b8 STATISTICS **HT 2002**

Problem Sheet 9

1. Three separate samples, one from each of four different suspensions of bacteria A, B, C and D, were prepared. Technician I examined under a microscope one sample from each suspension in random order. Similarly the other samples were tested by Technicians II and III.

The recorded number of organisms from each sample are summarised in the following table. Interest principally centres on how suspensions A, B, C and D affect the recorded number of organisms.

		Suspension			
		A	B	C	D
	I	67	84	77	60
Technician	II	71	77	74	70
	III	54	67	65	56

(i) What is this type of experimental design called? State briefly its advantages.

(ii) The analysis of variance table is of the form

	Df	Sum of Sq	Mean Sq	F Value
Suspensions		393		
Technicians		386		
Residual		100		

Fill in the remaining entries in the table.

(iii) Test the hypothesis that there is no difference between the suspensions and state your conclusions.

Is there evidence of difference between the technicians?

(iv) Suspensions B and C are of one type and A and D are of another. Test the hypothesis that there is no difference between the mean of B and C and the mean of A and D.

2. Consider a randomised block design with m blocks and p treatments, so that in the usual notation,

$$Y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij}, \qquad i = 1, \ldots, p, \quad j = 1, \ldots, m.$$

Suppose $\widehat{\theta}_T = \sum_{i=1}^{p} c_i \overline{Y}_{i.}$ is an estimator of the treatment contrast $\sum_{i=1}^{p} c_i \tau_i$, and $\widehat{\theta}_B = \sum_{j=1}^{m} d_j \overline{Y}_{.j}$ is an estimator of the block contrast $\sum_{j=1}^{m} d_j \beta_j$, where $\sum_{i=1}^{p} c_i = \sum_{j=1}^{m} d_j = 0$.

(i) Show that $\widehat{\theta}_T$ is an unbiased estimator of $\sum_{i=1}^{p} c_i \tau_i$, and that $\widehat{\theta}_B$ is unbiased estimator of $\sum_{j=1}^{m} d_j \beta_j$.

(ii) Show that $\mathrm{cov}(\widehat{\theta}_T, \widehat{\theta}_B) = 0$, and deduce that $\widehat{\theta}_T$ and $\widehat{\theta}_B$ are independent normal random variables.

3. In an experiment to study soil fumigants, four blocks of a field were used, each of which contained 8 experimental plots. Within each block no fumigant was used on 4 units and a standard dose of fumigants A, B, C and D on one unit each. The following results were obtained for the density of pests.

	None	A	B	C	D
Treatment averages	350	265	232	350	225
Block averages	365	255	396	220	
Residual SS	270,000 on 24 d.f.				

(i) Explain why there are 24 (not, say, 21) residual degrees of freedom.

(ii) Complete the following ANOVA table for these data.

	Df	Sum of Sq	Mean Sq	F Value
Treatments		93,304		
Blocks		172,336		
Residual	24	270,000		

(iii) Estimate the contrast fumigant B with no fumigant, and test whether it is non-zero.