Screening Designs

A good reference for this (and many other parts of the mechanics of designs) is

Box, G. E. P., Hunter, W. G. and Hunter, J. S. (1977) *Statistics for Experimenters*, Wiley.

Suppose we have several (two-level) factors, few of which we expect to be effective: this is often the case in sensitivity studies.

Fractional factorial designs

For a 2^p factorial design it is easy to generate a set of 2^q runs for $q < p$ that is a fraction of the design: write out a complete factorial in the first q factors, and associate the rest with the interactions. E.g. ($p = 7, q = 3$)

1	2	3	4	5	6	7
-	-	-	+	+	+	-
+	-	-	-	-	+	+
-	+	-	-	+	-	+
+	+	-	+	-	-	-
-	-	+	+	-	-	+
+	-	+	-	+	-	-
-	+	+	-	-	+	-
+	+	+	+	+	+	+

We can use this design to screen 7 factors. In the (hypothetical) example in Box, Hunter and Hunter only factors 2 and 4 had significant main effects: note that the design contains two replicates of the 2^2 design for those factors.

Resolution III designs like this confound second-order interactions with main effects, so we cannot conclude that 4 has a main effect without (in this example) assuming that the 12, 37 and 56 interactions are negligible. We may be prepared to assume that from subject-matter considerations, but we could add runs to resolve the ambiguity. One simple way to do so in this case is to add a second fraction. There are 16 fractions, and we choose the one with the sign of 4 reversed:

1	2	3	4	5	6	7
-	-	-	-	+	+	-
+	-	-	+	-	+	+
-	+	-	+	+	-	+
+	+	-	-	-	-	-
-	-	+	-	-	-	+
+	-	+	+	+	-	-
-	+	+	+	-	+	-
+	+	+	-	+	+	+

Plackett–Burman designs

Resolution III fractional factorial designs enable us to test out up to $n - 1$ factors in $n = 2^q$ runs. Once $n > 8$, the gaps between the possible values become large: we might want to do this for 9 factors, and we need 16 runs. Plackett–Burman designs provide the same property whenever n is a multiple of 4 (at least up to 100), so we could screen 9 factors in 12 runs using the first 9 columns of

+	-	+	-	-	-	+	+	+	-	+
+	+	-	+	-	-	-	+	+	+	-
-	+	+	-	+	-	-	-	+	+	+
+	-	+	+	-	+	-	-	-	+	+
+	+	-	+	+	-	+	-	-	-	+
+	+	+	-	+	+	-	+	-	-	-
-	+	+	+	-	+	+	-	+	-	-
-	-	+	+	+	-	+	+	-	+	-
-	-	-	+	+	+	-	+	+	-	+
+	-	-	-	+	+	+	-	+	+	-
-	+	-	-	-	+	+	+	-	+	+
-	-	-	-	-	-	-	-	-	-	-

the rows being runs and the columns factors.

Plackett–Burman designs also exist for factors all with $L > 2$ levels, for L prime or a power of a prime and $N = L^q$.