly larger than zero.

With respect to the random effects, first consider Model 1 in Table 2. The variances of the random effects indicate the differences between families. Families as a whole have no influence, implying that there is not a shared perception of reality within the families. The variances of actors and partners tell most of the story, whereas the family, the dyad, and the individual observation have much smaller variances. The fathers have the largest actor variance and are the only role with any partner variance. Gerlsma et al. (1997) gave the interpretation that the recalled warmth of the parent

Table 2. *Estimated effects and standard errors (SE) for recalled affection in families, for models without (Model 1) and with (Model 2) reciprocity effects*

	Model 1		Model 2	
Effect	Estimate	*SE*	Estimate	*SE*
Fixed Effects				
Constant term (fixed effect) μ	27.62	0.54	27.60	0.54
Father as actor (fixed effect) β_{FA}	1.72	0.86	1.75	0.86
Mother as actor (fixed effect) β_{MA}	1.60	0.63	1.60	0.57
Father–mother difference as partners β_{FP}–β_{MP}	−1.69	0.37	−1.70	0.37
Random Effects				
Family variance (F_k)	0	*	0	*
Father as actor variance (A^F_{sk})	16.77	4.30	16.43	4.38
Mother as actor variance (A^M_{sk})	6.63	1.91	7.05	1.91
Child as actor variance (A^C_{sk})	13.56	2.57	13.80	3.15
Father–Mother actor covariance (A^F_{sk}, A^M_{sk})	−0.22	2.02	−0.51	2.03
Father partner variance (B^F_{sk})	20.27	5.32	19.78	5.45
Mother partner variance (B^M_{sk})	0	*	0.30	2.45
Child partner variance (B^C_{sk})	0	*	0	*
Father actor-partner covariance (A^F_{sk}, B^F_{sk})	9.05	3.68	8.00	3.76
Mother actor-partner covariance (A^M_{sk}, B^M_{sk})	0	*	1.66	1.43
Child actor-partner covariance (A^C_{sk}, B^C_{sk})	0	*	0	*
Dyad variance relationship with father ($R^F_{(ij)k}$)	#	#	2.02	1.25
Dyad variance relationship with mother ($R^M_{(ij)k}$)	#	#	0.24	0.85
Residual variance (E_{ijk})	5.27	0.69	4.19	0.89
Deviance	1994.92		1989.18	

[#]Not estimated. *No standard error.

is mainly a tale of the rater (i.e., told by the actor), and about the father. The actor–partner correlation for fathers also is large, being equal to $9.05/\sqrt{(16.77 \times 20.27)} = .49$. Thus, if fathers see themselves as warm, their children also tend to do so. The estimated child partner variance is estimated as zero, so there is not much in common between what the father and mother report about a given child. It can be concluded that the parents do not differentiate between their two children in the same way. The residual variance is small, which emphasizes again the strength of actor and partner effects.

The results about the differences between fathers and mothers can be summarized as follows. The recalled warmth of the fathers is on average less than that of the mothers (the corresponding dummy variable is defined as 1 for fathers and −1 for mothers, and has a fixed effect −1.69; hence, the warmth given by the mothers exceeds the fathers' warmth on average by 3.38), but based on the father partner effect, the fathers differ strongly from each other (standard deviation of fathers' warmth in their children's memory is $\sqrt{20.27} = 4.5$). The variance of the mothers as partners is estimated as zero. Assuming a normal distribution of partner effects, this means that the difference between mother and father in warmth as recalled by the children has a normal distribution with a mean of 3.38 and a standard deviation of 4.5, which implies that in about 77% of the families, the mother is rated as warmer than the father and in 23% the situation is reversed. Because these percentages are based on the distribution of the partner effects only, they are the net of residual variability.

Now consider Model 2 presented in Table 2, which differs from Model 1 only in the dyad variance, (i.e., the reciprocity effect). The other parameter estimates differ only slightly between the two models. The reciprocity effect is marginally significant (subtracting the deviances yields $\chi^2(2) = 5.74$, $p < .10$), providing some support to the presence of dyadic reciprocity. The dyad variance is estimated as larger for the father than for the mother. This result shows that fathers' warmth is more specific for one of their children than the mothers' warmth. However, this difference is small, so even if fathers see themselves as warmer toward one child than toward the other, the children do not necessarily see it the same way.

Covariates

Covariates can be measured at the level of the person, the dyad, and the group. We consider here a covariate measured at the level of the individual, (e.g., a measure of the individual's empathic qualities). For such a covariate, we can test four hypotheses:

The covariate correlates with the actor effect.
The covariate correlates with the partner effect.
Dissimilarity of an actor–partner pair on the covariate, defined by the absolute value of the difference, correlates with the dyadic effect.
The mean of covariate for the group correlates with the group effect.

