

Bits with an asterisk (*) should be tackled after completing the other exercises.

1. **Sequences.** Generate the following sequences using `rep()`, `seq()` and arithmetic:

- (a) 1, 3, 5, 7, ..., 21.
- (b) 1, 10, 100, ..., 10^9 .
- (c) 0, 1, 2, 3, 0, ..., 3, 0, 1, 2, 3 [with each entry appearing 6 times]
- (d) 0, 0, 0, 1, 1, 1, 2, ..., 4, 4, 4.
- (e)* 50, 47, 44, ..., 14, 11.
- (f)* 1, 2, 5, 10, 20, 50, 100, ..., 5×10^4 .

Can any of your answers be simplified using recycling?

2. **Arithmetic.** Create a vector containing each of the following sequences:

- (a) $\cos\left(\frac{\pi n}{3}\right)$, for $n = 0, \dots, 10$.
- (b) 1, 9, 98, 997, ..., 999994.
- (c) $e^n - 3n$, for $n = 0, \dots, 10$.
- (d)* $3n \bmod 7$, for $n = 0, \dots, 10$.

Let

$$S_n = \sum_{i=1}^n \frac{(-1)^{i+1}}{2i-1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{(-1)^{n+1}}{2n-1}.$$

You will recall that $\lim_n S_n = \pi/4$.

- (e) Evaluate $4S_{10}$, $4S_{100}$ and $4S_{1000}$. [Hint: use the `sum()` function.]
- (f) Create a vector with entries $S_i - \frac{\pi}{4}$, for $i = 1, \dots, 1000$. [Hint: try creating the vector with entries S_i first; the function `cumsum()` may be useful.]

3. **Subsetting**

Create a vector `x` of normal random variables as follows:

```
> set.seed(123)
> x <- rnorm(100)
```

The `set.seed()` fixes the random number generator so that we all obtain the same `x`; changing the argument 123 to something else will give different results. This is useful for replication.

Give commands to select a vector containing:

- (a) the 25th, 50th and 75th elements;
- (b) the first 25 elements;
- (c) all elements except those from the 31st to the 40th.

Recall the logical operators `|`, `&` and `!`. Give commands to select:

- (d) all values larger than 1.5 (how many are there?);
- (e) what about the entries that are either > 1.5 or < -1 ?

4. **Monte Carlo Integration.** Now let's try some simple examples related to what you've studied in lectures. Suppose we have $Z \sim N(0, 1)$ and want to estimate $\theta = \mathbb{E}\phi(Z)$: we can generate a large number of independent normals, Z_1, \dots, Z_n and then look at the sample mean:

$$\frac{1}{n} \sum_{i=1}^n \phi(Z_i).$$

Let's try this for $\phi(x) = x^4$; generate $n = 10\,000$ standard normal random variables in a vector called `Z`.

Now, find the sample mean of Z^4 , and have a look at the values you get. Try the `summary()` and `hist()` functions to help you understand the data:

```
> mean(Z^4)
> summary(Z^4)
> hist(Z^4, breaks = 250) # notice how skewed this is!
```

Using the central limit theorem we also know that a $(1 - \alpha)$ -confidence interval is given by

$$\hat{\theta}_n \pm c_\alpha \frac{S_{\phi(Z)}}{\sqrt{n}}.$$

where

$$S_{\phi(Z)}^2 \equiv \frac{1}{n-1} \sum_{i=1}^n (\phi(Z_i) - \hat{\theta}_n)^2$$

Calculate $S_{\phi(Z)}^2$ using the mean and sum functions. Check that using `var(Z^4)` gives the same answer.

You can get the quantiles of a normal distribution using `qnorm()`. For example:

```
> qnorm(0.975)
## [1] 1.959964
```

Use this function with your work above to obtain a 99% confidence interval for the value of $\mathbb{E}\phi(Z)$.

5. **Records.*** Let X_1, X_2, \dots be independent and identically distributed continuous random variables. Call i a **record** if $X_i > X_j$ for all $j < i$ (trivially including $i = 1$). Let R_t be the index of the t th such record.

Suppose we have a vector `x` and want to find the indices that correspond to records. Using the `cummax()` function with `which()` and `==`, work out commands to give you a vector of the indices of records.

Thinking about the inversion method of random variables, can you explain why the distribution of R_t does not depend upon the distribution of the X_j s?