Time Series Problem Sheet 1 HT 2010

1. Let $\{X_t\}$ be the ARMA(1, 1) process,

$$X_t - \phi X_{t-1} = \epsilon_t + \theta\epsilon_{t-1}, \qquad \{\epsilon_t\} \sim \mathrm{WN}(0, \sigma^2),$$

where $|\phi| < 1$ and $|\theta| < 1$. Show that the autocorrelation function of $\{X_t\}$ is given by

$$\rho(1) = \frac{(1+\phi\theta)(\phi+\theta)}{1+\theta^2+2\phi\theta}, \qquad \rho(h) = \phi^{h-1}\rho(1) \quad \text{for } h \geqslant 1.$$

2. Consider a process consisting of a linear trend plus an additive noise term, that is,

$$X_t = \beta_0 + \beta_1 t + \epsilon_t$$

where β_0 and β_1 are fixed constants, and where the ϵ_t are independent random variables with zero means and variances σ^2. Show that X_t is non-stationary, but that the first difference series $\nabla X_t = X_t - X_{t-1}$ is second-order stationary, and find the acf of ∇X_t.

3. Let $\{S_t, t = 0, 1, 2, \dots\}$ be the random walk with constant drift μ, defined by $S_0 = 0$ and

$$S_t = \mu + S_{t-1} + \epsilon_t, \qquad t = 1, 2, \dots,$$

where $\epsilon_1, \epsilon_2, \dots$ are independent and identically distributed random variables with mean 0 and variance σ^2. Compute the mean of S_t and the autocovariance of the process $\{S_t\}$. Show that $\{\nabla S_t\}$ is stationary and compute its mean and autocovariance function.

4. If

$$X_t = a\cos(\lambda t) + \epsilon_t$$

where $\epsilon_t \sim \mathrm{WN}(0, \sigma^2)$, and where a and λ are constants, show that $\{X_t\}$ is not stationary.