If the covariate measures the individual's empathic qualities, and the *Y* variables refer to how helpful the actor is toward the partner, then positive values of these four covariate effects would be interpreted as follows: Empathic actors are more helpful; empathic partners receive more help; a person whose empathic score differs from the partner's helps the partner more; and families with, on average, high empathic qualities tend to help each other more. Especially for dependent variables that measure affection or cooperation, similarity may be theoretically quite important.

Though perhaps not obvious, the roles of mother, father, and child could also be regarded as covariates. There is no essential difference in the statistical analysis between the roles and other covariates. However, very often there is substantive interest that actor and partner variances vary by role.

In the study of parental rearing styles, among the measured characteristics were the total scores of the family members on the SCL-90 Symptom Checklist (Derogatis, 1975; Dutch form by Arrindell & Ettema, 1986), which gives a general indication of the level of psychological and physical well-being or distress of the individual. The range is from 90 to 450, with high values pointing to high distress.

The SCL-90 score was used to calculate four covariates as indicated above: actor distress, partner distress, dyadic distress similarity (absolute difference), and family average distress. Fixed effects of these covariates (not distinguishing between the roles of father, mother, or child) were added to Model 2 of Table 2. Results are presented in Table 3. We do not again present the results for the parameters previously included because they are not very different from the values in Table 2.

The coefficients have small values because distress has a wide range, of 360 points. The four covariates jointly result in a significant improvement of model fit ($\chi^2(4) = 12.86$, $p < .02$). The sizes of the estimated coefficients and their t-ratios show that the distress dissimilarity has the greatest and most significant ($t = -3.5$) contribution: A larger discrepancy between actor and partner leads to less perceived warmth. There is also a significant ($t = 2.2$) contribution of partner distress: If the partner is more distressed, the actor reports more warmth. Actor's distress and family-level average distress do not have significant contributions ($t = 0.9$ and 0.5, respectively).

Table 3. *Estimated effects and standard errors (SE) of well-being on recalled affection in families*

Effect	Estimate	*SE*
Actor distress	0.014	0.014
Partner distress	0.023	0.011
Distress dissimilarity	−0.038	0.011
Family distress	0.014	0.023
Deviance	1976.32	

Discussion

In family research, data about relations between family members are potentially very informative but hard to analyze. Both the potential and the difficulties rest on the complicated correlational structure between such relational data, owing to the fact that each individual is implied in various different relations with other individuals. The Social Relations Model (SRM) is an established model for relational data, but earlier existing estimation methods posed stringent requirements on the data (e.g., balanced designs) and generally needed special software. This article extends the SRM to relational data in families (or other groups) and proposes the use of a multilevel approach for parameter estimation. The extended SRM provides a detailed representation of the correlation structure of the relational data in well-interpretable parameters such as the actor variance, the partner variance, the family variance, and the dyadic reciprocity variance. Effects of the roles of the individuals (father, mother, child) and of covariates can also be included.

Covariates can be attributes of the individual or of the actor–partner pair. It was discussed how the effect of individual attributes can be distinguished into effects associated with the actor, the partner, with dyadic similarity, or with the family mean. An attractive feature of the multilevel approach is that different numbers of respondents in families, and missing data about certain persons or relations within families, do not lead to any technical problems as long as the incompleteness is random. The presentation in this article was about directed relations, but the same approach can be applied to undirected relations (i.e., the relation between i and j is necessarily equal to the relation between j and i).

Alternative approaches are the use of analysis of variance formulae derived especially for these models and for which specific software is available (Kenny, 1994) or the use of structural equations modeling (Kashy & Kenny, 1990). The random part of the multilevel models presented above can be regarded as a special instance of the general structural equation model (Bollen, 1989, pp. 319–321; Kenny, 1979, pp. 200–205). However, the analysis of variance and the structural equations approaches require complete data, or data that are balanced in some other way (e.g., a block design in families of fixed size). The multilevel approach is much more flexible about missing data. The inclusion of explanatory variables that are fixed and covariates is straightforward in the multilevel approach, but cannot be handled so easily by ANOVA or structural equation modeling approaches.

A further advantage of the multilevel approach is the easy estimation of restricted models, where some of the parameters are set to zero or assumed to be equal to other parameters. For instance, one might want to test the hypothesis that group means do not vary or that actor and partner variances are equal. Another possibility is to test whether the actor–partner covariance is zero. These kinds of nonsaturated models are not very easily estimated within standard SRM methods. Much more complicated models are possible in the multilevel approach. For instance, one can specify that the actor variance for each person is a linear function of a covariate or that the residual variance varies by actor or partner (cf. Snijders & Bosker, 1999, Chapter 8).

The multilevel approach to estimating the SRM offers a quite flexible data analysis strategy, parameters that are clearly interpretable, and estimation by software that is beginning to be widely available. Although we admit the approach that we have presented is complex, we are confident that advances in computer software will make this approach more practical. With the help of the *MLn* macros mentioned above, it is already now accessible to users of *MLn* and *MLwiN*.

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